Wavelength Analysis Using Equivalent Circuits in a Fast and Slow Wave Waffle-Iron Ridge Guide

Hideki KIRINO⁺, Kazuhiro HONDA⁺, Members, Kun LI, Student Member, and Koichi OGAWA⁺, Member

SUMMARY In this paper we use equivalent circuits to analyze the wavelengths in a Fast and Slow wave Waffle-Iron Ridge Guide (FS-WRG). An equivalent circuit for the transverse direction is employed and the transverse resonance method is used to determine the fast wave wavelength. Another equivalent circuit, for the inserted series reactance in the waveguide, is employed for the fast and slow wave wavelength. We also discuss the physical system that determines the wavelengths and the accuracy of this analysis by comparing the wavelengths with those calculated by EM-simulation. Furthermore, we demonstrate use of the results obtained in designing an array antenna.

topics: waffle-iron, ridge, waveguide, fast wave, slow wave, wavelength, array antenna

1. Introduction

Our designation for a waveguide combining waffle-iron and ridge structures, presented by us in references [1] and [2], is a Waffle-Iron Ridge Guide (WRG). This is the same as the Gap Ridge Waveguide (GRW) presented by other researchers [3–5]. The WRG, or GRW, has two metal plates. On one of the plates, there are two-dimensional periodic arrays of metal rods on either side of the central ridge. The other metal plate has a flat surface, and is positioned such that there is an air gap between the two plates.

Feeding the circuits of array antennas, and guiding and distributing RF signals to each antenna element are essential functions, and these have typically been accomplished by employing hollow waveguides and micro strip lines. However, at frequencies around 100 GHz, hollow waveguides are unsuitable because many screws are needed to fix the upper and lower plates, and micro strip lines are unsuitable because of the dielectric loss and radiation from the waveguide. The WRG (or GRW) has advantages in such frequency bands because there is no contact between the upper and lower metal plates and the absence of a dielectric.

The fast and slow wave waffle-iron ridge guide (FS-WRG) uses a novel technique, in that it has periodic steps on the ridge in order to shorten the wavelength in the WRG.

Generally, a rectangular hollow tube is used to represent a waveguide, where the fundamental wavelength in the waveguide is defined by the boundary conditions at the waveguide walls. These days, EM-simulators are employed to accurately and very easily predict the outcome when designing microwave and millimeter-wave devices including waveguides. On the other hand, before the advent of EM-simulators, scientists and engineers used many different theoretical methods and formulas to optimize the configurations of waveguides. An example, given in reference [6], is calculating the wavelength in a periodic structure similar to the WRG, where the transverse resonance method is employed. The authors point out that the accuracy of the calculation in this case is determined by the formulas used to describe the periodic structure. They also mention that, even with careful selection of the formulas, the size and type of periodic structure that can be analyzed may be limited. As stated above, EM-simulation is superior to theoretical methods such as the transverse resonance method as it can be more easily and generally applied.

In this paper, we adopt the transverse resonance method to analyze the wavelengths of the WRG and the FS-WRG in order to reveal the physical basis for the wavelengths. Although, the accuracy of the results is not guaranteed, we plan to use the results to predict variations that we can make to the WRG and the FS-WRG, such as modulating the height and arrangement of the waffle-iron rods, in order to widen the frequency band or have dual frequency bands.

Furthermore, we use the results obtained to design an array antenna. Even state of the art EM-simulators need an initial configuration to begin the calculations. We demonstrate the design of an array antenna using the results obtained from the wavelength analysis to define the initial configuration, and then evaluate the result. The EM-simulator used in this paper is Femtet from Murata Software.

2. Configuration of the FS-WRG

Figure 1 shows the configuration of a FS-WRG [7]–[9]. There are waffle-iron regions on both sides of the ridge. The height of the waffle-iron rod is \( \lambda /4 \), where \( \lambda \) is wavelength in free space. The upper and lower plates are not in contact and are separated by a gap of \( \lambda /8 \), which is determined by the essential condition of preventing parallel plate propagation in the waffle-iron region. By preventing propagation in the waffle-iron region, the RF signals propagate only on the ridge. The waffle-iron rod sizes and the spaces between adjacent rods in the \( x \)- and \( y \)-directions are both \( \lambda /8 \). There are trelches on each side of the ridge of depth \( D \) from the top of the ridge, and there are periodic steps of depth \( d \) on the ridge that generate the slow wave.

There are no cut off frequencies in a WRG or a FS-WRG...
Fig. 1  Configuration of the FS-WRG.

Fig. 2  Set up for measuring the isolation between two parallel WRGs with \( N \) waffle-iron lines between them.

Table 1  Isolation between two WRGs arranged in parallel with \( N \) waffle-iron lines between them defined for \( \omega / \omega_0 = 0.83 \) to 1.1.

<table>
<thead>
<tr>
<th>( N )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isolation [dB]</td>
<td>15</td>
<td>25</td>
<td>35</td>
<td>45</td>
</tr>
</tbody>
</table>

as there are in hollow waveguides. As described in the literature [2], the propagation frequency band and the prevention of parallel plate mode characteristics can be evaluated by isolating two WRGs arranged in parallel with \( N \) waffle-iron lines between them. The set up for measuring the isolation is shown in Fig. 2, where a TEM wave is input in the normal direction to the ridge and leaked to the other ridge. The isolation with respect to \( N \) is given in Table 1.

3. Analysis of the Fast Wave

As mentioned in the previous section the periodic steps in Fig. 1 are for generating the slow wave, so here we consider a configuration with \( d = 0 \), which is for the fast wave analysis in a WRG. As is generally known, the fast wave is the motion of the wavefront of a TEM wave tilted from the direction of the waveguide. A well-known conventional method for analyzing the fast wave is the transverse resonance method, which has also been employed in periodic structures [6]. The left hand side of Fig. 3 shows the relationship between the wavelength of the TEM wave, \( \lambda \), and the wavelength of the fast wave, \( \lambda_f \), and an equivalent circuit for the WRG in transverse resonance. As shown in Fig. 3, there is an Equivalent Magnetic Boundary (EMB) on each side of the ridge formed by waffle-iron rods with height \( \lambda / 4 \). The phase \( \phi \) is the only parameter used to evaluate the wavelength. In Fig. 3, to make it easier to understand transverse resonance in a WRG, we introduce an Equivalent Short Wall (ESW), where the ESW is a virtual wall with phase shift \( \pi \) (or \( -\pi \)), the same as in a hollow waveguide. The relationship between the wavelength and the distance between the EMB walls is then given by

\[
\frac{1}{\lambda^2} = \frac{1}{(\lambda + 2a)^2} + \frac{1}{\lambda_f^2} 
\]

(1)

\( \lambda \): Wavelength in free space

\( \lambda_f \): Wavelength of the fast wave

\( a \): Distance between the EMB walls

EMB: Equivalent Magnetic Boundary

ESW: Equivalent Short Wall

\( \phi \): Reflection phase of TEM wave in the transverse direction

\( Z_t \): Equivalent impedance looking in the transverse direction

\( Z_o \): Signal source impedance for the TEM wave

\( \theta \): Angle between the TEM wave and the waveguide axis

where all the parameters mentioned above correspond to those in Fig. 3.

In Fig. 3, \( \phi \) is the phase generated by the round trip between ESW and the centre of the ridge, where the TEM wave has been injected in the transverse direction from the centre of the ridge. When we make the phase shift at the ESW \( \pi \) rather than \( -\pi \), and use \( \beta = 2\pi / \lambda \), the phase \( \phi \) is given by

\[
\phi = \pi - 2\beta \left( \frac{\lambda}{4} + \frac{a}{2} \right) = -2\pi \frac{a}{\lambda} 
\]

(2)

where the range of \( \phi \) is \(-2\pi < \phi < 0\). Having a phase shift
\( \pi \) at the ESW means the phase shift at the EMB is zero.

Substituting \( a \) from Eq. (2) into Eq. (1), we obtain the wavelength ratio \( \lambda_f / \lambda \) in terms of \( \phi \) as follows:

\[
\frac{\lambda_f}{\lambda} = \frac{1}{\sqrt{1 - \left( \frac{\pi}{2\phi} \right)^2}}
\]

The right hand side of Fig. 3 shows an equivalent circuit for analyzing the fast wave. When the input impedance in the direction of the transverse resonance at the centre of the ridge is \( Z_t \) as shown in the figure, the reflection phase \( \phi \) can be written in another form as follows:

\[
\phi = \text{Ang}(\Gamma) \quad \Gamma = \frac{Z_t/Z_o - 1}{Z_t/Z_o + 1}
\]

where \( Z_o \) is the characteristic impedance of the parallel plate waveguide between the ridge and the upper plate, for which the width is infinite and the height is \( \frac{\lambda}{2} \).

In order to obtain \( Z_t \), we use an approximation for the field distribution in the WRG. Fig. 4 shows an illustration of the field in the WRG and an approximation to the field where the region \( S \) has been replaced by metal, with field lines normal to the metal surface. Comparing Fig. 4(a) and Fig. 4(b), we find that the field distributions are similar because there is almost no field in the region \( S \) and also no field in the region \( S' \), which justifies the use of this approximation.

To find \( Z_t \), we use this approximation to develop the equivalent circuit shown in Fig. 5. In Fig. 5, all the characteristic impedances of the transmission lines in each part have the same value \( Z_o \) because all the lines have the same dimensions when viewed as a parallel plate waveguide. An important point for obtaining \( Z_t \) is how many waffle-iron lines are included in the equivalent circuit. In order to evaluate the effect of the number of waffle-iron lines, we analyzed three equivalent circuits having open-ends at positions P1, P2 and P3 as shown in Fig. 5, where the boundary condition expressed by the reflection coefficient at P1 to P3 is “1”.

In addition, we evaluated the effect of the parasitic reactance at the corners of the ridge and waffle-iron rods as shown in Fig. 5. The parasitic reactance values can be obtained from the simple equations given in Sect. 6.1 of Chapter 6 in reference [10]. The values obtained for the dimensions in Fig. 1 are given in Table 2.

![Fig. 4](image_url) Field approximation in the WRG.

![Fig. 5](image_url) Equivalent circuit in the transverse resonance direction.

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Parasitic reactance on the Ridge and the Waffle-iron.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_a )</td>
<td>( X_b )</td>
</tr>
<tr>
<td>3.0 ( Z_o )</td>
<td>9.2 ( Z_o )</td>
</tr>
</tbody>
</table>

![Fig. 6](image_url) Wavelengths of the fast wave obtained using the equivalent circuits where the number of waffle-iron lines is three for EM-simulation.
length characteristics of the WRG when its size or shape is changed. Moreover, the result means that the wavelength characteristics can be predicted just by analyzing the reflection characteristics in the transverse direction of the WRG. Understanding this is very important for our future work where we plan to propose new WRG variations in order, for example, to widen the frequency band or have dual frequency bands.

The difference of the values to the EM-simulation is around +15%. We consider that this difference is caused by rough approximations and adopting simple formulas for the complex WRG structure. However, this accuracy is sufficient to apply to an initial design of EM-simulation. The curve when P3 is open-ended can be written in the following polynomial form.

\[
\frac{\lambda_f}{\lambda} = -49.23 \left( \frac{D}{\lambda} \right)^3 + 53.61 \left( \frac{D}{\lambda} \right)^2 - 20.37 \left( \frac{D}{\lambda} \right) + 3.84
\]

(6)

4. Analysis of the Fast and Slow Waves

Basically, propagation in the WRG is by the fast wave. The slow wave function must be added to the WRG, and the analysis of the slow wave is based on that of the fast wave.

In general, the slow wave effect can be obtained from a periodic structure, which increases the equivalent length of the waveguide by inserting a series of inductances or shunt capacitances into the waveguide. Fig. 7 shows cross sections and equivalent circuits of the FS-WRG, where Fig. 7(a) is for \( d = 0 \) and Fig. 7(b) is for \( d \neq 0 \). As shown in Fig. 7(b), a periodic step with depth \( d \) (refer to Fig. 1) is equivalent to a line of impedance \( Z_p \) terminated by a short. Therefore, under the condition that the parasitic reactance produced at a corner of the step shown in Fig. 7(b) is small enough and \( d < \lambda/4 \), the equivalent reactance of the periodic step becomes inductive which generates a slow wave effect.

By focusing on the difference between the distributed reactance for \( d = 0 \) and \( d \neq 0 \), the wavelength ratio between them can be expressed in terms of the distributed reactance as follows:

\[
\frac{\lambda_{fs}}{\lambda_f} = \sqrt{\frac{L_f C_f}{L_{fs} C_{fs}}}
\]

(7)

where \( \lambda_f \), \( L_f \) and \( C_f \) are the wavelength, the distributed series inductance and the distributed shunt capacitance per unit length for \( d = 0 \), respectively, while \( \lambda_{fs} \), \( L_{fs} \) and \( C_{fs} \) are those for \( d \neq 0 \). The distributed reactance due to \( L_{fs} \) and \( C_{fs} \) in Eq. (7) can be substituted by the average reactance in the period \( T \) in Fig. 7. In order to determine the average reactance of the period \( T \), the equivalent circuit is approximated by that shown in Fig. 8. In Fig. 8 \( Z_{EL} \) is the impedance of the periodic step when looking in the \( -z \) direction from the top of the ridge and \( Z_{EC} \) is the impedance for the approximation from Fig. 8(a) to Fig. 8(b). Using the approximate equivalent circuit in Fig. 8(b), Eq. (7) can be rewritten as follows:

\[
\frac{\lambda_{fs}}{\lambda_f} = \sqrt{\frac{\Delta L \Delta C}{\frac{\Delta L}{2} + \frac{Z_{EL}}{j\omega}} \left( \frac{\Delta C}{2} + \frac{1}{j\omega Z_{EC}} \right)}
\]

(8)
where $\Delta C$ and $\Delta L$ are fragments of the distributed reactance $L_f$ and $C_f$ in a period of the periodic step $T$.

Equation (8) means that the wavelength ratio of the fast and slow waves to the fast wave $\lambda_{fs}/\lambda_f$, can be obtained from $\Delta C$, $\Delta L$, $Z_{EL}$ and $Z_{EC}$. From Eqs. (6) and (8), the wavelength ratio $\lambda_{fs}/\lambda$ is given by

$$\frac{\lambda_{fs}}{\lambda} = \frac{\lambda_{fs}}{\lambda_f} = \frac{\lambda_f}{\lambda}$$

(9)

$\Delta C$ and $\Delta L$ can be obtained as follows. We make another approximation that most of the electric field is confined to the region G as shown in the upper-right hand side of Fig. 7. Using this approximation, $\Delta C$ is equal to the parallel plate capacitance of a $\lambda/8$ cube, which is given by

$$\Delta C = \varepsilon \frac{(\lambda/8)^2}{\lambda/8} = \varepsilon \frac{\lambda}{8}$$

(10)

$\varepsilon$: permittivity of the air

The characteristic impedance, $Z_f$, for the fast wave in Fig. 7(a) is given by

$$Z_f = \sqrt{\frac{\Delta L}{\Delta C}}$$

(11)

The height and width of the cross section in region G are the same, and we can introduce a virtual initial value for $Z_f$ of $Z_{fo} = 120\pi$ which is the characteristic impedance of a TEM wave propagating in the direction of the ridge. In the actual fast wave, however, the TEM wave propagates at an angle $\theta$ as shown in Fig. 3. Hence the magnetic field normal to the ridge, $H'$, is given by

$$H' = H \sin \theta$$

(12)

where $H$ is the magnetic field of the TEM wave in free space. This means that $Z_f$ can be expressed in terms of the ratio between the wavelength of the fast wave and that in free space as follows:

$$Z_f = \frac{E}{H'} = \frac{E}{H \sin \theta} = Z_{fo} \frac{\lambda_f}{\lambda} = 120\pi \frac{\lambda_f}{\lambda}$$

(13)

where $E$ is the electric field of the TEM wave in free space. From Eqs. (10), (11) and (13) $\Delta L$ is given by

$$\Delta L = \varepsilon \frac{\lambda}{8} \left(120\pi \frac{\lambda_f}{\lambda}\right)^2$$

(14)

We can obtain $\Delta C$ and $\Delta L$ from Eqs. (10) and (14). $Z_{EL}$ and $Z_{EC}$ in Eq. (8) can be obtained from the following considerations. The equivalent circuit of the periodic step is a shorted transmission line in the $-z$ direction with the parasitic reactance as shown in Fig. 7(b) and Fig. 8. We denote the characteristic impedance of the shorted transmission line of the periodic step by $Z_P$. The cross section of the shorted transmission line is square with side length $\lambda/8$ and has no metal sidewall. Therefore, the TEM wave propagates in the shorted transmission line and $Z_P = 120\pi$. In order to obtain the parasitic reactance, we employ an approximation $Z_P = Z_0$ which is based on the fact that the dimensions of the cross sections are similar. With these considerations, the parasitic reactance values are obtained as shown in Table 3 from equations given in reference [10].

![Fig. 9](image)

**Fig. 9** Wavelength of the Fast and Slow wave, where the number of waffle-iron lines is three for the EM-simulation.

<table>
<thead>
<tr>
<th>$\lambda/P$</th>
<th>$Xas$</th>
<th>$Xbs$</th>
<th>$Xcs$</th>
<th>$Xds$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3.0 Z_P$</td>
<td>$9.2 Z_P$</td>
<td>$0.8 Z_P$</td>
<td>$5.1 Z_P$</td>
<td></td>
</tr>
</tbody>
</table>

Table 3  Parasitic reactance of the periodic steps on the ridge.

Now we have $Z_P$, $d$ and the parasitic reactance in Fig. 8(b), $Z_{EL}$ and $Z_{EC}$ can be calculated. Finally, the wavelength ratio $\lambda_{fs}/\lambda_f$ can be calculated from Eq. (8) using the values of $\Delta C$, $\Delta L$, $Z_{EL}$, $Z_{EC}$ and the parasitic reactance.

Figure 9 shows the wavelength ratio of the fast and slow waves obtained using Eqs. (6), (8) and (9) as a function of the depth of the periodic steps $d$. In Fig. 9, the wavelength ratio determined by EM-simulation is also displayed. The wavelength ratio curves calculated from the equivalent circuits and by EM-simulation have similar trends, which verifies that the analysis using the equivalent circuits in Figs. 5 and 8 is effective in order for us enough to predict the way in which the wavelength characteristics change with the shape of the ridge surface in the FS-WRG. This is also important for our future work on proposing new FS-WRG variations. In the case of $D = 0.25\lambda$, the difference of the calculated values to the EM-simulation is between $-5\%$ and $+7\%$ which is improved from that in the fast wave. We consider the reason for this improvement is that, the influence of rough approximations and adopting simple formulas for the complex WRG structure canceled each other between the calculations of the fast wave and the fast and slow wave.

In Fig. 9, we selected the curve for $D = 0.25\lambda$ to use in our array antenna design in the following section. The selected curve can be written in the following polynomial form.
\[
\frac{\lambda_{fs}}{\lambda} = -61.33 \left(\frac{d}{\lambda}\right)^3 + 11.03 \left(\frac{d}{\lambda}\right)^2 - 2.41 \left(\frac{d}{\lambda}\right) + 1.13
\] (15)

5. Application to Array Antenna Design

In this section, as an example, we demonstrate the use of Eqs. (6) and (15) in designing an array antenna. If only in-phase excitation over a big aperture is needed, a long pyramidal horn is the best solution. A benefit of the array antenna is to reduce the thickness of the antenna. The greater the number employed in the array for the same aperture the thinner the antenna becomes. Increasing the number in the array for the same aperture is equivalent to shortening the distance between the element ports, which requires a shorter wavelength on the feed line. As mentioned in the above discussion, the benefit of a FS-WRG is its ability to shorten the wavelength compared to that without trenches or periodic steps. Therefore, we select a shorter element distance of \( \lambda_{fs} = \lambda \) rather than \( \lambda_f \) by which the benefit of the array antenna is used.

In this paper, we demonstrate an example of an array antenna with two elements in series in which the aperture is \( 2\lambda \). The initial configuration of the feeding circuit for EM-simulation calculated using Eqs. (6) and (15) is shown in Fig. 10, in which \( D = 0.25\lambda \) and \( d = 0.06\lambda \). The pyramidal horns are outlined in Fig. 10. Therefore, the EM-simulation model used to obtain just the distribution characteristics to the element ports can be represented by the structure without the lines showing the pyramidal horns. As shown in Fig. 10, the shape of element ports in the model to obtain the distribution characteristics are straight slots.

Figure 11 shows an equivalent circuit of the design goal of an array antenna with two elements in series. As shown in Fig. 11, the elements in the design goal are \( \lambda \)-length line sources in phase and of the same magnitude, which corresponds to the prepared initial configuration. The impedance has no reactance, such as for long pyramidal horns, and the space between the elements is \( \lambda \), which is the same as in the initial configuration. The dashed line in Fig. 11 encompasses the distribution circuit, which corresponds to the EM-simulation model used to obtain the distribution characteristics, as explained above.

First, we calculated the distribution characteristics by EM-simulation as two vectors, one from the Input Port to Element Port 1 and the other from the Input Port to Element Port 2. The vector of Element Port 1 is normalized by the vector of Element Port 2. The normalized vector is compared to that of the Design Goal in Table 4.

Next we calculated the whole antenna configuration in Fig. 10. In Fig. 12, two directivity curves are plotted. Design Goal is the product of an array factor and an element factor in which the array factor is in-phase and the same magnitude and the element factor is the directivity of a \( \lambda \)-length line source. Whole Antenna directivity is the result of an EM-simulation of the whole antenna structure in Fig. 10 including the horns.

![Fig. 10](image1)

**Fig. 10** An example of an array antenna with two elements in series and the initial configuration of the feeding circuit for EM-simulation calculated using Eqs. (6) and (15).

<table>
<thead>
<tr>
<th></th>
<th>Magnitude</th>
<th>Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design Goal</td>
<td>0dB</td>
<td>0deg</td>
</tr>
<tr>
<td>EM-simulation</td>
<td>0.2dB</td>
<td>−27deg</td>
</tr>
</tbody>
</table>
It can be surmised that the difference between the Whole Antenna directivity and the Design Goal directivity is generated by the following effects.

i) The coupling reactance between the ridge and the horns.

ii) The phase difference at the aperture of the short pyramidal horn.

As shown in Fig. 12, the difference between the direction of the main lobe in the Whole Antenna directivity and that of the Design Goal is around 5 degrees. This result means that the remaining work for the designer is to adjust the trench depth $D$ and the periodic step depth $d$ to approach the Design Goal.

6. Conclusion

We proposed a new method using equivalent circuits for analyzing the wavelengths in a WRG and a FS-WRG. The traditional transverse resonance method was also employed for the fast wave. From this analysis we obtained information sufficient for us to predict how the wavelength characteristics change when the shapes of the WRG or FS-WRG are changed. This information will be very important for our future work in which we intend to design new WRGs and FS-WRGs with, for example, wider or dual frequency bands.

We demonstrated the use of the results obtained in defining an initial configuration for an array antenna design with two elements in series. This method makes the design work for WRGs and FS-WRGs easier and faster.

References


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Koichi Ogawa received his M.S. degree in Electrical Engineering from Shizuoka University in 1981. He received a Ph.D. degree in Electrical Engineering from the Tokyo Institute of Technology in 2000. In 2005, he was a Visiting Professor with the Antennas and Propagation Division, Aalborg University, Denmark. Dr. Ogawa is currently a Professor at Toyama University, Japan.