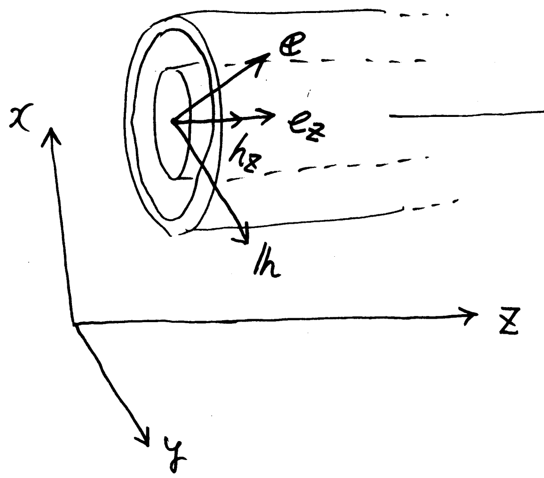
Transmission lines and waveguides, Content:

- infinitely long transmission line
- field equations for TEM, TE, TM waves
- rectangular waveguides
  - dispersion relation
  - group velocity and phase velocity
- surface wave
- dielectric slab waveguide

Transmission lines and waveguides

Consider "infinitely long transmission lines".



$\longrightarrow$ wave propagation, $e^{-j\beta z} \cdot e^{j\omega t}$

> "Express all the field components with E_z and H_z".

Consider time-harmonic fields (調和振動電界,磁界)

$$
\begin{cases}
\mathcal{E}(x, y, z, t) = \mathbb{E}(x, y, z)\, e^{j\omega t} & (1a) \\
\mathcal{H}(x, y, z, t) = \mathbb{H}(x, y, z)\, e^{j\omega t} & (1b)
\end{cases}
$$

$$
\begin{cases}
\mathbb{E}(x, y, z) = [\mathbb{e}(x, y) + \hat{z}\, e_z(x, y)]\, e^{-j\beta z} & (2a) \\
\mathbb{H}(x, y, z) = [\mathbb{h}(x, y) + \hat{z}\, h_z(x, y)]\, e^{-j\beta z} & (2b)
\end{cases}
$$

$\mathbb{e}, \mathbb{h}$: transversal fields. (横方向電磁界) (x.y 方向)

e_z, h_z: longitudinal fields (進行方向電磁界) (z 方向)

β: propagation const. (伝搬定数)

$\downarrow$ substitute into Maxwell's eqs. in freq. domain

$$
\begin{cases}
\nabla \times \mathbb{E} = -j\omega\mu \mathbb{H} & (3a) \\
\nabla \times \mathbb{H} = j\omega\varepsilon \mathbb{E} & (3b)
\end{cases}
$$

rewriting using components, $\mathbb{E} = (E_x, E_y, E_z)$, $\mathbb{H} = (H_x, H_y, H_z)$

$$\left\{\begin{array}{ll}
\dfrac{\partial E_z}{\partial y} - \dfrac{\partial E_y}{\partial z} = -j\omega\mu H_x & (4a) \\
\dfrac{\partial E_x}{\partial z} - \dfrac{\partial E_z}{\partial x} = -j\omega\mu H_y & (4b) \\
\dfrac{\partial E_y}{\partial x} - \dfrac{\partial E_x}{\partial y} = -j\omega\mu H_z & (4c) \\
\dfrac{\partial H_z}{\partial y} - \dfrac{\partial H_y}{\partial z} = j\omega\varepsilon E_x & (4d) \\
\dfrac{\partial H_x}{\partial z} - \dfrac{\partial H_z}{\partial x} = j\omega\varepsilon E_y & (4e) \\
\dfrac{\partial H_y}{\partial x} - \dfrac{\partial H_x}{\partial y} = j\omega\varepsilon E_z & (4f)
\end{array}\right.$$

,

then, the z-derivatives are $\dfrac{\partial e^{-j\beta z}}{\partial z} = -j\beta\, e^{-j\beta z}$, i.e,
(4a~f) are rewritten as

$$\left\{\begin{array}{ll}
\dfrac{\partial E_z}{\partial y} + j\beta E_y = -j\omega\mu H_x & (5a) \\
-j\beta E_x - \dfrac{\partial E_z}{\partial x} = -j\omega\mu H_y & (5b) \\
\dfrac{\partial E_y}{\partial x} - \dfrac{\partial E_x}{\partial y} = -j\omega\mu H_z & (5c) \\
\dfrac{\partial H_z}{\partial y} + j\beta H_y = j\omega\varepsilon E_x & (5d) \\
-j\beta H_x - \dfrac{\partial H_z}{\partial x} = j\omega\varepsilon E_y & (5e) \\
\dfrac{\partial H_y}{\partial x} - \dfrac{\partial H_x}{\partial y} = j\omega\varepsilon E_z & (5f)
\end{array}\right.$$

substitute

(5a) :

$$\frac{\partial E_z}{\partial y} + j\beta \cdot \underbrace{\frac{1}{j\omega\varepsilon}\left(-j\beta H_x - \frac{\partial H_z}{\partial x}\right)}_{E_y} = -j\omega\mu H_x \qquad (6)$$

$$\Downarrow$$

$$-\frac{j\beta^2}{\omega\varepsilon} H_x - \frac{\beta}{\omega\varepsilon}\frac{\partial H_z}{\partial x}$$

$$\therefore \quad \frac{\partial E_z}{\partial y} - \frac{\beta}{\omega\varepsilon}\frac{\partial H_z}{\partial x} = -j\omega\mu H_x + \frac{j\beta^2}{\omega\varepsilon} H_x \qquad (7)$$

$\times(\omega\varepsilon)$

$$\omega\varepsilon\frac{\partial E_z}{\partial y} - \beta\frac{\partial H_z}{\partial x} = \left(-j\omega^2\varepsilon\mu + j\beta^2\right) H_x , \qquad (8)$$

consider that $\omega^2\varepsilon\mu = \frac{\omega^2}{c_0^2} = k_0^2$ (9)

k_0 : wave number in vacuume free space

right hand side,

$$RHS = -j\,(k_0^2 - \beta^2)\, H_x$$

$$= -j\,k_c^2\, H_x ,$$

$$\underline{\underline{k_c^2 = k_0^2 - \beta^2}}$$: cut off wave number (10)

Finally (5a) is rewritten as

$$\omega\varepsilon\frac{\partial E_z}{\partial y} - \beta\frac{\partial H_z}{\partial x} = -j\,k_c^2\, H_x$$

or

$$\boxed{H_x = \frac{j}{k_c^2}\left(\omega\varepsilon\frac{\partial E_z}{\partial y} - \beta\frac{\partial H_z}{\partial x}\right)} \qquad (11)$$

From (5 b),

$$-j\beta \cdot \frac{1}{j\omega\varepsilon}\left(\frac{\partial H_z}{\partial y} + j\beta H_y\right) - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$$

$$-\frac{\beta}{\omega\varepsilon}\frac{\partial H_z}{\partial y} + \frac{j\beta^2}{\omega\varepsilon} H_y - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$$

$$-\beta\frac{\partial H_z}{\partial y} - \omega\varepsilon\frac{\partial E_z}{\partial x} = (-j\omega^2\varepsilon\mu - j\beta^2) H_y$$

$$= -j(k_0^2 - \beta^2) H_y$$

$$= -j k_c^2 H_y$$

$$\therefore \boxed{H_y = \frac{-j}{k_c^2}\left(\omega\varepsilon\frac{\partial E_z}{\partial x} + \beta\frac{\partial H_z}{\partial y}\right)} \qquad (12)$$

Same for the electric fields, we get

$$\boxed{E_x = \frac{-j}{k_c^2}\left(\beta\frac{\partial E_z}{\partial x} + \omega\mu\frac{\partial H_z}{\partial y}\right)} \qquad (13)$$

,

$$\boxed{E_y = \frac{j}{k_c^2}\left(-\beta\frac{\partial E_z}{\partial y} + \omega\mu\frac{\partial H_z}{\partial x}\right)} \qquad (14)$$

.

	E_z	H_z	
TEM	$=0$	$=0$	---- transvers electro-magnetic
TE	$=0$	$\neq 0$	---- transvers electric
TM	$\neq 0$	$=0$	---- transvers magnetic

※. For infinitely long transmission line, all the transversal fields can be expressed by the longitudinal fields.
(H_x, H_y, E_x, E_y) (H_z, E_z)

TEM wave

$$\mathbb{E} = (E_x,\ E_y,\ 0), \quad \mathbb{H} = (H_x,\ H_y,\ 0)$$

Maxwell's equations reduce to

$$\begin{cases} -\dfrac{\partial E_y}{\partial z} = -j\omega\mu H_x & ② \\ \dfrac{\partial E_x}{\partial z} = -j\omega\mu H_y & ① \\ -\dfrac{\partial H_y}{\partial z} = j\omega\varepsilon E_x & ① \\ \dfrac{\partial H_x}{\partial z} = j\omega\varepsilon E_y & ② \end{cases}$$

Two pair of equations (①, ②)

$\Downarrow$

two "scalar wave equations,"

$\Downarrow$

two plane-wave solutions

TE wave

$\mathbb{E} = (E_x, E_y, 0)$. $\mathbb{H} = (H_x, H_y, H_z)$

Helmholtz equation in terms of H_z

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k_0^2 \right) H_z = 0 \qquad (15)$$

$$k_0^2 = \omega^2 \varepsilon \mu$$

$$\downarrow \quad H_z = h_z(x, y)\, e^{-j\beta z}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \beta^2 + k_0^2 \right) H_z = 0$$

$$\therefore \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2 \right) H_z = 0 \qquad (16)$$

where $k_c^2 = k_0^2 - \beta^2$, (17)

$\downarrow$ Solve for H_z, then, we will find

from (11) ... $H_x = \dfrac{-j}{k_c^2} \beta \dfrac{\partial H_z}{\partial x}$ (18)

" (12) ... $H_y = \dfrac{-j}{k_c^2} \beta \dfrac{\partial H_z}{\partial y}$ (19)

" (13) ... $E_x = \dfrac{-j}{k_c^2} \cdot \omega\mu \dfrac{\partial H_z}{\partial y}$ (20)

" (14) ... $E_y = \dfrac{j}{k_c^2} \cdot \omega\mu \dfrac{\partial H_z}{\partial x}$ (21)

give all the field components.

TM wave $\quad \mathbb{E} = (E_x, E_y, E_z)$, $\mathbb{H} = (H_x, H_y, 0)$

Helmholtz equation in terms of E_z

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k_0^2 \right) E_z = 0 \qquad (22)$$

$$\Big\downarrow \; E_z = e_z(x, y)\, e^{-j\beta z}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \beta^2 + k_0^2 \right) E_z = 0$$

$$\therefore \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2 \right) E_z = 0 , \qquad (23)$$

where $k_c^2 = k_0^2 - \beta^2 \quad (24)$

slove for E_z, then we will find

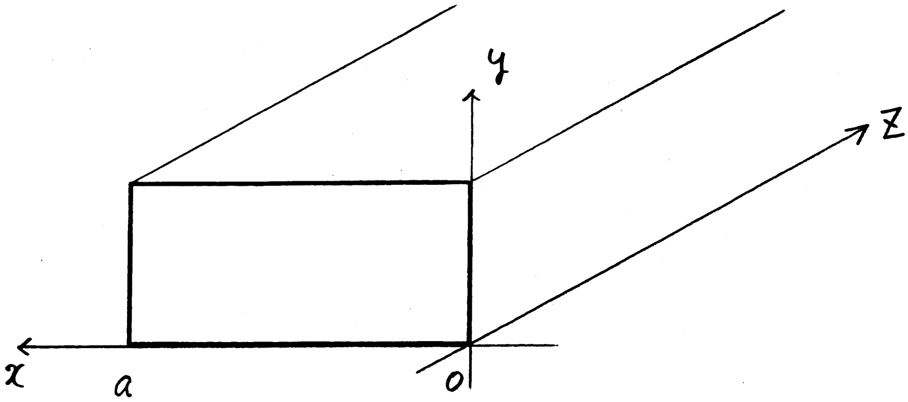
from (11) ... $\quad H_x = \frac{j}{k_c^2} \omega \varepsilon \frac{\partial E_z}{\partial y} \qquad (25)$

" (12) -- $\quad H_y = \frac{-j}{k_c^2} \omega \varepsilon \frac{\partial E_z}{\partial x} \qquad (26)$

" (13) ... $\quad E_x = \frac{-j}{k_c^2} \cdot \beta \frac{\partial E_z}{\partial x} \qquad (27)$

" (14) ... $\quad E_y = \frac{-j}{k_c^2} \cdot \beta \frac{\partial E_z}{\partial y} \qquad (28)$

Rectangular waveguide.



A rectangular waveguide supports TE and TM modes.
We consider here the TE mode.

$$\mathbb{E} = (E_x,\ E_y,\ 0), \qquad \mathbb{H} = (H_x,\ H_y,\ \underline{\underline{H_z}}) \tag{29}$$

find this!

Starting with the Helmholtz equation in terms of H_z,

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k_0^2 \right) H_z = 0 \tag{30}$$

where $k_0^2 = \omega^2 \varepsilon_0 \mu_0$, (31)

k_0 is the propagation const. in free space

Assume

$$H_z(x, y, z) = H_{z_0}(x, y)\, e^{-j\beta z}, \tag{32}$$

the Helmholtz eq. is rewritten as

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \underbrace{- \beta^2 + k_0^2}_{\downarrow} \right) H_z(x, y, z) = 0 \tag{33}$$

or

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2 \right) H_{z_0}(x, y)\, \cancel{e^{-j\beta z}} = 0 \tag{34}$$

k_0 : propagation const. in free space (wave number)

β : propagation const. of the guided wave

k_c : cut-off wave number

Rewrite H_{z_0} as

$$H_{z_0}(x, y) = X(x) \cdot Y(y), \tag{35}$$

and

$$k_c^2 = k_x^2 + k_y^2, \tag{36}$$

then the Helmholtz eq is

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2 \right) X(x)\, Y(y) = 0, \tag{37}$$

$$\frac{\partial^2 X(x)}{\partial x^2} \cdot Y(y) + \frac{\partial^2 Y(y)}{\partial y^2} X(x) + k_c^2 X(x)\, Y(y) = 0$$

or

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + k_x^2 + k_y^2 = 0 \tag{38}$$

method of variable separation

General solution:

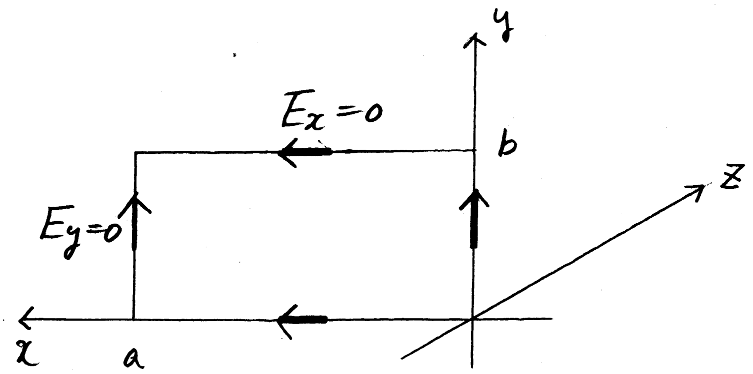
$$\boxed{H_{z_0}(x, y) = \{A\cos(k_x x) + B\sin(k_x x)\}\{C\cos(k_y y) + D\sin(k_y y)\}} \tag{39}$$

Now we impose boundary conditions to the general solution.

Suppose
$$E_x(x, y, z) = E_{x_0}(x, y)\, e^{-j\beta z}, \tag{40}$$
$$E_y(x, y, z) = E_{y_0}(x, y)\, e^{-j\beta z}, \tag{41}$$

B.C.:

$$\begin{cases} E_{x0}(x,y) = 0 \quad \text{at} \quad y = 0,\ b & (42) \\ E_{y0}(x,y) = 0 \quad \text{at} \quad x = 0,\ a & (43) \end{cases}$$



From (20), (p43)

$$E_{x0}(x,y) = \frac{-j}{k_c^2} \cdot \omega\mu_0 \frac{\partial H_{z0}(x,y)}{\partial y}$$

$$= \frac{-j}{k_c^2} \omega\mu_0 \cdot (A \cos k_x x + B \sin k_x x) \cdot k_y(-C \sin k_y y + D \cos k_y y) \qquad (44)$$

From (21), (p.43)

$$E_{y0}(x,y) = \frac{j}{k_c^2} \omega\mu_0 \frac{\partial H_{z0}(x,y)}{\partial x}$$

$$= \frac{j}{k_c^2} \omega\mu_0 \cdot k_x(-A \sin k_x x + B \cos k_x x) \cdot (C \cos k_y y + D \sin k_y y) \qquad (45)$$

From (44), at $y = 0$, $E_{x0} = (\cdots)(-C \cdot 0 + D \cdot 1) = 0$

therefore, $D = 0$ (46)

at $y = b$, $E_{x0} = (\cdots)(-C \cdot \sin k_y \cdot b) = 0$

therefore,

$$k_y = \frac{n\pi}{b}, \quad n = 1, 2, \cdots \qquad (47)$$

From (45), at $x=0$, $E_{y0}(x,y) = (\cdots)(-A\cdot 0 + B\cdot 1) = 0$

therefore, $B = 0$. (48)

at $x=a$, $E_{y0}(x,y) = (\cdots)(-A\sin k_x a) = 0$

therefore

$$k_x = \frac{m\pi}{a}, \quad m = 1, 2, \cdots \tag{49}$$

To summarize,

$$\begin{cases} A \neq 0, & B = 0, & k_x = \dfrac{m\pi}{a}, & m = 1, 2, \cdots \\ C \neq 0, & D = 0, & k_y = \dfrac{n\pi}{b}, & n = 1, 2, \cdots \end{cases},$$

and the final solution for H_z is

$$H_z(x,y,z) = A_{mn}\cos(k_x x)\cos(k_y y)\, e^{-j\beta z} \tag{50}$$

$$k_x = \frac{m\pi}{a}, \quad k_y = \frac{n\pi}{b},$$

$$m, n = 1, 2, \cdots$$

From (18); $H_x = \dfrac{-j}{k_c^2}\beta\dfrac{\partial H_z}{\partial x} = \dfrac{-j\beta}{k_c^2} A_{mn} k_x (-1)\sin k_x x\cdot\cos k_y y\cdot e^{-j\beta z}$ (51)

(19); $H_y = \dfrac{-j}{k_c^2}\beta\dfrac{\partial H_z}{\partial y} = \dfrac{-j\beta}{k_c^2} A_{mn} k_y (-1)\cos k_x x\cdot\sin k_y y\cdot e^{-j\beta z}$ (52)

(20); $E_x = \dfrac{-j}{k_c^2}\omega\mu\dfrac{\partial H_z}{\partial y} = \dfrac{-j\omega\mu}{k_c^2} A_{mn} k_y (-1)\cos k_x x\cdot\sin k_y\cdot y\, e^{-j\beta z}$ (53)

(21); $E_y = \dfrac{j}{k_c^2}\omega\mu\dfrac{\partial H_z}{\partial x} = \dfrac{j\omega\mu}{k_c^2} A_{mn} k_x (-1)\sin k_x x\cos k_y y\cdot e^{-j\beta z}$ (54)

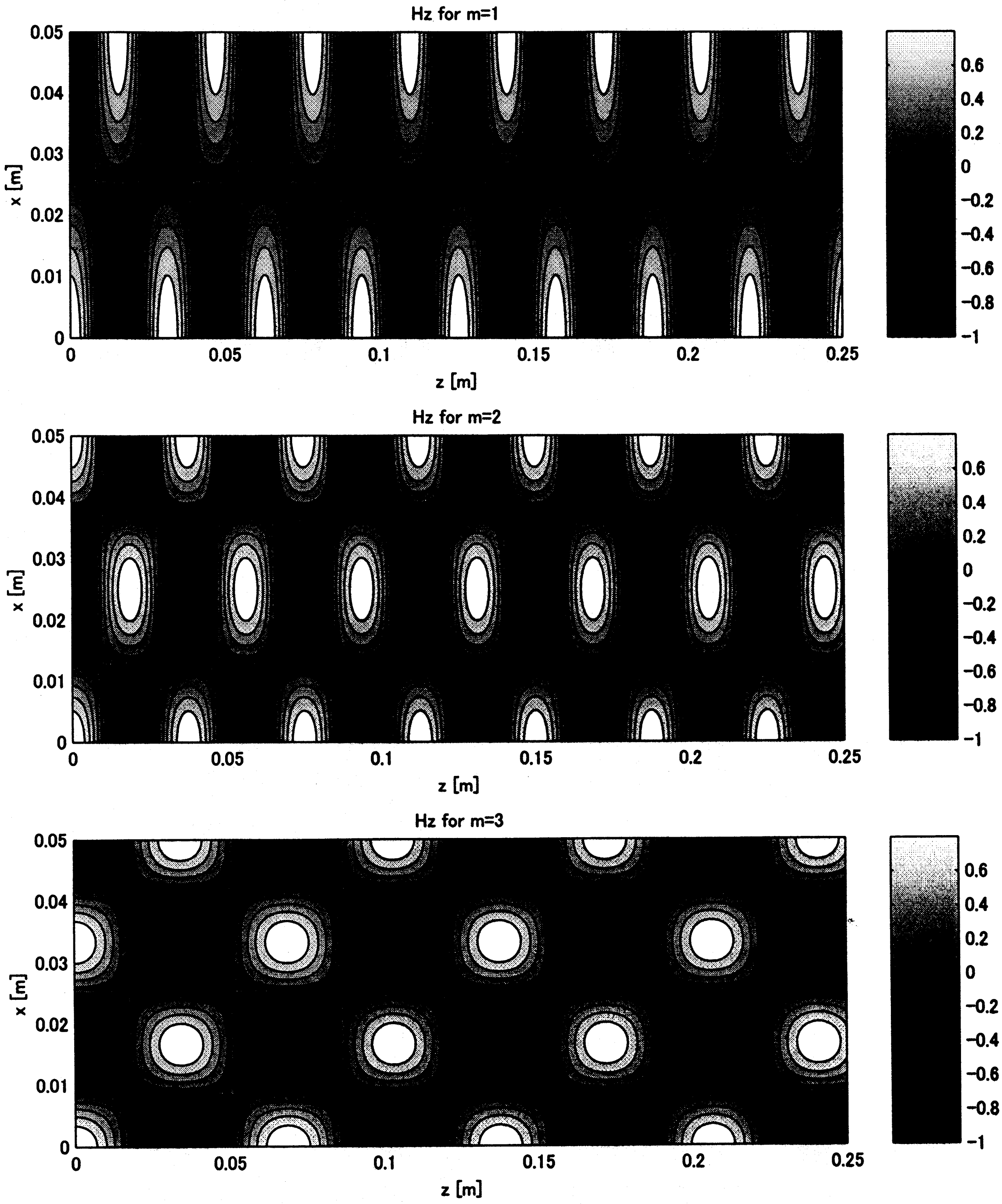
and

$$\beta = \sqrt{k_0^2 - k_c^2} = \sqrt{k_0^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} \tag{55}$$

Rectangular waveguide (TE mode)
P48. (50) plotted.

$f = 10 \times 10^9$ Hz, $a = 50$ mm. $k_y = 0$ $(n=0)$, $\beta = \sqrt{\frac{\omega^2}{c^2} - k_x^2 - k_y^2}$

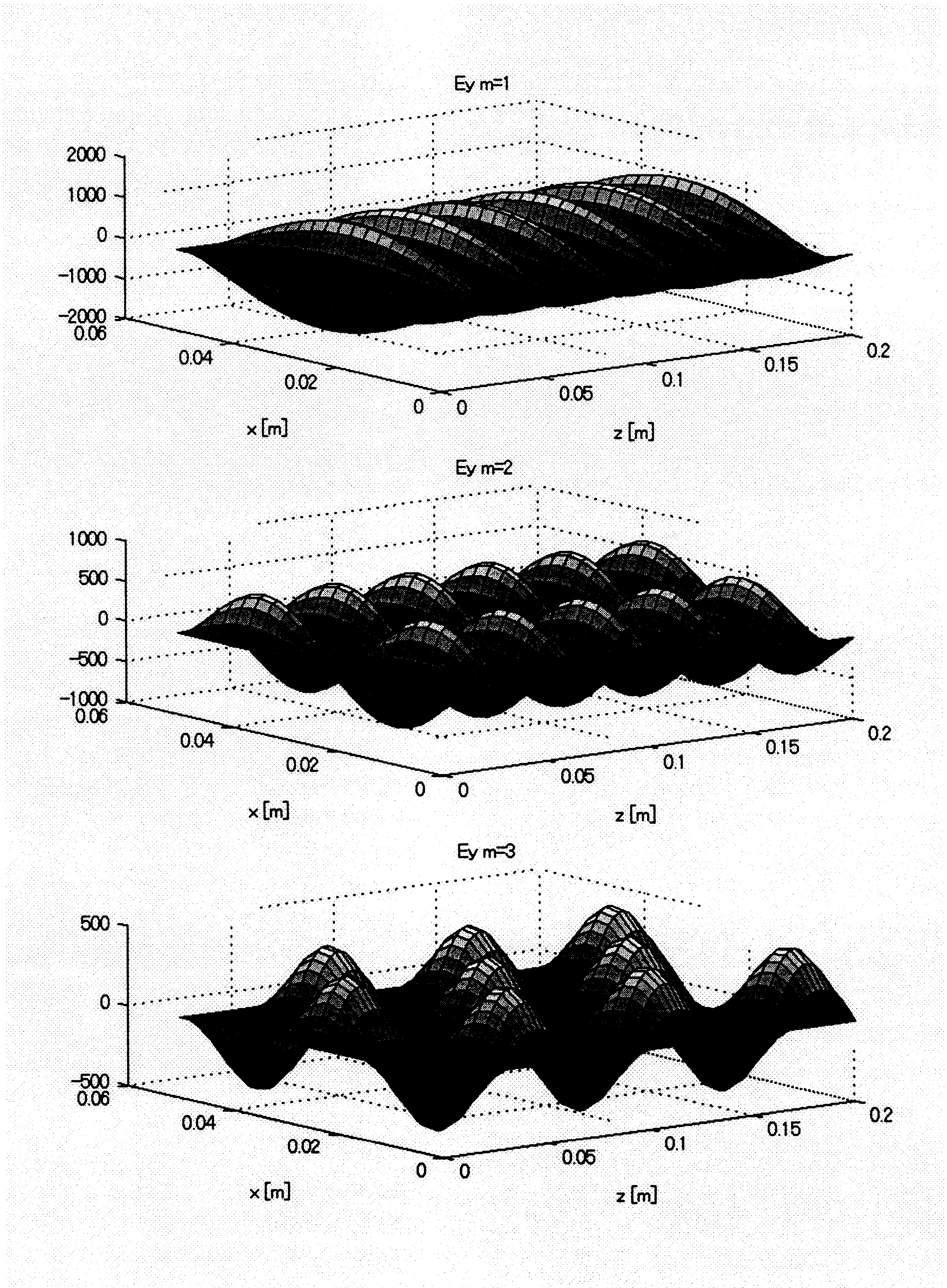
Hz field components



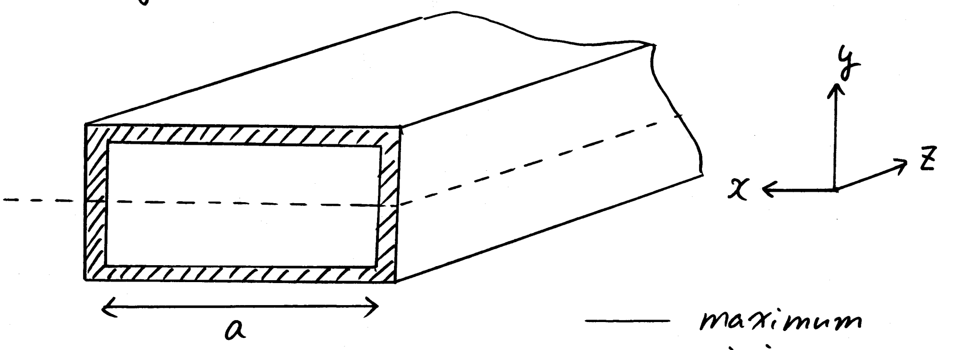
Rectangular waveguide. (TE mode)
p48. (54),

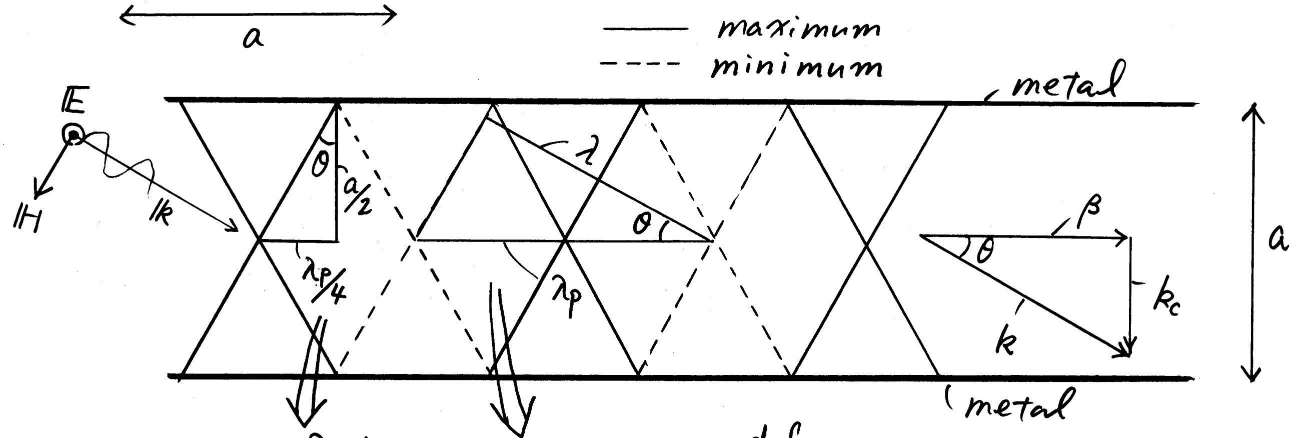
Same as p48.1

Ey field components, (surface plot)



Rectangular waveguide (TE wave)





$$\tan\theta = \frac{\lambda_p/4}{a/2} = \frac{\lambda_p}{2a}$$

$$\cos\theta = \frac{\lambda}{\lambda_p}$$

def

λ ; free space wave length

λ_p ; guided wave length

λ_c ; cut-off wave length
$(=2a)$

$\Downarrow$

$\beta = \frac{2\pi}{\lambda_p}$; propagation constant

$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{\lambda f} = \frac{\omega}{c}$; free-space wave number,

$k_c = \frac{2\pi}{\lambda_c} = \frac{\pi}{a}$

$\Downarrow$

$$1 = \sin^2\theta + \cos^2\theta$$

$$\frac{1}{\cos^2\theta} = \tan^2\theta + 1$$

$$\left(\frac{\lambda_p}{\lambda}\right)^2 = \left(\frac{\lambda_p}{2a}\right)^2 + 1$$

$$\left(\frac{1}{\lambda}\right)^2 = \left(\frac{1}{2a}\right)^2 + \left(\frac{1}{\lambda_p}\right)^2$$

$\times 4\pi^2 \downarrow$

$$k^2 = k_c^2 + \beta^2 \qquad (56)$$

$\Downarrow$

$$\begin{cases} \text{propagation const ; } \beta = \sqrt{k^2 - k_c^2} = \sqrt{\left(\frac{2\pi f}{c}\right)^2 - \left(\frac{\pi}{a}\right)^2} \\ \text{guided wave length ; } \lambda_p = \frac{2\pi}{\beta} \end{cases}$$

determined by "f" and "a".

Dispersion relation (ω-β diagram) for rec. w.g.

from (56),

$$k^2 = k_c^2 + \beta^2, \quad \rightarrow \omega = c\sqrt{k_c^2 + \beta^2}$$

with $k = \dfrac{\omega}{c}$,

$$k_c = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

: cut-off wave number

$$\Rightarrow \beta = \sqrt{k^2 - k_c^2}$$

plot

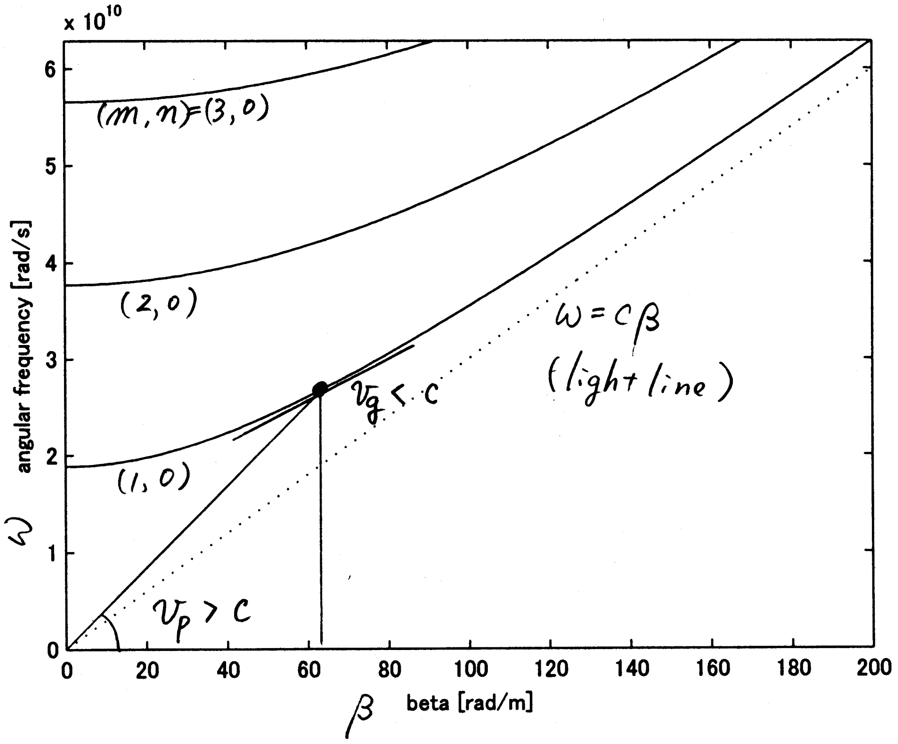


Fig. ω-β diagram

- phase velocity : (apparent velocity)

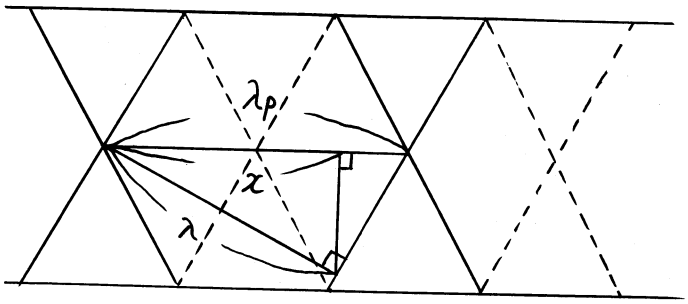
$$v_p = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\frac{\omega^2}{c^2} - k_c^2}} = \frac{c}{\sqrt{1 - \frac{c^2}{\omega^2}k_c^2}} \quad > c$$

- group velocity : (speed of energy flow)

$$v_g = \frac{d\omega}{d\beta} = \frac{1}{\frac{d\beta}{d\omega}} = \frac{1}{\frac{1}{2}\left(\frac{\omega^2}{c^2} - k_c^2\right)^{-1/2} \cdot \frac{2\omega}{c^2}} = c\sqrt{1 - \frac{c^2}{\omega^2}k_c^2}$$

$$< c$$

Group velocity and Phase velocity



$\lambda : x = \lambda_p : \lambda$
$\Rightarrow x = \dfrac{\lambda^2}{\lambda_p}$

Phase velocity : (apparent velocity, 位相速度)

$$v_p = \frac{\lambda_p}{t}, \quad t = \frac{\lambda}{c}$$
$$= \frac{\lambda_p}{\lambda} \cdot c \qquad \left(\text{or, } v_p = \frac{\omega}{\beta} = \frac{ck}{\beta} = c \cdot \frac{2\pi/\lambda}{2\pi/\lambda_p} \right)$$

where

$$\frac{\lambda_p}{\lambda} = \frac{1}{\lambda} \cdot \frac{1}{\sqrt{\left(\frac{1}{\lambda}\right)^2 - \left(\frac{1}{2a}\right)^2}} = \frac{1}{\sqrt{1 - \left(\frac{\lambda}{2a}\right)^2}} = \frac{1}{\sqrt{1 - \left(\frac{f_c}{f}\right)}},$$

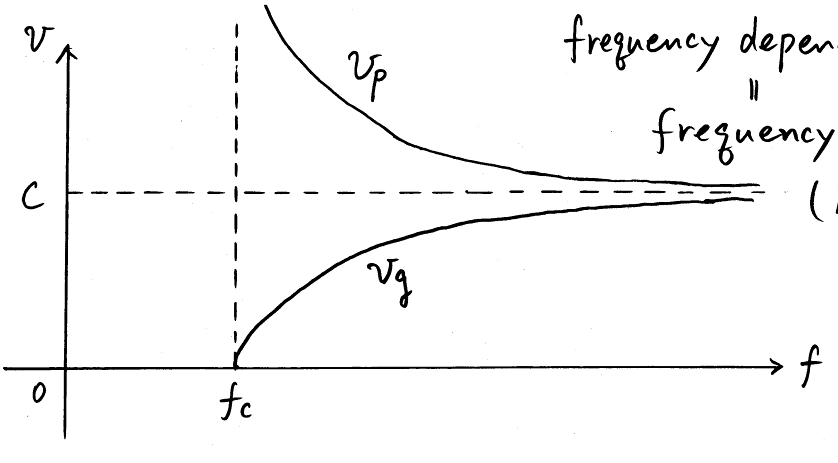
$\lambda_c = 2a$

$$\Rightarrow v_p = \frac{c}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \quad > c$$

speed of light !

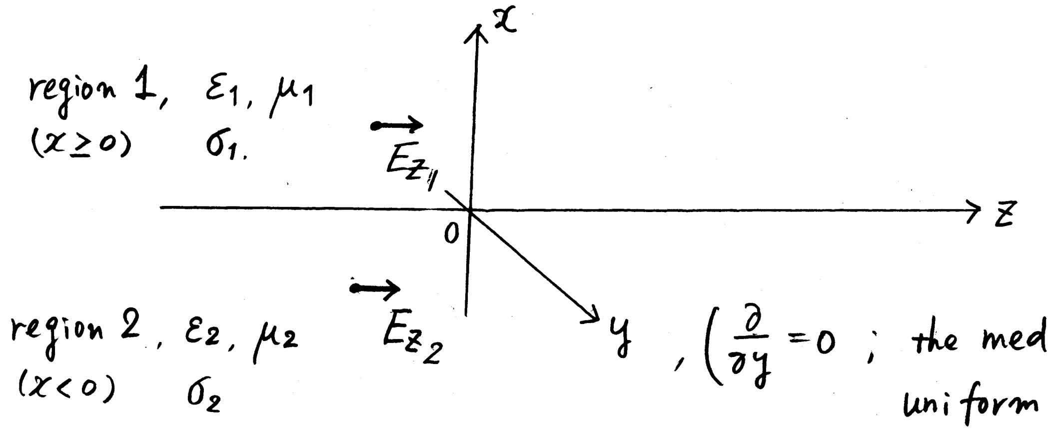
Group velocity ; (actual velocity of energy carried by the wave)
実際のエネルギーの速度

$$v_g = \frac{x}{t} = \frac{\lambda^2}{\lambda_p} \cdot \frac{1}{\lambda/c} = \frac{\lambda}{\lambda_p} \cdot c$$
$$= c\sqrt{1 - \left(\frac{f_c}{f}\right)^2} \quad < c$$



frequency dependent velocity
||
frequency dispersion
(周波数分散)

Surface wave (表面波)



, $\left(\frac{\partial}{\partial y} = 0\right.$; the media is uniform in y-direc. $\left.\right)$

A fundamental mode of a surface wave is TM.

Helmholtz eq. in terms of E_z in region "i", $i = 1$ or 2;

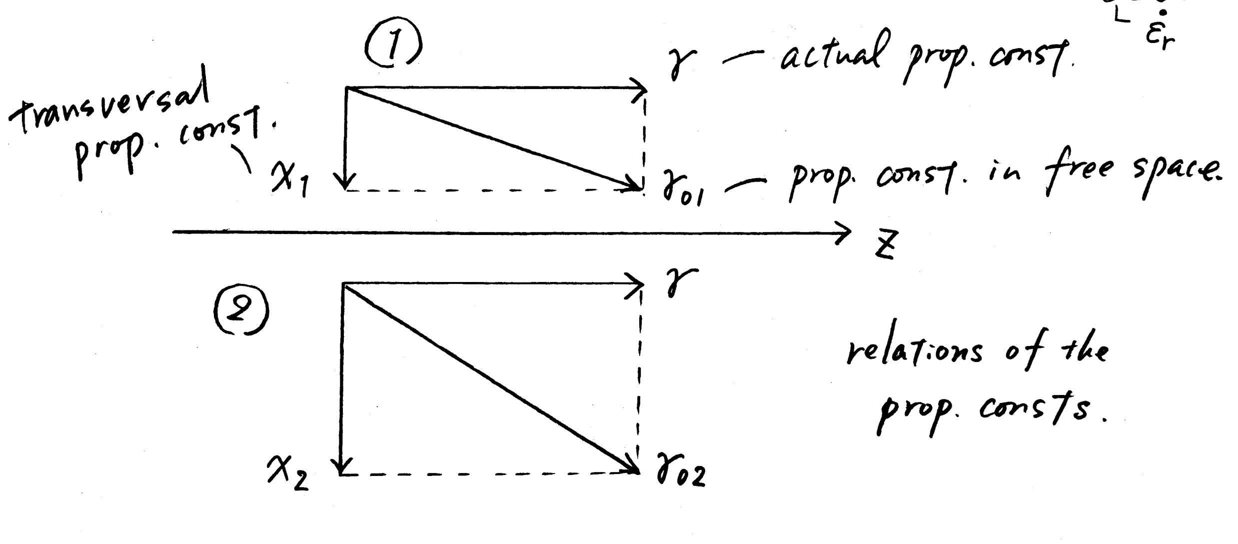
$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + \omega^2 \varepsilon_i \mu_i \right) E_{z_i} = 0 \qquad (60)$$

Consider lossy dielectric media;

$$\nabla \times \mathbb{H} = j\omega\varepsilon \mathbb{E} + \sigma \mathbb{E} = j\omega \underbrace{\varepsilon \left(1 - j\frac{\sigma}{\omega\varepsilon}\right)}_{\dot{\varepsilon}} \mathbb{E}$$

$$\begin{aligned} \dot{\varepsilon} &= \varepsilon' - j\varepsilon'' \\ &= \varepsilon - j\frac{\sigma}{\omega} \\ &= \varepsilon_0 \underbrace{\left(\varepsilon_r - j\frac{\sigma}{\varepsilon_0 \omega}\right)}_{\dot{\varepsilon}_r} \qquad (61) \end{aligned}$$

(Also, "μ" could be lossy.)



relations of the prop. consts.

lossless case,	lossy case.

Assume

$$E_{z_i}(x, z) = E_{zo_i}(x)\, e^{-j\beta z}, \qquad E_{z_i}(x, z) = E_{zo_i}(x)\, e^{-\gamma z} \qquad i = 1, 2 \qquad (62)$$

Helmholtz eq.

$$\Big(\frac{\partial^2}{\partial x^2} - \beta^2 + \underbrace{\omega^2 \varepsilon_i \mu_i}_{k_{o_i}^2}\Big) E_{zo_i} = 0 \qquad \Big(\frac{\partial^2}{\partial x^2} + \gamma^2 + \underbrace{\omega^2 \dot{\varepsilon}_i \mu_i}_{-\gamma_{o_i}^2}\Big) E_{zo_i} = 0 \qquad (63)$$

$\Downarrow$ moreover we rewrite them as $\Downarrow$

$$\Big(\frac{\partial^2}{\partial x^2} \underbrace{- \beta^2 + k_{o_i}^2}_{k_{c_i}^2}\Big) E_{zo_i} = 0, \qquad \Big(\frac{\partial^2}{\partial x^2} + \underbrace{\gamma^2 - \gamma_{o_i}^2}_{-\chi_i^2}\Big) E_{zo_i} = 0 \qquad (64)$$

$\Downarrow$ $\Downarrow$

$$\Big(\frac{\partial^2}{\partial x^2} + k_{c_i}^2\Big) E_{zo_i} = 0, \qquad \Big(\frac{\partial^2}{\partial x^2} - \chi_i^2\Big) E_{zo_i} = 0 \qquad (65)$$

General solution

$$E_{zo_i}(x) = A_i\, e^{+jk_{c_i} x} + B_i\, e^{-jk_{c_i} x} \qquad E_{zo_i}(x) = A_i\, e^{\chi_i x} + B_i\, e^{-\chi_i x} \qquad i = 1, 2 \qquad (66)$$

Fields in each region:

i) In region 1, $E_{z0_1}(x) = \cancel{A_1 e^{\chi_1 x}} + B_1 e^{-\chi_1 x}$ (67)
must be finite as $x \to \infty$, therefore $A_1 = 0$.

ii) In region 2. $E_{z0_2}(x) = A_2 e^{\chi_2 x} + \cancel{B_2 e^{-\chi_2 x}}$ (68)
must be finite as $x \to -\infty$, therefore $B_2 = 0$.

For TM mode,

from (26) (p.44), $$H_y = \frac{-j}{k_{c_i}^2} \omega \dot{\varepsilon}_i \frac{\partial E_{z_i}}{\partial x} \tag{69}$$

from (27), $$E_x = \frac{-j}{k_{c_i}^2} \cdot \beta \frac{\partial E_{z_i}}{\partial x} \tag{70}$$

$\Downarrow$ replace $\begin{cases} j\beta \to \gamma \\ k_{c_i}^2 \to -\chi_i^2 \end{cases}$ (71)

$$\boxed{\begin{aligned} H_y &= \frac{j}{\chi_i^2} \omega \dot{\varepsilon}_i \frac{\partial E_{z_i}}{\partial x} \qquad (72) \\ E_x &= \frac{\gamma}{\chi_i^2} \frac{\partial E_{z_i}}{\partial x} \qquad (73) \end{aligned}}$$

Therefore, in region 1,

$$\begin{cases} E_{z_1}(x, z) = B_1 e^{-\chi_1 x - \gamma z} & (74) \\ H_{y_1}(x, z) = \dfrac{-j\omega \dot{\varepsilon}_1}{\chi_1} B_1 e^{-\chi_1 x - \gamma z} & (75) \\ E_{x_1}(x, z) = \dfrac{-\gamma}{\chi_1} B_1 e^{-\chi_1 x - \gamma z} & (76) \end{cases}$$

In region 2,

$$\begin{cases} E_{z2}(x,z) = A_2\, e^{\chi_2 x - \gamma z} & (77) \\ H_{y2}(x,z) = \dfrac{j\omega\dot{\varepsilon}_2}{\chi_2} A_2\, e^{\chi_2 x - \gamma z} & (78) \\ E_{x2}(x,z) = \dfrac{\gamma}{\chi_2} A_2\, e^{\chi_2 x - \gamma z} & (79) \end{cases}$$

Now, we impose boundary conditions, i.e. the field components tangential to the boundary must be continuous.

At $x=0$, $E_{z1} = E_{z2}$

$$\begin{cases} E_{z1} = B_1 e^{-\gamma z} \\ E_{z2} = A_2 e^{-\gamma z} \end{cases} \Rightarrow B_1 = A_2 = E_{z0}, \qquad (80)$$

also $H_{y1} = H_{y2}$

$$\begin{cases} H_{y1} = -\dfrac{j\omega\dot{\varepsilon}_1}{\chi_1} B_1 \\ H_{y2} = \dfrac{j\omega\dot{\varepsilon}_2}{\chi_2} A_2 \end{cases} \Rightarrow -\frac{\dot{\varepsilon}_1}{\chi_1} = \frac{\dot{\varepsilon}_2}{\chi_2}. \qquad (81)$$

From (81) we get "γ" as follows:

since $\chi_i^2 = -\gamma^2 + \gamma_{0i}^2$,

(81): $$-\frac{\dot{\varepsilon}_1}{\sqrt{-\gamma^2 + \gamma_{01}^2}} = \frac{\varepsilon_2}{\sqrt{-\gamma^2 + \gamma_{02}^2}},$$

or

$$\frac{\dot{\varepsilon}_1^2}{\gamma_{01}^2 - \gamma^2} = \frac{\dot{\varepsilon}_2^2}{\gamma_{02}^2 - \gamma^2},$$

consider that $\omega^2 \dot{\varepsilon}_i \mu_i = -\gamma_{0i}^2$,

$\times\omega^4$; $$\frac{\omega^4 \dot{\varepsilon}_1^2 \mu_1^2}{\gamma_{01}^2 - \gamma^2} \cdot \frac{1}{\mu_1^2} = \frac{\omega^4 \dot{\varepsilon}_2^2 \mu_2^2}{\gamma_{02}^2 - \gamma^2} \cdot \frac{1}{\mu_2^2}$$

$$\frac{\gamma_{01}^4}{\gamma_{01}^2 - \gamma^2} \cdot \frac{1}{\mu_1^2} = \frac{\gamma_{02}^4}{\gamma_{02}^2 - \gamma^2} \cdot \frac{1}{\mu_2^2}$$

$$\frac{\gamma_{01}^4}{\mu_1^2} (\gamma_{02}^2 - \underline{\gamma^2}) = \frac{\gamma_{02}^4}{\mu_2^2} (\gamma_{01}^2 - \underline{\underline{\gamma^2}})$$

$$\left(\frac{\gamma_{02}^4}{\mu_2^2} - \frac{\gamma_{01}^4}{\mu_1^2} \right) \gamma^2 = \frac{\gamma_{02}^4}{\mu_2^2} \gamma_{01}^2 - \frac{\gamma_{01}^4}{\mu_1^2} \gamma_{02}^2$$

$$\gamma^2 = \frac{\dfrac{\gamma_{02}^4 \gamma_{01}^2}{\mu_2^2} - \dfrac{\gamma_{01}^4 \gamma_{02}^2}{\mu_1^2}}{\dfrac{\gamma_{02}^4}{\mu_2^2} - \dfrac{\gamma_{01}^4}{\mu_1^2}}$$

$$= \gamma_{01}^2 \gamma_{02}^2 \cdot \frac{\dfrac{\gamma_{02}^2}{\mu_2^2} - \dfrac{\gamma_{01}^2}{\mu_1^2}}{\dfrac{\gamma_{02}^4}{\mu_2^2} - \dfrac{\gamma_{01}^4}{\mu_1^2}} \quad \begin{array}{l} \left(\times \dfrac{\mu_1^2}{\gamma_{01}^4} \right) \\ \left(\times \dfrac{\mu_1^2}{\gamma_{01}^4} \right) \end{array}$$

$$= \gamma_{02}^2 \cdot \frac{\dfrac{\gamma_{02}^2}{\gamma_{01}^2} \cdot \dfrac{\mu_1^2}{\mu_2^2} - 1}{\dfrac{\gamma_{02}^4}{\gamma_{01}^4} \cdot \dfrac{\mu_1^2}{\mu_2^2} - 1} \quad , \qquad (82)$$

For the special case of $\mu_1 = \mu_2$,

$$\gamma^2 = \gamma_{02}^2 \frac{\gamma_{02}^2 \gamma_{01}^2 - \gamma_{01}^4}{\gamma_{02}^4 - \gamma_{01}^4}$$

$$= \gamma_{01}^2 \gamma_{02}^2 \frac{\gamma_{02}^2 - \gamma_{01}^2}{\gamma_{02}^4 - \gamma_{01}^4}$$

$$= \frac{\gamma_{01}^2 \gamma_{02}^2}{\gamma_{01}^2 + \gamma_{02}^2} \quad , \qquad (83)$$

Once γ is known, we get χ_i as

$$\chi_i = \sqrt{\gamma_{oi}^2 - \gamma^2} \quad , \quad i = 1, 2 \tag{84}$$

where
$$\gamma_{oi}^2 = -\omega^2 \dot{\varepsilon}_i \mu_i \quad , \tag{85}$$

$$\dot{\varepsilon}_i = \varepsilon_i \left(1 - j \frac{\sigma_i}{\omega \varepsilon_i}\right) \tag{86}$$

(see p.14, (51) or p51, (61))

In summary, the field components are

in region 1,

$$\begin{cases} E_{z_1}(x, z) = E_{z0}\, e^{-\chi_1 x - \gamma z} & (87) \\ H_{y_1}(x, z) = -\dfrac{j\omega\dot{\varepsilon}_1}{\chi_1} E_{z0}\, e^{-\chi_1 x - \gamma z} & (88) \\ E_{x_1}(x, z) = -\dfrac{\gamma}{\chi_1} E_{z0}\, e^{-\chi_1 x - \gamma z} \quad , & (89) \end{cases}$$

and in region 2,

$$\begin{cases} E_{z2}(x, z) = E_{z0}\, e^{\chi_2 x - \gamma z} & (90) \\ H_{y2}(x, z) = \dfrac{j\omega\varepsilon_2}{\chi_2} E_{z0}\, e^{\chi_2 x - \gamma z} & (91) \\ E_x(x, z) = \dfrac{\gamma}{\chi_2} E_{z0}\, e^{\chi_2 x - \gamma z} & (92). \end{cases}$$

Surface wave

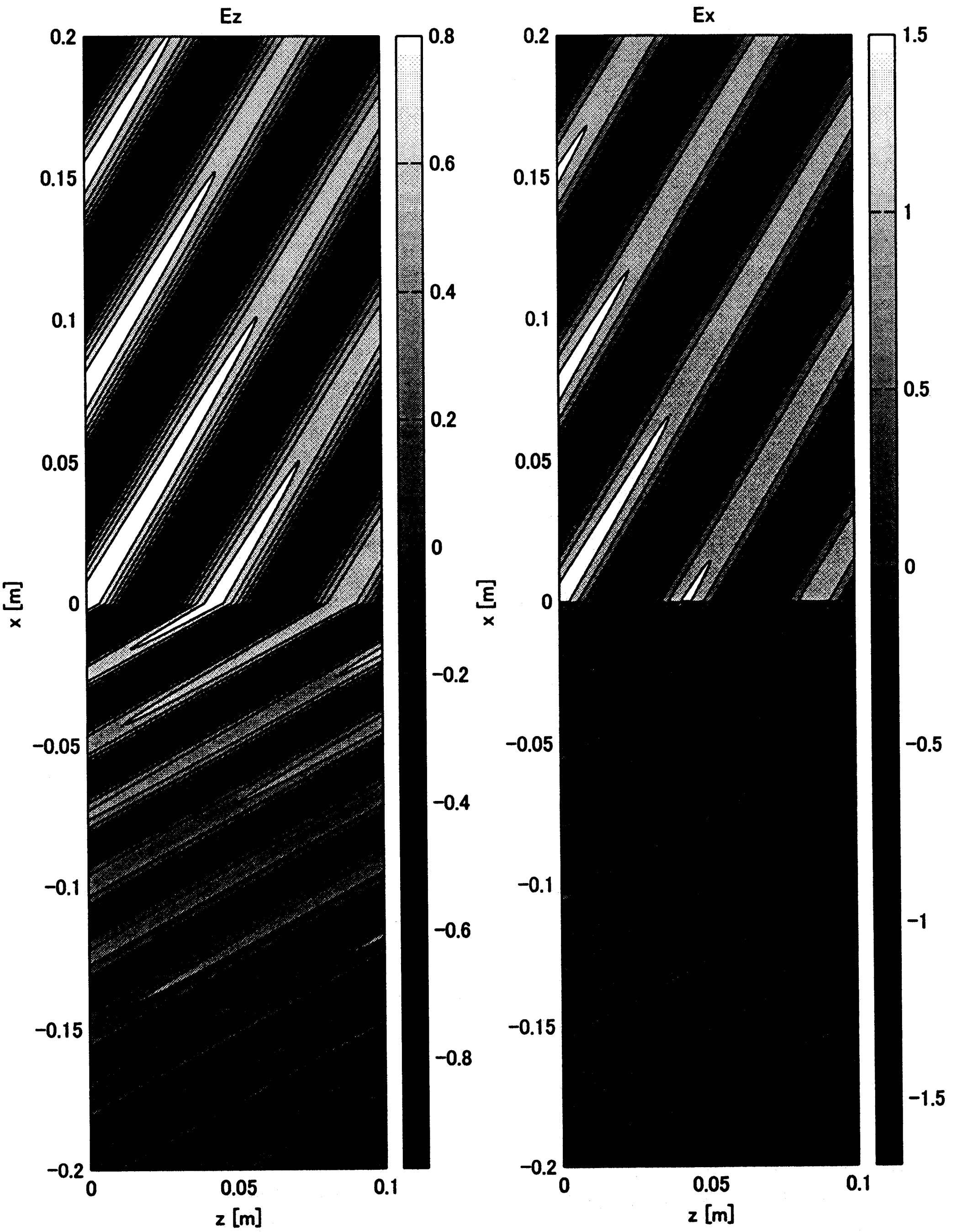
P.56 (87) (89) (90) (92)

$f = 8 \times 10^9$ Hz

$$\begin{cases} \varepsilon_{r_1} = 1, \ \mu_{r_1} = 1, \ \sigma_1 = 0.01 \text{ S/m} \\ \varepsilon_{r_2} = 3, \ \mu_{r_2} = 1, \ \sigma_2 = 0.1 \text{ S/m} \end{cases} \Rightarrow \begin{cases} \gamma = 2.58 + j1.45 \times 10^2 \\ \chi_1 = 0.703 - j\,83.8 \\ \chi_2 = 1.12 \times 10^1 + j\,2.52 \times 10^2 \end{cases}$$

γ is obtained by p55 (82) or (83),

χ_i, $i = 1, 2$ are from (84).



Now we consider a more special case

region 1 : air , $\varepsilon_1 = \varepsilon_0$, $\mu_1 = \mu_0$

region 2 : metal (non-magnetic)

$\omega \varepsilon_2 \ll \sigma_2$, $\mu_2 = \mu_0$

In this case,

$$\gamma_{01} = j\omega\sqrt{\varepsilon_0 \mu_0} = jk_0 = j\frac{\omega}{c_0} \qquad (93)$$

k_0 : phase constant in free space

c_0 : speed of light in free space.

$$\gamma_{02} = j\omega\sqrt{\varepsilon_2\left(1 - j\frac{\sigma_2}{\omega\varepsilon_2}\right)\cdot\mu_0}$$

$$= j\omega\sqrt{\left(\varepsilon_2 - j\frac{\sigma_2}{\omega}\right)\mu_0} \qquad \left(\sqrt{j} = \sqrt{e^{j\frac{\pi}{2}}} = e^{j\frac{\pi}{4}} = \frac{1+j}{\sqrt{2}}\right)$$

$$= j\omega\sqrt{-j\frac{\sigma_2}{\omega}\mu_0}$$

$$= \omega\sqrt{j\frac{\sigma_2}{\omega}\mu_0} = \sqrt{j\omega\sigma_2\mu_0}$$

$$= \sqrt{\frac{\omega\sigma_2\mu_0}{2}}\cdot(1+j) \qquad (94)$$

$$\gamma = \frac{\gamma_{01}\cdot\gamma_{02}}{\sqrt{\gamma_{01}^2 + \gamma_{02}^2}} = \frac{\gamma_{01}}{\sqrt{1 + \frac{\gamma_{01}^2}{\gamma_{02}^2}}} = \frac{jk_0}{\sqrt{1 + \frac{-k_0^2}{j\omega\sigma_2\mu_0}}}$$

$$= \frac{jk_0}{\sqrt{1 + j\frac{\omega\varepsilon_0}{\sigma_2}}} \qquad (95)$$

$$\chi_2 = \sqrt{\gamma_{02}^2 - \gamma^2}$$
$$\cong \sqrt{\gamma_{02}^2}$$
$$= \gamma_{02}$$
$$= \sqrt{\frac{\omega \sigma_2 \mu_0}{2}} (1+j) \qquad (96)$$

From boundary condition (81),

$$\chi_1 = -\chi_2 \frac{\varepsilon_1}{\dot{\varepsilon}_2}$$
$$= -\chi_2 \frac{\omega^2 \varepsilon_1 \mu_0}{\omega^2 \dot{\varepsilon}_2 \mu_0} = -\chi_2 \frac{\gamma_{01}^2}{\gamma_{02}^2}$$
$$= -\frac{\gamma_{01}^2}{\gamma_{02}} = -\frac{-k_0^2}{\sqrt{j\omega\sigma_2\mu_0}}$$
$$= k_0 \frac{\omega\sqrt{\varepsilon_0\mu_0}}{\sqrt{j\omega\sigma_2\mu_0}} = k_0 \sqrt{\frac{\omega\varepsilon_0}{j\sigma_2}}$$
$$= k_0 \sqrt{\frac{\omega\varepsilon_0}{\sigma_2}} \cdot \frac{\sqrt{2}}{1+j}$$
$$= k_0 \sqrt{\frac{\omega\varepsilon_0}{\sigma_2}} \frac{\sqrt{2}(1-j)}{(1+j)(1-j)}$$
$$= k_0 \sqrt{\frac{\omega\varepsilon_0}{2\sigma_2}} (1-j) \qquad (97)$$

$$\delta = \frac{1}{Re\{\chi_2\}} = \sqrt{\frac{2}{\omega\sigma_2\mu_0}} \quad : \text{ skin depth} \qquad (98)$$

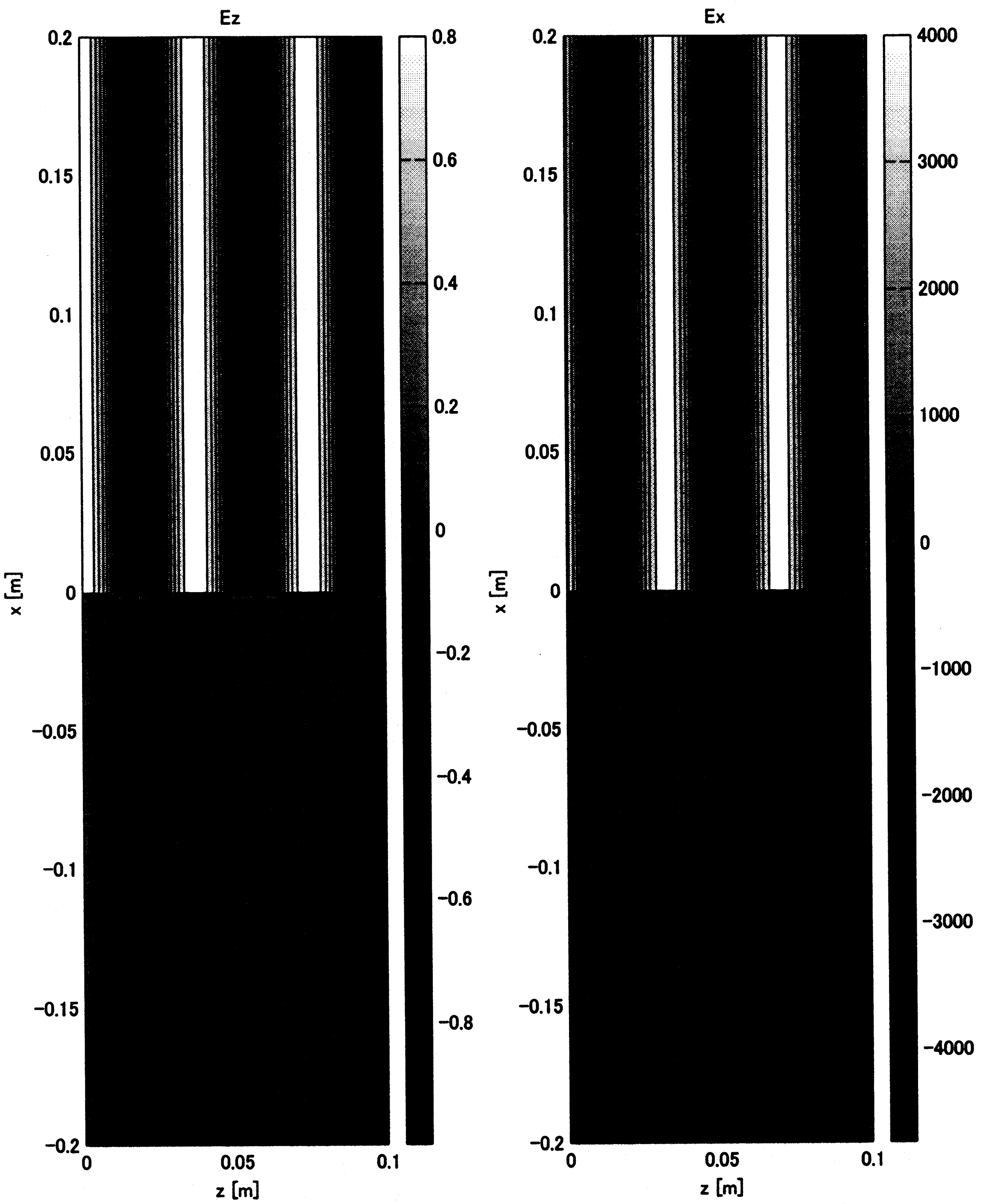
The field decays in the conductor by a factor of $\frac{1}{e}$ as it penetrates by the distance δ.

Surface wave for a good conductor

$f = 8 \times 10^9\ Hz$

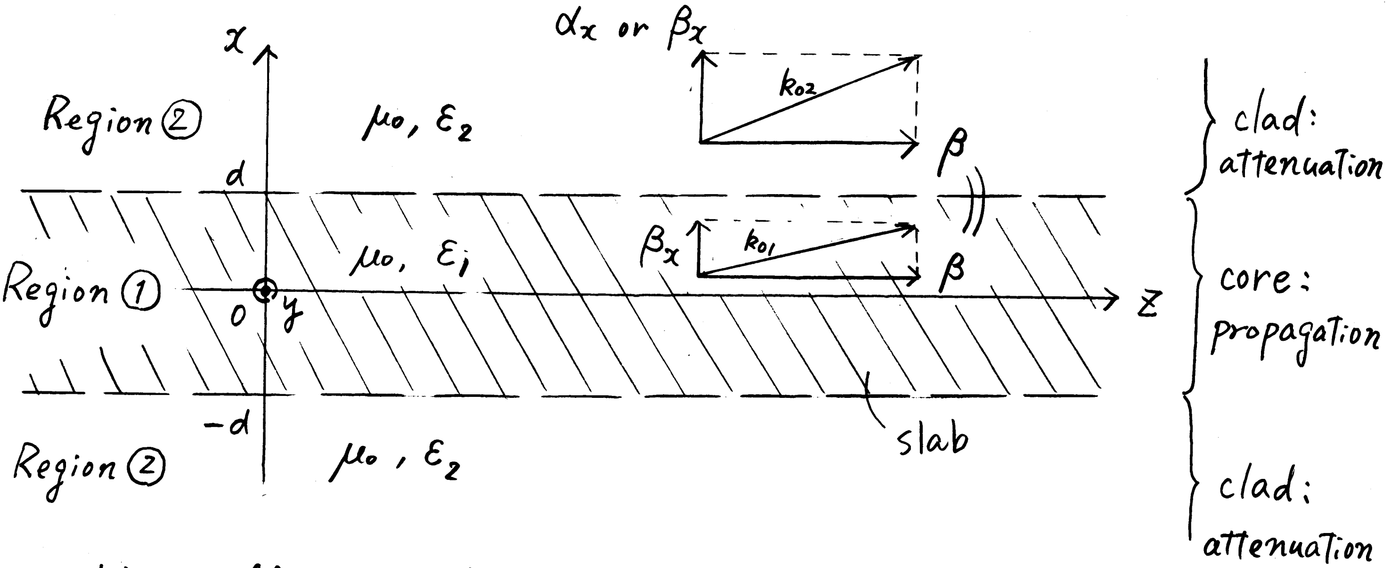
$$\begin{cases} \varepsilon_{r_1} = 1,\ \mu_{r_1} = 1,\ \sigma_1 = 0 \quad (\text{air}) \\ \varepsilon_{r_2} = 1,\ \mu_{r_2} = 1,\ \underline{\underline{\sigma_2 = 1 \times 10^7\ S/m}} \end{cases} \Rightarrow \begin{cases} \gamma = 3.73 + j1.67 \times 10^2 \\ \chi_1 = 0.0250 - j0.0250 \\ \chi_2 = 5.62 \times 10^5 + j5.62 \times 10^5 \end{cases}$$

~Ag, Cu etc.



Dielectric slab waveguide

This type of waveguide is considered as a two-dimensional approximation of an optical fiber.



Use different notations:

β : phase const. of the guided wave

β_x : transversal phase const.
(eq. to "k_c"; cut-off wave number)

α_x : transversal attenuation const.

$k_0 = \omega\sqrt{\varepsilon_0 \mu_0}$: free space wave number.
(or phase const. in free space)

$\Downarrow$

$$k_{01} = \omega\sqrt{\varepsilon_1 \mu_0} \qquad (99)$$

$$k_{02} = \omega\sqrt{\varepsilon_2 \mu_0} \qquad (100)$$

Slab waveguide	TE	E_y : continuous (fundamental mode)
	TM	E_y : discontinuous.
c.f. Surface wave	TE	large attenuation.
	TM	smaller energy (fundamental mode)

We now consider TE mode propagation with the field components

$$\begin{cases} \mathbb{E} = (0,\ E_y,\ 0) \\ \mathbb{H} = (H_x,\ 0,\ H_z)\ . \end{cases} \tag{101}$$

Maxwell's equations

$$\nabla \times \mathbb{E} = -\mu \frac{\partial \mathbb{H}}{\partial t} = -j\omega\mu \mathbb{H} \tag{102}$$

$$\nabla \times \mathbb{H} = \varepsilon \frac{\partial \mathbb{E}}{\partial t} = j\omega\varepsilon \mathbb{E} \tag{103}$$

are written down as

$$\begin{cases} \cancel{\dfrac{\partial E_z}{\partial y}} - \dfrac{\partial E_y}{\partial z} = -j\omega\mu H_x & (104a) \\ \dfrac{\partial E_x}{\partial z} - \dfrac{\partial E_z}{\partial x} = -j\omega\mu H_y & (104b) \\ \dfrac{\partial E_y}{\partial x} - \cancel{\dfrac{\partial E_x}{\partial y}} = -j\omega\mu H_z & (104c) \\ \cancel{\dfrac{\partial H_z}{\partial y}} - \dfrac{\partial H_y}{\partial z} = j\omega\varepsilon E_x & (104d) \\ \dfrac{\partial H_x}{\partial z} - \dfrac{\partial H_z}{\partial x} = j\omega\varepsilon E_y & (104e) \\ \dfrac{\partial H_y}{\partial x} - \cancel{\dfrac{\partial H_x}{\partial y}} = j\omega\varepsilon E_z & (104f) \end{cases}$$

Since $\dfrac{\partial}{\partial y} = 0$, these can be reduced to

$$\left\{\begin{array}{ll} \dfrac{\partial E_y}{\partial z} = j\omega\mu H_x & (105a) \\ \dfrac{\partial E_y}{\partial x} = -j\omega\mu H_z & (105b) \\ \dfrac{\partial H_x}{\partial z} - \dfrac{\partial H_z}{\partial x} = j\omega\varepsilon E_y & (105c) \end{array}\right.$$

substitute.

From those, we get the <u>Helmholtz equation</u> by eliminating e.g. H_x and H_z ;

(105a) $$H_x = \frac{1}{j\omega\mu}\frac{\partial E_y}{\partial z}$$

(105b) $$H_z = -\frac{1}{j\omega\mu}\frac{\partial E_y}{\partial x}$$,

substituting into (105c),

$$\frac{\partial}{\partial z}\left(\frac{1}{j\omega\mu}\frac{\partial E_y}{\partial z}\right) - \frac{\partial}{\partial x}\left(-\frac{1}{j\omega\mu}\frac{\partial E_y}{\partial x}\right) = j\omega\varepsilon E_y$$

or

$$\underline{\frac{\partial^2 E_y}{\partial z^2} + \frac{\partial^2 E_y}{\partial x^2} + \omega^2\varepsilon\mu E_y = 0} \qquad (106)$$

(same eq. for H_z)

As usual, we can assume that

$$E_y(x,z) = E_{y0}(x)\, e^{-j\beta z} , \qquad (107)$$

and then (106) is rewritten by

$$\frac{\partial^2 E_{y0}}{\partial x^2} - \beta^2 E_{y0} + k_0^2 E_{y0} = 0, \quad k_0^2 = \omega^2\varepsilon\mu \qquad (108)$$

or

$$\frac{\partial^2 E_{y0}}{\partial x^2} + \underline{\underline{(k_0^2 - \beta^2)}} E_{y0} = 0 \, , \qquad (109)$$

$$k_0^2 - \beta^2 = \beta_x^2 \qquad (110)$$

$$\Big\downarrow \text{ extention}$$

$$j\beta_x \longrightarrow \gamma_x = \alpha_x + j\beta_x \qquad (111)$$

$$\text{or} \quad \underline{\underline{\beta_x^2 \longrightarrow -\gamma_x^2}} \qquad (112)$$

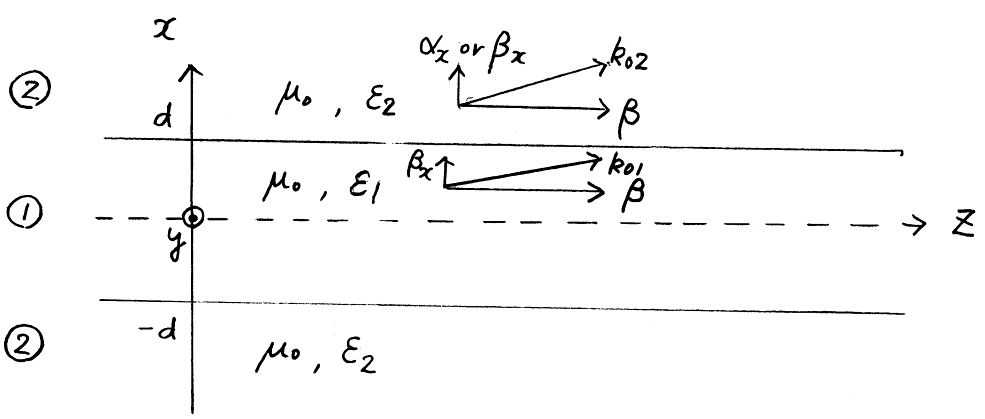
then the Helmholtz wave equation is extended to a lossy case as

$$\frac{\partial^2 E_{y0}(x)}{\partial x^2} - \gamma_x^2 E_{y0}(x) = 0 \, . \qquad (113)$$

The general solution of this equation is readily obtained as

$$\begin{aligned} E_{y0}(x) &= A e^{\gamma_x x} + B e^{-\gamma_x x} \\ &= A e^{(\alpha_x + j\beta_x) x} + B e^{-(\alpha_x + j\beta_x) x} \\ &= A e^{\alpha_x x}(\cos\beta_x x + j \sin\beta_x x) \\ &\quad + B e^{-\alpha_x x}(\cos\beta_x x - j \sin\beta_x x) \, . \end{aligned} \qquad (114)$$

Dielectric slab waveguide (2D approx. of optical fiber)



TE mode, symmetric mode (TE_0, TE_2, ...) (E_y, H_x, H_z only)

We assume

$$E_y = \begin{cases} A \cos \beta_x x \, e^{-j\beta z} & |x| < d \quad (1) \text{ ① core} \\ B \, e^{-\alpha_x x} \, e^{-j\beta z} & x > d \quad (2) \text{ ② clad} \\ B \, e^{\alpha_x x} \, e^{-j\beta z} & x < -d \quad (3) \text{ ② clad} \end{cases}$$

$\nabla \times \mathbb{E} = -j\omega\mu_0 \mathbb{H}$, $\left(d\mathcal{E} = -j\omega\mu_0 * \mathcal{H} \right) \rightarrow H_z = \dfrac{1}{-j\omega\mu_0} \dfrac{\partial E_y}{\partial x}$

$$H_z = \begin{cases} -\dfrac{j\beta_x}{\omega\mu_0} A \sin \beta_x x \, e^{-j\beta z} & |x| < d \quad (4) \text{ ① core} \\ -\dfrac{j\alpha_x}{\omega\mu_0} B \, e^{-\alpha_x x} \, e^{-j\beta z} & x > d \quad (5) \text{ ② clad} \\ \dfrac{j\alpha_x}{\omega\mu_0} B \, e^{\alpha_x x} \, e^{-j\beta z} & x < -d \quad (6) \text{ ② clad} \end{cases}$$

Boundary condition:
Continuity of fields at $x = d$:

$$\left. \frac{E_y}{H_z} \right|_{x=d} = \frac{\overset{(1)}{A \cos \beta_x d \, e^{-j\beta z}}}{\underset{(4)}{-\dfrac{j\beta_x}{\omega\mu_0} A \sin \beta_x d \, e^{-j\beta z}}} = \frac{\overset{(2)}{B \, e^{-\alpha_x d} \, e^{-j\beta z}}}{\underset{(5)}{-\dfrac{j\alpha_x}{\omega\mu_0} B \, e^{-\alpha_x d} \, e^{-j\beta z}}}$$

then we get

$$\frac{1}{\beta_x \tan \beta_x d} = \frac{1}{\alpha_x}, \text{ or, } \tan \beta_x d = \frac{\alpha_x}{\beta_x} \quad (7)$$

Propagation Constant Relation

Helmholtz equation :

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right) E_y + k_{0i}^2 E_y = 0 \quad , \quad k_{0i}^2 = \omega^2 \varepsilon_i \mu_0 \ (i = 1, 2) \qquad (8)$$

① For $|x| < d$, from the relation between the transversal and the longitudinal wave numbers,

$$\underbrace{\beta_x^2}_{\text{trans.}} + \underbrace{\beta^2}_{\text{longi.}} = k_{01}^2 \quad , \quad k_{01}^2 = \omega^2 \varepsilon_1 \mu_0 \quad . \qquad (9)$$

② For $x > d$,

ATT!

$$\ominus \underbrace{\alpha_x^2}_{\text{trans.}} + \underbrace{\beta^2}_{\text{longi.}} = k_{02}^2 \qquad (10) \longleftarrow$$

ATT.!

$$\gamma_x = \alpha_x + j\beta_x$$

$$\therefore -\gamma_x^2 = -(\alpha_x + j\beta_x)^2$$

Substituted into β_x^2 to get "$-\alpha_x^2$"

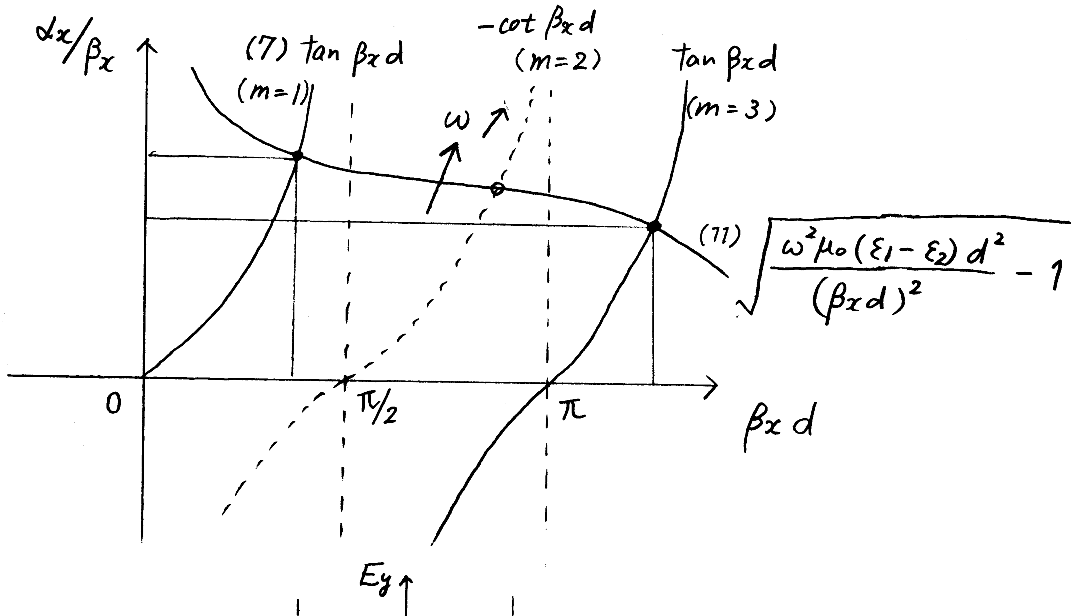
(9) − (10) to eliminate β

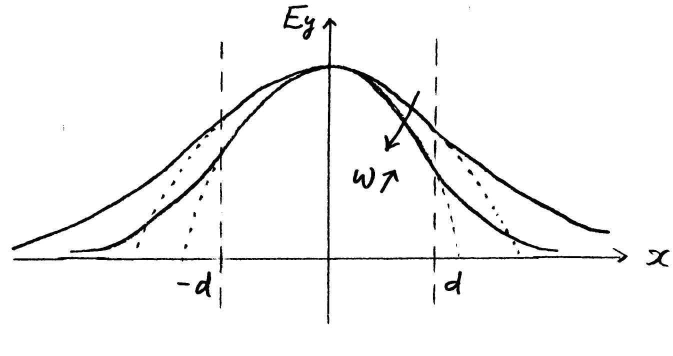
$$\beta_x^2 + \alpha_x^2 = k_{01}^2 - k_{02}^2$$

$$= \omega^2 \mu_0 (\varepsilon_1 - \varepsilon_2) \quad : \text{ determinantal equation}$$

or

$$1 + \frac{\alpha_x^2}{\beta_x^2} = \frac{\omega^2 \mu_0 (\varepsilon_1 - \varepsilon_2)}{\beta_x^2} \quad , \text{ or } \quad \frac{\alpha_x}{\beta_x} = \sqrt{\frac{\omega^2 \mu_0 (\varepsilon_1 - \varepsilon_2)}{\beta_x^2} - 1} \qquad (11)$$





Dielectric slab waveguide

$d = 0.2\ \mu m$, $\left(\begin{array}{l} \varepsilon_1 = 4, \\ \varepsilon_2 = 1 \end{array} \right.$ $\left(\begin{array}{l} \text{Solve (7)=(11) (p63, 64), i.e.} \\ \tan(\beta_x \cdot d) = \sqrt{\dfrac{\omega^2 \mu_0 (\varepsilon_1 - \varepsilon_2) d^2}{(\beta_x \cdot d)^2} - 1} \quad (12) \end{array} \right.$

$\Downarrow$

$\beta_x \cdot d = 0.8518\ (1.55\,\mu m)$
$\qquad\quad\ \ 0.9504\ (1.3\,\mu m)$

$\Downarrow$

calculate $\left\{ \begin{array}{ll} k_{01},\ k_{02} & (8) \\ \beta & (9) \\ \alpha_x & (10) \end{array} \right.$ $\Rightarrow$ Solution of the form p63 (1), (2)