Metamaterial, 人工物質 (meta／人工の, 自然界に存在しない)

artificial, non-existant in nature.

Original paper:

V.G. Veselago, "The electrodynamics of substances with simultaneously negative values of ϵ and μ", Soviet physics uspekhi, vol. 10, no. 4, pp. 509-514, 1968.

"Simultaneously negative ϵ, μ"

or

"Negative index material (NIM)"

「負の屈折率をもつ物質」

For simplicity, isotropic substances (等方性物質);

$$k^2 = \frac{\omega^2}{c^2} n^2$$

where $k = |\mathbf{k}|$: phase constant / wave number

$\mathbf{k}$: wave vector

c : speed of light

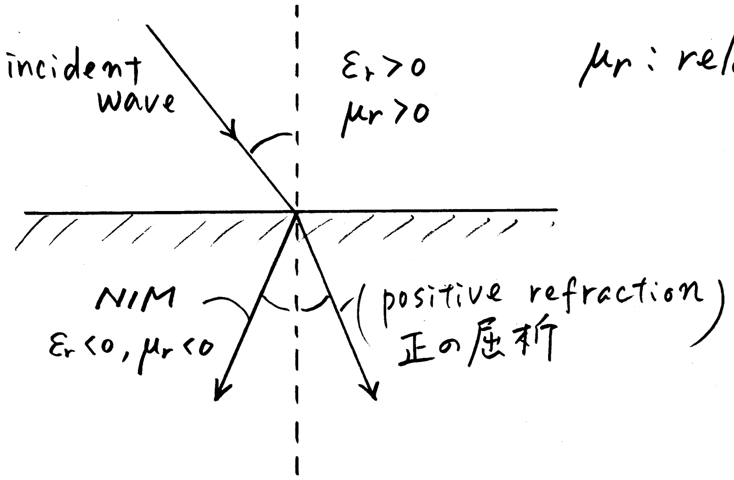
n : refractive index

ω : angular frequency

$$n^2 = \varepsilon_r \mu_r$$

ε_r : relative permittivity (dielectric constant)

μ_r : relative permeability



For a negative index material (NIM), refractive index can be interpreted as

$$n = \sqrt{\varepsilon_r'} \cdot \sqrt{\mu_r'}$$

when $\varepsilon_r' = -\varepsilon_r$, $\mu_r' = -\mu_r$, $\varepsilon_r, \mu_r > 0$,

$$
\begin{aligned}
n &= \sqrt{-\varepsilon_r} \cdot \sqrt{-\mu_r} \\
&= i\sqrt{\varepsilon_r} \cdot i\sqrt{\mu_r} \\
&= -\sqrt{\varepsilon_r}\sqrt{\mu_r}
\end{aligned}
$$

for rigorous proof, need

negative sign ← causality / 因果律

in Maxwell eqs.

For a monochromatic wave of angular frequency ω,

$$
\begin{cases}
\mathbb{E} \xrightarrow{\text{replace}} \mathbb{E}\, e^{i(\mathbb{k}\cdot\mathbb{r} - \omega t)} & \mathbb{k} = (k_x, k_y, k_z) \\
\mathbb{H} \longrightarrow \mathbb{H}\, e^{i(\mathbb{k}\cdot\mathbb{r} - \omega t)} & \mathbb{r} = (x, y, z)
\end{cases}
$$

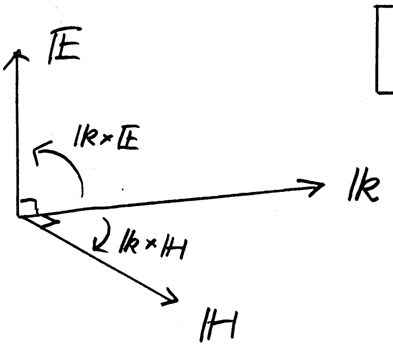
,

Maxwell's equations are replaced as

$$
\begin{cases}
\nabla \times \mathbb{E} = -\dfrac{1}{c}\dfrac{\partial \mathbb{B}}{\partial t} \longrightarrow i\,\mathbb{k} \times \mathbb{E} = i\dfrac{\omega}{c}\mathbb{B} = i\dfrac{\omega}{c}\mu_0\mu_r\mathbb{H} \\
\nabla \times \mathbb{H} = \dfrac{1}{c}\dfrac{\partial \mathbb{D}}{\partial t} \longrightarrow i\,\mathbb{k} \times \mathbb{H} = -i\dfrac{\omega}{c}\mathbb{D} = -i\dfrac{\omega}{c}\varepsilon_0\varepsilon_r\mathbb{E} \\
\mathbb{B} = \mu_0\mu_r\mathbb{H} \\
\mathbb{D} = \varepsilon_0\varepsilon_r\mathbb{E}
\end{cases}
$$

$$\begin{cases} \mathbb{k} \times \mathbb{E} = \frac{\omega}{c} \mu_0 \mu_r \mathbb{H} \\ \mathbb{k} \times \mathbb{H} = -\frac{\omega}{c} \varepsilon_0 \varepsilon_r \mathbb{E} \end{cases}$$

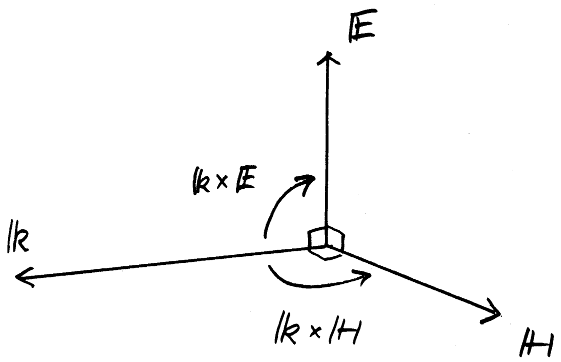
□ If $\varepsilon_r > 0$, $\mu_r > 0$,



right-handed triplet

(3要素)

□ If $\varepsilon_r < 0$, $\mu_r < 0$,



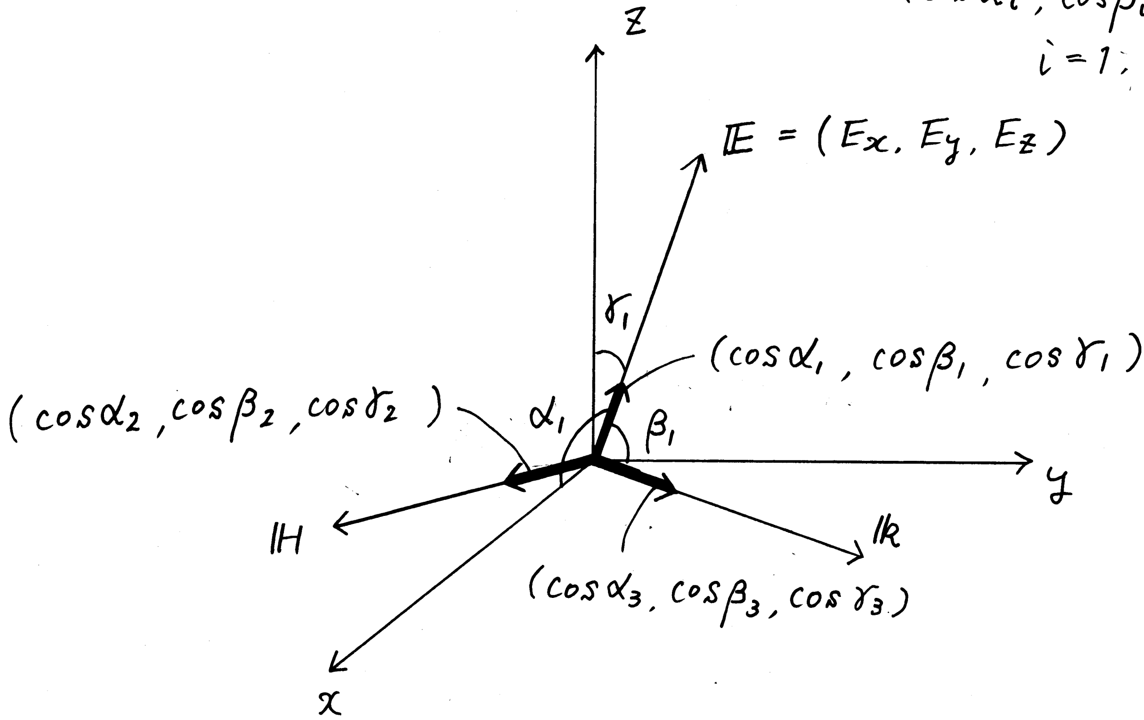
left-handed triplet

Orthogonality, rightness factor

For the vectors $\mathbb{E}$, $\mathbb{H}$ and $\mathbb{k}$, define direction cosine (方向余弦)

$(\cos\alpha_i, \cos\beta_i, \cos\gamma_i)$

$i = 1, 2, 3$



$$(\cos\alpha_1, \cos\beta_1, \cos\gamma_1) = \left(\frac{E_x}{|\mathbb{E}|}, \frac{E_y}{|\mathbb{E}|}, \frac{E_z}{|\mathbb{E}|}\right), \quad \text{etc.}$$

$$\Downarrow$$

$$\cos^2\alpha_1 + \cos^2\beta_1 + \cos^2\gamma_1 = 1$$

$$G = \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{pmatrix}$$

Define a rightness factor "p" as the determinant of G

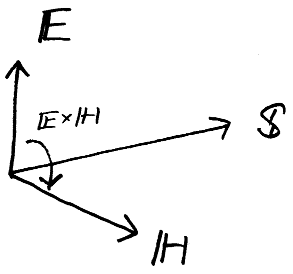
$$p = |G| = \begin{vmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{vmatrix}$$

then

$$\begin{cases} \text{right-hand set of } \mathbb{E}, \mathbb{H}, \mathbb{k} \longrightarrow p = 1 \\ \text{left} \quad \text{〃} \longrightarrow p = -1 \end{cases}$$

Energy flux, Poynting vector

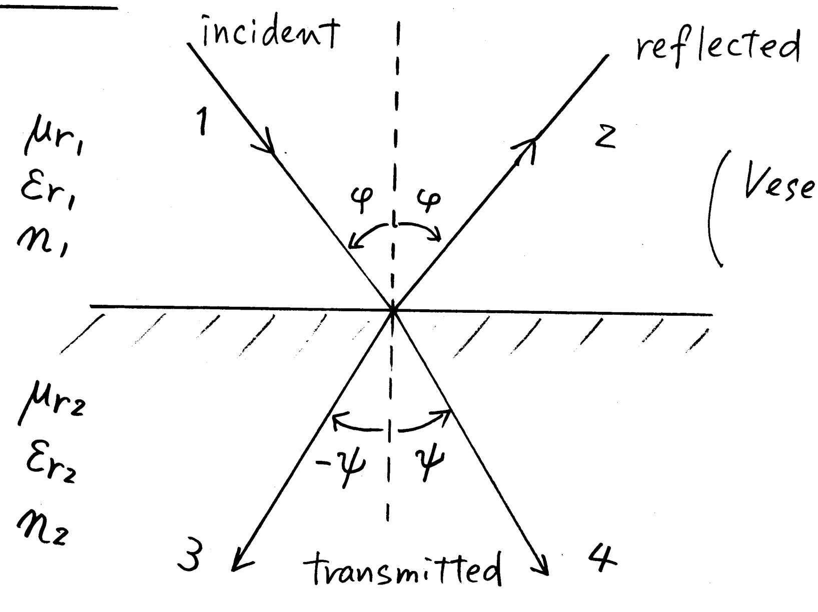
$$\mathbb{S} = \mathbb{E} \times \mathbb{H}$$



$\left\{ \begin{array}{l} \text{right-hand set of } \mathbb{E}, \mathbb{H}, \mathbb{k} \longrightarrow \mathbb{S}, \mathbb{k} : \text{same direction} \\ \text{left} \quad \text{"} \quad \longrightarrow \mathbb{S}. \mathbb{k} : \text{opposite "} \end{array} \right.$

↑ for this case, we have a <u>negative velocity</u>.

Snell's law

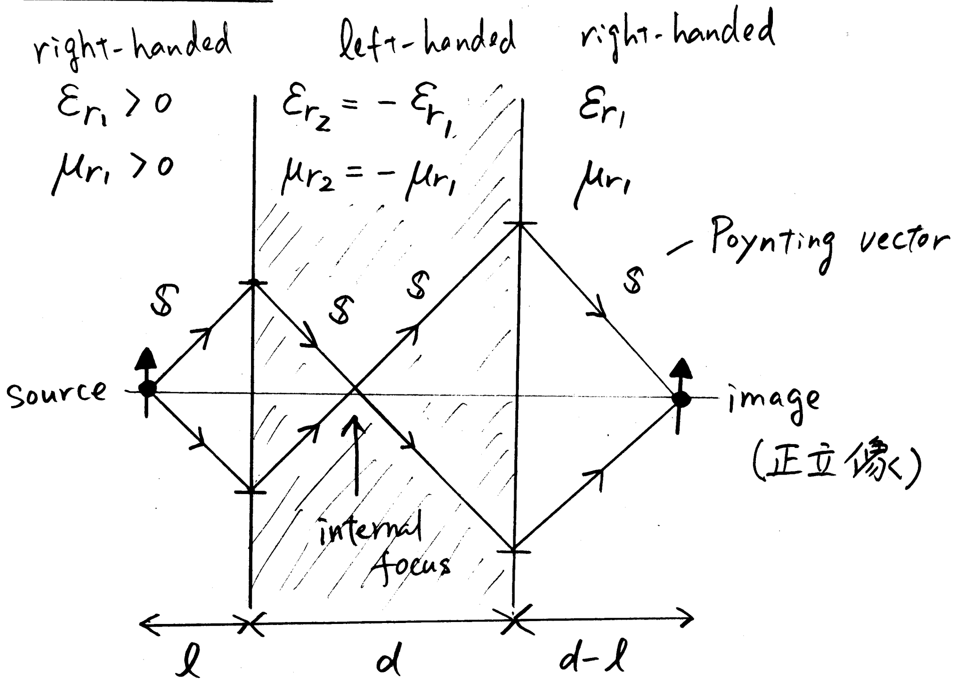


(Veselago, 1968, Fig. 3)

$$\frac{\sin\varphi}{\sin\psi} = \frac{n_2}{n_1} = \frac{p_2}{p_1} \left| \sqrt{\frac{\varepsilon_{r2}\,\mu_{r2}}{\varepsilon_{r1}\,\mu_{r1}}} \right|$$

p_1, p_2 : rightness factor of the media 1, 2

Veselago lens



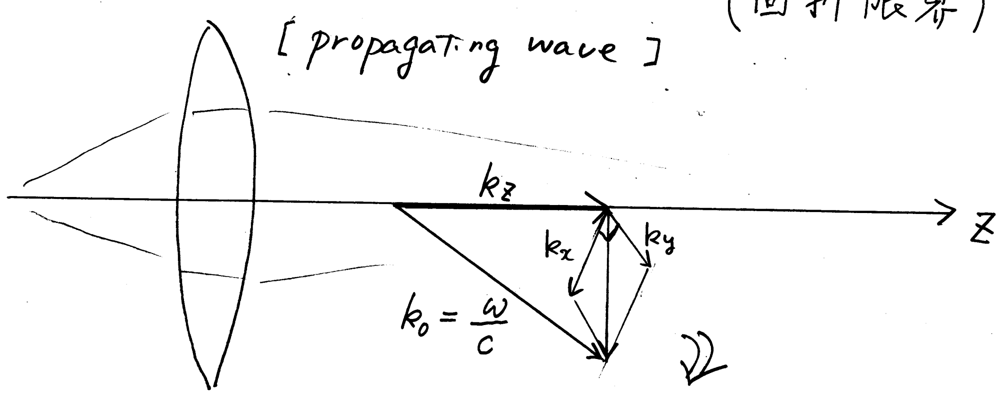
See also Pendry, MTT 47, p2075, 1999

Superlens — perfect lens for sub-diffraction imaging

original paper :

J. B. Pendry, "Negative refraction makes a perfect lens", Phys. rev. lett., vol. 85, no. 18, pp3966−3969, 2000.

Conventional optical lenses have "<u>sub-diffraction limit</u>".
(回折限界)



$$k_x^2 + k_y^2 + k_z^2 = \frac{\omega^2}{c^2}$$

wave vector $\mathbb{k} = (k_x, k_y, k_z)$

ω : angular freq.
c : speed of light

The electric component is expanded into Fourier series

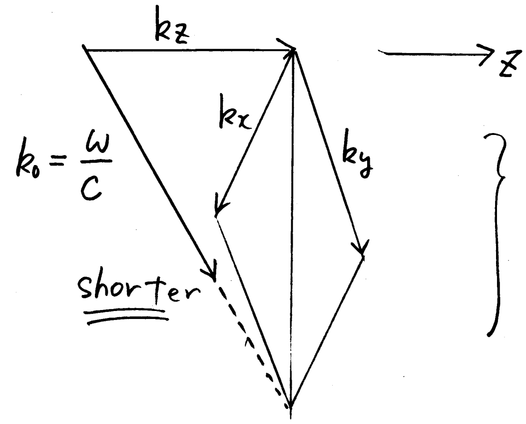
$$\mathbb{E}(\mathbb{r}, t) = \sum_{\sigma, k_x, k_y} \mathbb{E}_\sigma(k_x, k_y)\, e^{i(\mathbb{k}\cdot\mathbb{r} - \omega t)} \quad , (\sigma : \text{mode index})$$

Each component of the Fourier series is collected by the lens into a focal point.

propagating wave : $k_z = \sqrt{\frac{\omega^2}{c^2} - k_x^2 - k_y^2}$, $\frac{\omega^2}{c^2} > k_x^2 + k_y^2$

However for larger values of k_x and k_y ;

[evanescent wave]



Such waves decay exponentially with z.

evanescent wave:
$$k_z = i\sqrt{k_x^2 + k_y^2 - \frac{\omega^2}{c^2}} \quad , \quad \frac{\omega^2}{c^2} < k_x^2 + k_y^2$$

Propagating waves are limitted to the case of

$$k_x^2 + k_y^2 < \frac{\omega^2}{c^2}$$

Maximum resolution in the image can never be greater than

$$\Delta \approx \frac{2\pi}{k_{max}} = \frac{2\pi c}{\omega} = \lambda \quad \text{(wave length)}$$

$$k_{max} \cong \sqrt{k_x^2 + k_y^2}$$

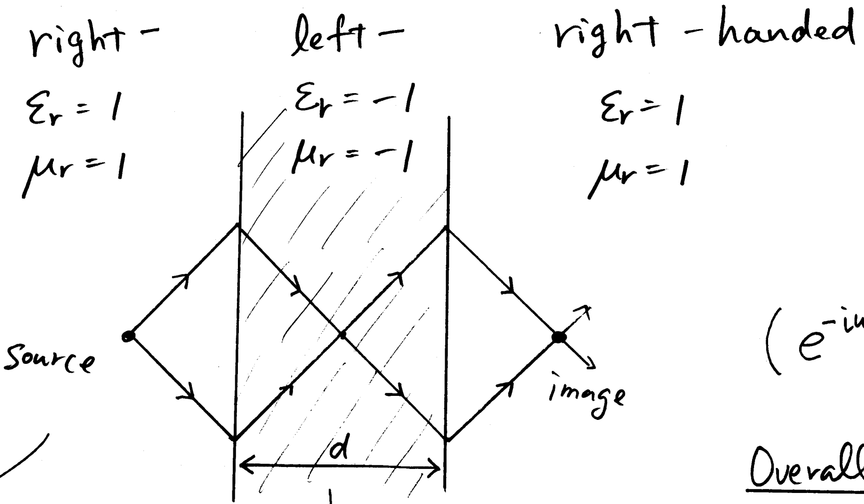
For Veselago lens with $\varepsilon_r = -1$, $\mu_r = -1$

$$\begin{cases} n = -\sqrt{\varepsilon_r \mu_r} = -1, \\ Z = \sqrt{\dfrac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}} = \sqrt{\dfrac{\mu_0}{\varepsilon_0}} = \eta \end{cases}$$

$\Downarrow$

perfect match to free space.

(no reflection at the interface)



($e^{-i\omega t}$ is assumed)

phase factor: $e^{ik_z \cdot z}$

$\boxed{\text{propagating wave:}}$ $k_z = \sqrt{\dfrac{\omega^2}{c^2} - k_x^2 - k_y^2}$

$$k_z' = -\sqrt{\frac{\omega^2}{c^2} - k_x^2 - k_y^2}$$

causality

Overall transmission

$$T = \exp(ik_z' d) = \exp\left(-i\sqrt{\frac{\omega^2}{c^2} - k_x^2 - k_y^2}\, d\right)$$

$\rightarrow$ negative phase velocity!!

$\boxed{\text{evanescent wave:}}$ $k_z = i\sqrt{k_x^2 + k_y^2 - \dfrac{\omega^2}{c^2}}$

$\left(k_x^2 + k_y^2 > \dfrac{\omega^2}{c^2}\right)$

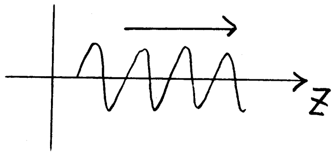
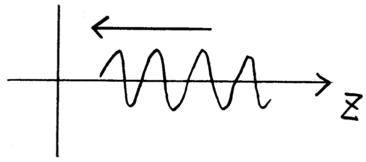
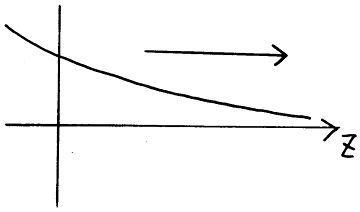
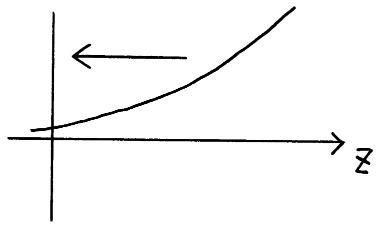
$$k_z' = -i\sqrt{k_x^2 + k_y^2 - \varepsilon_r \mu_r \frac{\omega^2}{c^2}} \quad \left(k_x^2 + k_y^2 > \varepsilon_r \mu_r \frac{\omega^2}{c^2}\right)$$

causality

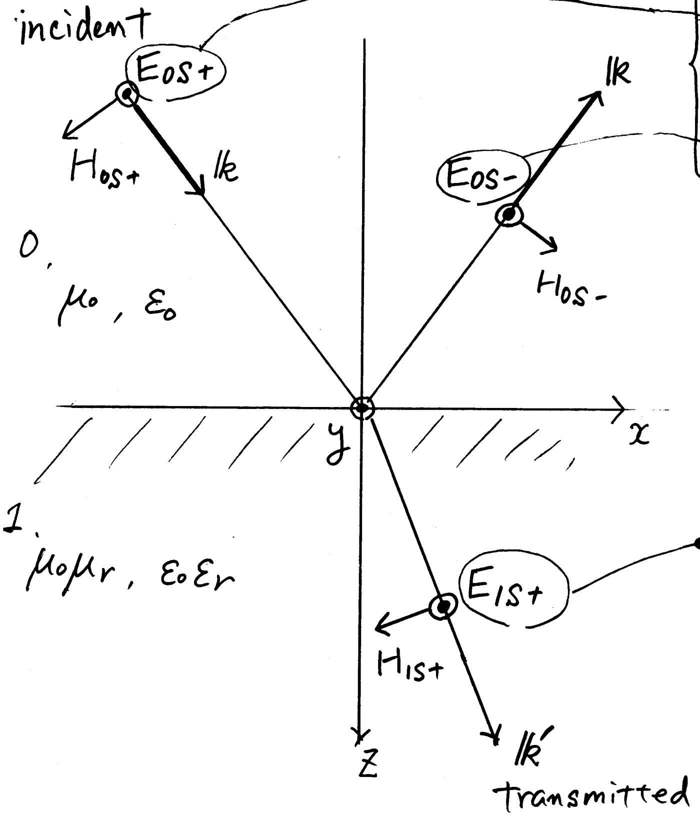
$$T = \exp(ik_z' d) = \exp(ik_z d) = \exp\left(+\sqrt{k_x^2 + k_y^2 - \frac{\omega^2}{c^2}}\, d\right)$$

$\Rightarrow$ Evanescent waves are amplified.

Summary : Assume $\boxed{e^{-i\omega t}}$ time dependency ,

	$e^{ik_z z}$, $k_z > 0$ Right-hand material	$e^{ik_z z}$, $k_z < 0$ Left-hand material (ε_r, μ_r)
Propagating wave	$e^{i\sqrt{\frac{\omega^2}{c^2}-k_x^2-k_y^2}\,z}$ 	$e^{-i\sqrt{\varepsilon_r\mu_r\frac{\omega^2}{c^2}-k_x^2-k_y^2}\,z}$  backward-wave
Evanescent wave	$e^{i\cdot i\sqrt{k_x^2+k_y^2-\frac{\omega^2}{c^2}}\,z}$ $=e^{-\sqrt{k_x^2+k_y^2-\frac{\omega^2}{c^2}}\,z}$  attenuation	$e^{-i\cdot i\sqrt{k_x^2+k_y^2-\varepsilon_r\mu_r\frac{\omega^2}{c^2}}\,z}$ $=e^{+\sqrt{k_x^2+k_y^2-\varepsilon_r\mu_r\frac{\omega^2}{c^2}}\,z}$  amplification (in terms of the positive z-direction)

Proof of evanescent waves, (S-poralized case).



phase factor

$$e^{ik_z z + ik_x x - i\omega t}$$

$$r e^{-ik_z z + ik_x x - i\omega t}$$

$$k_z = i\sqrt{\underbrace{k_x^2 + k_y^2}_{大} - \underbrace{\frac{\omega^2}{c^2}}_{小}}$$

(for evanescent wave)

$$t e^{ik'_z z + ik_x x - i\omega t}$$

for evanescent wave,

$$k'_z = -i\sqrt{\underbrace{k'^2_x + k'^2_y}_{大} - \underbrace{\varepsilon_r\mu_r \frac{\omega^2}{c^2}}_{小}}$$

I. at $z=0$, $E_{0S+} + E_{0S-} = E_{1S+}$ (Tangential E field)

$$e^{ik_x x - \cancel{i\omega t}} + r e^{ik_x x - \cancel{i\omega t}} = t e^{ik'_x x - \cancel{i\omega t}}$$

$$(1+r) e^{ik_x x} = t e^{ik'_x x}$$

II. at $z=0$ phase must be continuous

$$ik_x x - \cancel{i\omega t} = ik'_x x - \cancel{i\omega t}$$

$$k_x = k'_x$$

$$1 + r = t.$$

III. at $z=0$, $H_{x0S+} + H_{x0S-} = H_{x1S+}$ (Tangential H field)

from Faraday's law, $\frac{\partial}{\partial y}\cancel{E_z} - \frac{\partial}{\partial z}E_y = -\mu\frac{\partial}{\partial t}H_x$

for s-polarized wave, $-ik_z E_y = +i\omega\mu H_x$,

cont.

cont'd

To get magnetic field $\quad H_x = \dfrac{-k_z}{\omega\mu} E_y$

By using this relation, we get

$$-\frac{k_z}{\omega\mu_0} + r\frac{k_z}{\omega\mu_0} = -t\frac{k_z'}{\omega\mu_0\mu_r}$$

attention!

$$(1-r)\frac{k_z}{\omega\mu_0} = t\frac{k_z'}{\omega\mu}$$

Substituting $1+r=t$, we get

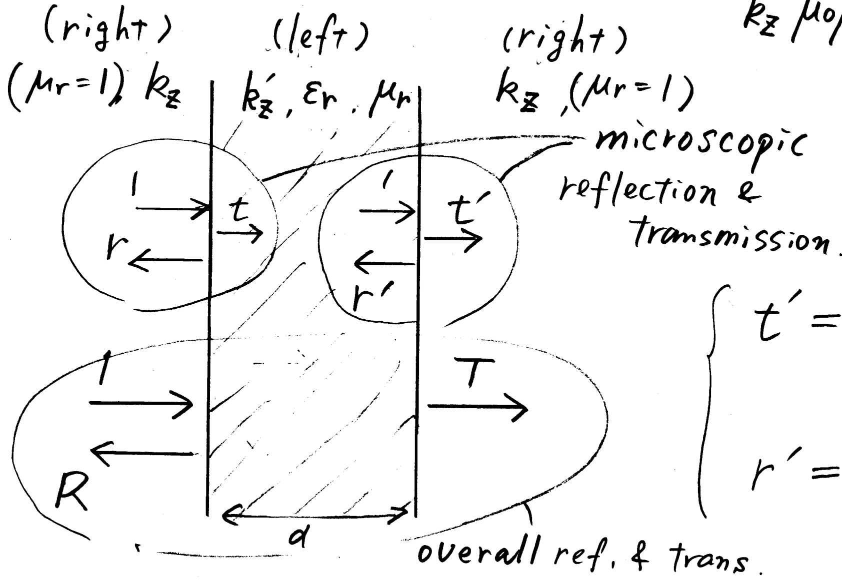
$$(1-\underbrace{t+1}_{=r})\frac{k_z}{\omega\mu_0} = t\frac{k_z'}{\omega\mu}$$

$$\frac{2k_z}{\omega\mu_0} = t\left(\frac{k_z}{\omega\mu_0} + \frac{k_z'}{\omega\mu_0\mu_r}\right)$$

s-polarized wave, then

$$t = \frac{2k_z\mu_0\mu_r}{k_z\mu_0\mu_r + k_z'\mu_0} = \frac{2k_z\mu_r}{k_z\mu_r + k_z'}$$

$$r = t-1 = \frac{k_z\mu_0\mu_r - k_z'\mu_0}{k_z\mu_0\mu_r + k_z'\mu_0} = \frac{k_z\mu_r - k_z'}{k_z\mu_r + k_z'}$$



$$\begin{cases} t' = \dfrac{2k_z'\cdot 1}{k_z' + k_z\mu_r} \\[2ex] r' = \dfrac{k_z' - k_z\mu_r}{k_z' + k_z\mu_r} \end{cases}$$

(c.f. for p-polarized wave,

$$t = \frac{2k_z\varepsilon_r}{k_z\varepsilon_r + k_z'}, \quad r = \frac{k_z\varepsilon_r - k_z'}{k_z\varepsilon_r + k_z'}$$
)

cont'd

The overall transmission T_s is given by (for s-pol. case)

$$T_s = tt'\exp(ik_z'd) + tt'r'^2\exp(3ik_z'd) + tt'r'^4\exp(5ik_z'd) + tt'r'^6\exp(7ik_z'd) + \cdots$$

$$= \frac{tt'\exp(ik_z'd)}{1 - r'^2\exp(2ik_z'd)}$$

$$\lim_{\substack{\mu_r\to-1\\ \varepsilon_r\to-1}} T_s = \lim_{\substack{\mu_r\to-1\\ \varepsilon_r\to-1}} \frac{\dfrac{2k_z\mu_r}{k_z\mu_r + k_z'}\cdot\dfrac{2k_z'}{k_z' + k_z\mu_r}\exp(ik_z'd)}{1 - \left(\dfrac{k_z' - k_z\mu_r}{k_z' + k_z\mu_r}\right)^2\exp(2ik_z'd)}$$

$$= \lim_{\substack{\mu_r\to-1\\ \varepsilon_r\to-1}} \frac{\cancel{2k_z}\,\mu_r^{\to -1}\cdot\cancel{2k_z'}\,\exp(ik_z'd)}{\cancel{(k_z' + k_z\mu_r)^2} - \underbrace{(k_z' - k_z\mu_r)^2}_{\to 0}\exp(2ik_z'd)}$$

$(\because\ k_z' \to -k_z\ (\mu_r\to-1,\ \varepsilon_r\to-1))$

$$= \exp(ik_z'd) = \exp(ik_zd)$$

recall that $k_z = -i\sqrt{k_x'^2 + k_y'^2 - \frac{\omega^2}{c^2}}$ for evanescent wave,

$$\lim_{\substack{\mu_r\to-1\\ \varepsilon_r\to-1}} T_s = \exp\left(+\sqrt{k_x'^2 + k_y'^2 - \frac{\omega^2}{c^2}}\,d\right)$$

$\square$ end of proof

Therefore, it is clear that the evanescent wave is amplified when it passes through negative index materials.

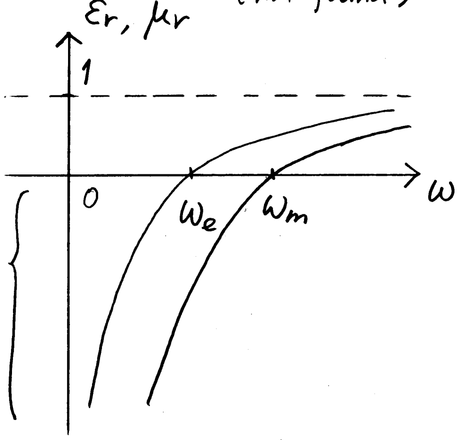
Dispersion relation in negative index materials

Consider lossless materials having the relative ε and μ as

$$\varepsilon_r = 1 - \frac{\omega_e^2}{\omega^2} \quad , \quad \text{(like electronic plasma)}$$

$$\underbrace{\mu_r = 1 - \frac{\omega_m^2}{\omega^2}}_{} \quad . \quad \text{(still non-existent in nature)}$$

(not found)



negative due to resonance (共振)

For TEM wave, phase constant (wave number) is given by

$$k_z = \pm \frac{\omega}{c}\sqrt{\varepsilon_r \mu_r} \qquad \left(\text{suppose } k_x = k_y = 0 \text{ in } k_x^2 + k_y^2 + k_z^2 = \frac{\omega^2}{c^2}\varepsilon_r\mu_r \right)$$

$$= \pm \frac{\omega}{c}\sqrt{\left(1 - \frac{\omega_e^2}{\omega^2}\right)\left(1 - \frac{\omega_m^2}{\omega^2}\right)} \qquad \text{—— } (\ast)$$

$k_z(\omega)$, (see ω-k diagram on page NIM-14)

When $\omega_e \gg \omega$, $\omega_m \gg \omega$,

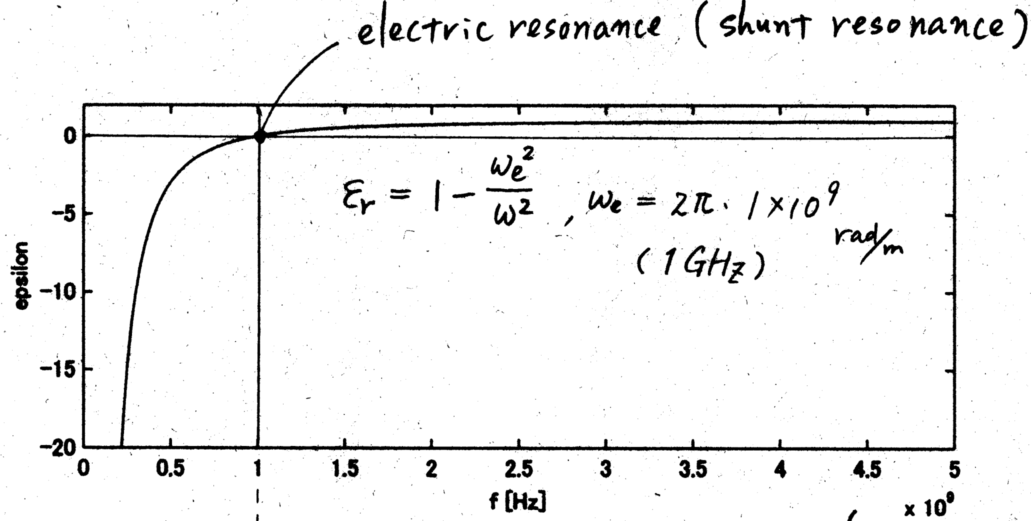
phase velocity $v_p = \dfrac{\omega}{k_z} = \dfrac{c}{\sqrt{\dfrac{\omega_e^2}{\omega^2}\cdot\dfrac{\omega_m^2}{\omega^2}}} = \dfrac{c\,\omega^2}{\omega_e\,\omega_m}$

group velocity $v_g = \dfrac{d\omega}{dk_z} = \dfrac{1}{\dfrac{d}{d\omega}\left(\dfrac{\omega}{c}\sqrt{\dfrac{\omega_e^2}{\omega^2}\cdot\dfrac{\omega_m^2}{\omega^2}}\right)} = -\dfrac{c\,\omega^2}{\omega_e\,\omega_m}$

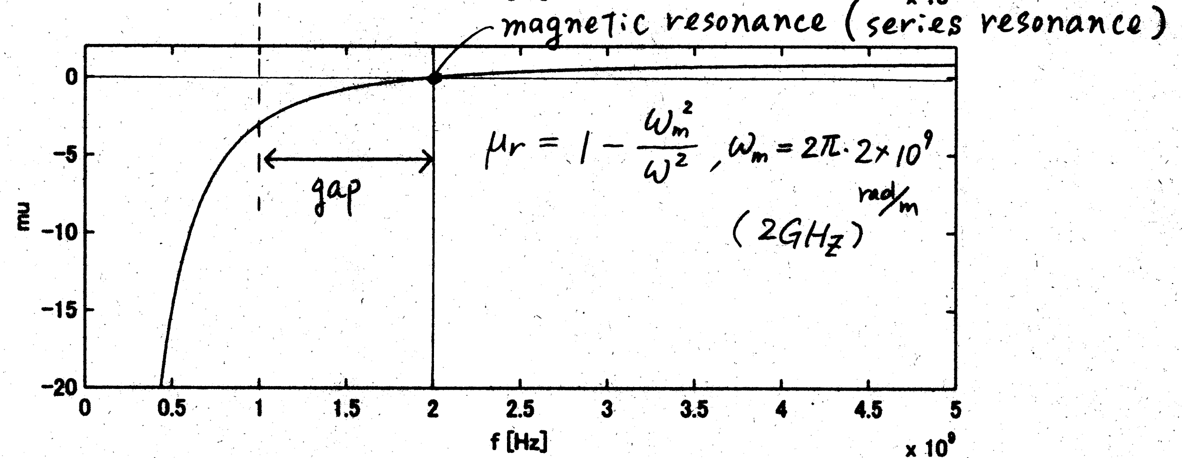
$\Rightarrow$ v_p , v_g have opposite directions.

Dispersion relation in NIM

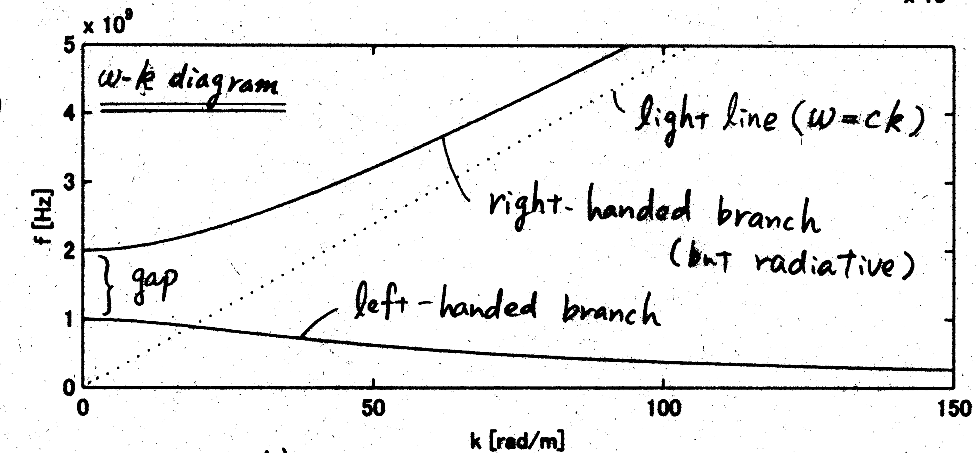
ε_r

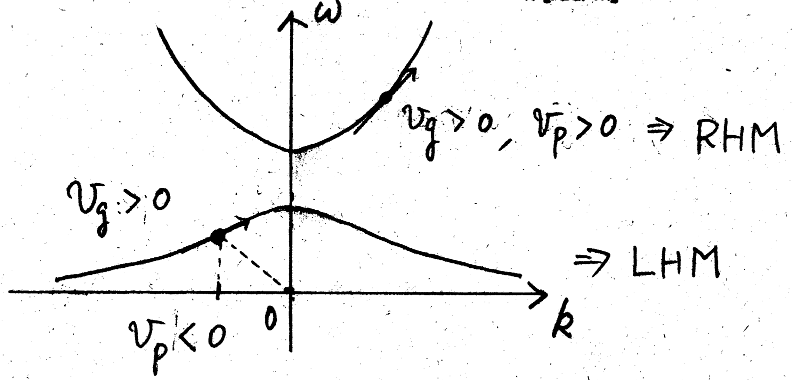


μ_r



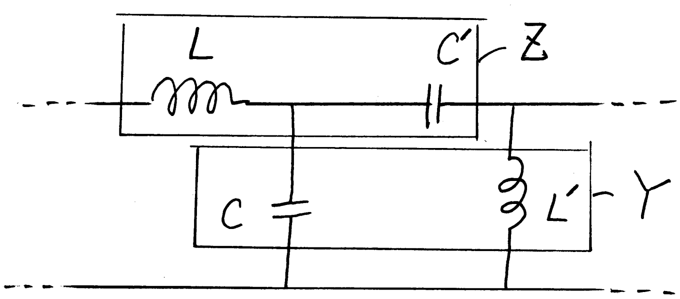
k_z
(eq. A)





Equivalent circuit of Negative Index Materials

NIMs are equivalent to a resonant transmition circuit,



ladder transmission circuit

For the ladder transmission circuit, the series Z and the shunt Y are approximated as

$$\begin{cases} Z = j\omega L + \dfrac{1}{j\omega C'} = j\omega L\left(1 - \dfrac{1}{\omega^2 LC'}\right) \to 0 : \text{series resonance } (Z) \\[2ex] Y = j\omega C + \dfrac{1}{j\omega L'} = j\omega C\left(1 - \dfrac{1}{\omega^2 CL'}\right) \to 0 : \text{shunt resonance } (Y) \end{cases}$$

From these values, the phase constant is calculated by

$$\beta = \frac{1}{j}\sqrt{ZY} = \omega\sqrt{LC}\sqrt{\left(1 - \frac{1}{\omega^2 LC'}\right)\left(1 - \frac{1}{\omega^2 CL'}\right)},$$

which has the same form as (☆) on p. NIM-13.

Numerical analysis of Negative Index Materials by the Finite-Difference Time-Domain method.

1D (one-dimensional) Maxwell's equations inside a dispersive material

$$
\begin{cases}
\dfrac{\partial E_x}{\partial z} = -\dfrac{\partial B_y}{\partial t} \\[2ex]
-\dfrac{\partial H_y}{\partial z} = \dfrac{\partial D_x}{\partial t} \\[2ex]
D_x = \varepsilon_0 \varepsilon_r E_x = \varepsilon_0 \left(\varepsilon_\infty + \chi^{(e)}_{(\omega)} \right) E_x \\[1ex]
B_y = \mu_0 \mu_r H_y = \mu_0 \left(\mu_\infty + \chi^{(m)}_{(\omega)} \right) H_y
\end{cases}
$$

(curl eqs.)

(const. eqs.)

electric susceptibility

magnetic susceptibility

where

$$
\begin{cases}
P_x = \varepsilon_0 \chi^{(e)}_{(\omega)} E_x & \text{--- dielectric polarization} \\[1ex]
Q_y = \mu_0 \chi^{(m)}_{(\omega)} H_y & \text{--- magnetic polarization (equivallent)}
\end{cases}
$$

and

$$
\begin{cases}
\chi^{(e)}_{(\omega)} = -\dfrac{\omega_e^2}{\omega^2}, \\[2ex]
\chi^{(m)}_{(\omega)} = -\dfrac{\omega_m^2}{\omega^2}.
\end{cases}
$$

Now, we consider the polarizations in frequency-domain

$$\tilde{P}(\omega) = \varepsilon_0 \left(-\frac{\omega_e^2}{\omega^2}\right) \tilde{E}(\omega)$$

$$\therefore \quad \omega^2 \tilde{P}(\omega) = -\varepsilon_0 \omega_e^2 \tilde{E}(\omega)$$

$\Downarrow$ by F^{-1} i.e, $j\omega \to \frac{\partial}{\partial t}$, $\omega^2 \to -\frac{\partial^2}{\partial t^2}$

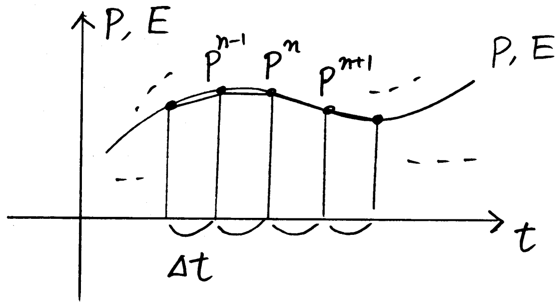
Time-domain equation

$$\frac{\partial^2 P(t)}{\partial t^2} = \varepsilon_0 \omega_e^2 E(t) ,$$

which is approximated by finite difference form,

$$\frac{p^{n+1} - 2 \cdot p^n + p^{n-1}}{\Delta t^2} = \varepsilon_0 \omega_e^2 E^n$$

p^n, E^n : at time "$n \cdot \Delta t$".



$\Downarrow$

Polarization at time $(n+1)\Delta t$ is predicted from past by

$$\boxed{p^{n+1} = 2 \cdot p^n - p^{n-1} + \varepsilon_0 \omega_e^2 \Delta t^2 \cdot E^n} \quad ⑤$$

Similarly for magnetic counterpart,

$$\boxed{Q^{n+1/2} = 2 \cdot Q^{n-1/2} - Q^{n-3/2} + \mu_0 \omega_m^2 \Delta t^2 \cdot H^{n-1/2}} \quad ②$$

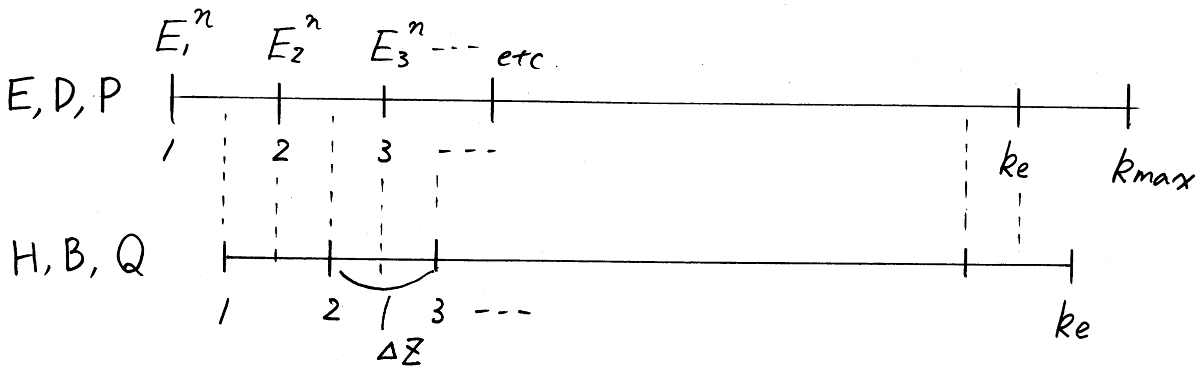


Fig. Definition of the field nodes.

For the curl equations, a similar finite-differencing leads to

$$\begin{cases} \dfrac{E_{k+1} - E_k}{\Delta z} = -\dfrac{B^{n+1/2} - B^{n-1/2}}{\Delta t} \\[2ex] \dfrac{H_{k+1/2} - H_{k-1/2}}{\Delta z} = -\dfrac{D^{n+1} - D^{n}}{\Delta t} \end{cases}.$$

These are solved in terms of $B^{n+1/2}$ and D^{n+1} as

$$\boxed{B_{k+1/2}^{n+1/2} = B_{k+1/2}^{n-1/2} - \frac{\Delta t}{\Delta z}\left(E_{k+1}^{n} - E_{k}^{n}\right)}$$ calc. ①

$$\boxed{D_{k}^{n+1} = D_{k}^{n} - \frac{\Delta t}{\Delta z}\left(H_{k+1/2}^{n+1/2} - H_{k-1/2}^{n+1/2}\right)}$$ ④

(ex. D_k^n means $D(k\Delta z, n\Delta t)$)

time

position

For the constitutive equations

- $D^{n+1} = \varepsilon_0 \varepsilon_\infty E^{n+1} + P^{n+1}$

$$\Downarrow$$

$$\boxed{E^{n+1} = \frac{D^{n+1} - P^{n+1}}{\varepsilon_0 \varepsilon_\infty}} \quad ⑥$$

- $B^{n+1/2} = \mu_0 \mu_\infty H^{n+1/2} + Q^{n+1/2}$

$$\Downarrow$$

$$\boxed{H^{n+1/2} = \frac{B^{n+1/2} - Q^{n+1/2}}{\mu_0 \mu_\infty}} \quad ③$$

Calculate eqs. ① → ⑥ to obtain E^{n+1} and $H^{n+1/2}$, and repeat this process.

$$\Downarrow$$

time evolution of the fields.

NIM-18.1


```
%***********************************************************************
%     1-D FDTD code with simple radiation boundary conditions
%***********************************************************************

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% This program has been largely modified for the implementation of
% negative refraction index materials through the auxiliary differential
% equation of lossless Drude type material dispersion in both dielectric
% permittivity and magnetic permeability.
%
% Modification by M.Fujii, May 31, 2007.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%***********************************************************************
%     Program author: Susan C. Hagness
%                     Department of Electrical and Computer Engineering
%                     University of Wisconsin-Madison
%                     1415 Engineering Drive
%                     Madison, WI 53706-1691
%                     608-265-5739
%                     hagness@engr.wisc.edu
%
%     Date of this version:  February 2000
%
%     This MATLAB M-file implements the finite-difference time-domain
%     solution of Maxwell's curl equations over a one-dimensional space
%     lattice comprised of uniform grid cells.
%
%     To illustrate the algorithm, a sinusoidal wave (1GHz) propagating
%     in a nonpermeable lossy medium (epsr=1.0, sigma=5.0e-3 S/m) is
%     modeled.  The simplified finite difference system for nonpermeable
%     media (discussed in Section 3.6.6 of the text) is implemented.
%
%     The grid resolution (dx = 1.5 cm) is chosen to provide 20
%     samples per wavelength.  The Courant factor S=c*dt/dx is set to
%     the stability limit: S=1.  In 1-D, this is the "magic time step."
%
%     The computational domain is truncated using the simplest radiation
%     boundary condition for wave propagation in free space:
%
%                     Ez(imax,n+1) = Ez(imax-1,n)
%
%     To execute this M-file, type "fdtd1D" at the MATLAB prompt.
%     This M-file displays the FDTD-computed Ez and Hy fields at every
%     time step, and records those frames in a movie matrix, M, which is
%     played at the end of the simulation using the "movie" command.
%
%***********************************************************************
clear
%***********************************************************************
%     Fundamental constants
%***********************************************************************

c0  = 2.99792458e8;             %speed of light in free space
mu0 = 4.0*pi*1.0e-7;            %permeability of free space
ep0 = 1.0/(c0*c0*mu0);          %permittivity of free space
```

18.2


```
freq   = 1.0e+9;              %frequency of source excitation
lambda = c0/freq;             %wavelength of source excitation
omega  = 2.0*pi*freq;

%***********************************************************************
%     Grid parameters
%***********************************************************************

ke = 400;                     %number of grid cells in z-direction
kmax = ke+1;
kb = kmax;

dz       = lambda/40.0;       %space increment of 1-D lattice
dt       = 0.95*dz/c0;        %time step (Magic time step does not work for NIM.)
omegadt = omega*dt;

zmax = dz*kb;                 %length of the calculation space
nmax = round(30.0e-9/dt)      %total number of time steps

%***********************************************************************
%     Material parameters and locations
%***********************************************************************

%Default values (initialization)
epi(1:ke) = 1.0;    %epsilon at infinite frequency
mui(1:ke) = 1.0;    %mu at inifinite frequency
omg_e(1:ke) = 0.0;  %plasma (angular) frequency for dielectric susceptibility
omg_m(1:ke) = 0.0;  %plasma (angular) frequency for equivalent magnetic susceptibility

%Enter structures to be analysed

%epi(0.3*ke:0.7*ke) = 2.0;
%mui(0.3*ke:0.7*ke) = 2.0;

omg_e(0.3*ke:0.7*ke) = 2*pi*sqrt(2)*1e9;  %at this frequency, ep = -1
omg_m(0.3*ke:0.7*ke) = 2*pi*sqrt(2)*1e9;  %at this frequency, mu = -1

%***********************************************************************
%     Updating coefficients for space region
%***********************************************************************

ca=dt/dz;
cp(1:ke) = ep0*omg_e(1:ke).^2*dt^2;
cq(1:ke) = mu0*omg_m(1:ke).^2*dt^2;

%***********************************************************************
%     Field arrays
%***********************************************************************

%Fields and flux densities
e(1:kb) = 0.0; %Ex, only this needs the 'kb'-th node.
d(1:ke) = 0.0; %Dx
h(1:ke) = 0.0; %Hy
b(1:ke) = 0.0; %By
```

18.3


```
%Polarizations (for 3 time steps, future (index 3), present (2) and previous time (1))
p(1:3,1:ke) = 0.0; %Px, dielectric polarization
q(1:3,1:ke) = 0.0; %Qy, equivalent magnetic polarization (dimension different from magnitization)

%***********************************************************************
%     Movie initialization
%***********************************************************************

z=linspace(dz,ke*dz,ke);

subplot(2,1,1), plot(z,e(1:ke),'r'), axis([0 zmax -1 1]);
ylabel('Ex');

subplot(2,1,2), plot(z,h,      'b'), axis([0 zmax -3.0e-3 3.0e-3]);
xlabel('z (meters)');ylabel('Hy');

rect=get(gcf,'Position');
rect(1:2)=[0 0];

M=moviein(nmax/2,gcf,rect);  %for Matlab Rev.11 and earliers

%***********************************************************************
%     BEGIN TIME-STEPPING LOOP
%***********************************************************************

for n=1:nmax

%***********************************************************************
%     Data structure (supplemented by M.Fujii, 3 June, 2002)
%***********************************************************************
%
%   Hy:     |-----+-----+-----+-----+-----+-----+-----|
%           1    2    3 ...                           ie
%
%
%   Ex: |-----+-----+-----+-----+-----+-----+-----+-----|
%       1    2    3 ...                          ie    ib
%       |                       |
%       |                                   |
%   hard excitation at ez(1)          radiation boundary condition (rbc)
%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%     Excitation of electric field
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

e(1)=sin(omegadt*n);  %excitation

%***********************************************************************
%     Update electric flux density
%***********************************************************************

d(2:ke) = d(2:ke) - ca*(h(2:ke)-h(1:ke-1));

%***********************************************************************
%     Update dielectric polarization for current time step
```

```
%***********************************************************************

p(3,2:ke) = 2*p(2,2:ke) - p(1,2:ke) + cp(2:ke).*e(2:ke);

%***********************************************************************
%     Update electric fields
%***********************************************************************

rbc = e(ke);  %radiation boundary condition
e(2:ke) = (d(2:ke)-p(3,2:ke))/ep0./epi(2:ke);
e(kb) = rbc;

%***********************************************************************
%     Update dielectric polarization for previous time steps
%***********************************************************************

p(1,:) = p(2,:);
p(2,:) = p(3,:);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%     Update magnetic flux density
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

b(1:ke) = b(1:ke) - ca*(e(2:kb)-e(1:ke));

%***********************************************************************
%     Update equivalent magnetic polarization for current time step
%***********************************************************************

q(3,1:ke) = 2*q(2,1:ke) - q(1,1:ke) + cq(1:ke).*h(1:ke);

%***********************************************************************
%     Update magnetic fields
%***********************************************************************

h(1:ke) = (b(1:ke)-q(3,1:ke))/mu0./mui(1:ke);

%***********************************************************************
%     Update magnetic polarization for previous time steps
%***********************************************************************

q(1,:) = q(2,:);
q(2,:) = q(3,:);

%***********************************************************************
%     Visualize fields
%***********************************************************************

if mod(n,2)==0;

rtime=num2str(round(n*dt/1.0e-9));

subplot(2,1,1), plot(z,e(1:ke),'r'), axis([0 zmax -1 1]);
title(['time = ',rtime,' ns']);
ylabel('Ex');
```

```
subplot(2,1,2), plot(z,h,      'b'), axis([0 zmax -3.0e-3 3.0e-3]);
title(['time = ',rtime,' ns']);
xlabel('z (meters)');ylabel('Hy');

M(:,n/2)=getframe(gcf,rect);  %extract movie frames from object 'gcf'

end

%***************************************************************************
%     END TIME-STEPPING LOOP
%***************************************************************************

end

%movie(gcf,M,0,10,rect);  %Repeat movie


%mpgwrite(M,'jet','nim1Dmovie.mpg')
```

Propagation of plane wave in a left-handed material

$$\left.\begin{aligned} \omega_e &= \omega_m = 2\pi \times \sqrt{2} \times 10^9 \ \text{rad/m} \\ \omega &= 2\pi \times 10^9 \ \text{rad/m} \end{aligned}\right\} \Rightarrow \varepsilon_r = -1,\ \mu_r = -1.$$