

FY 2018

Gravitational Waves

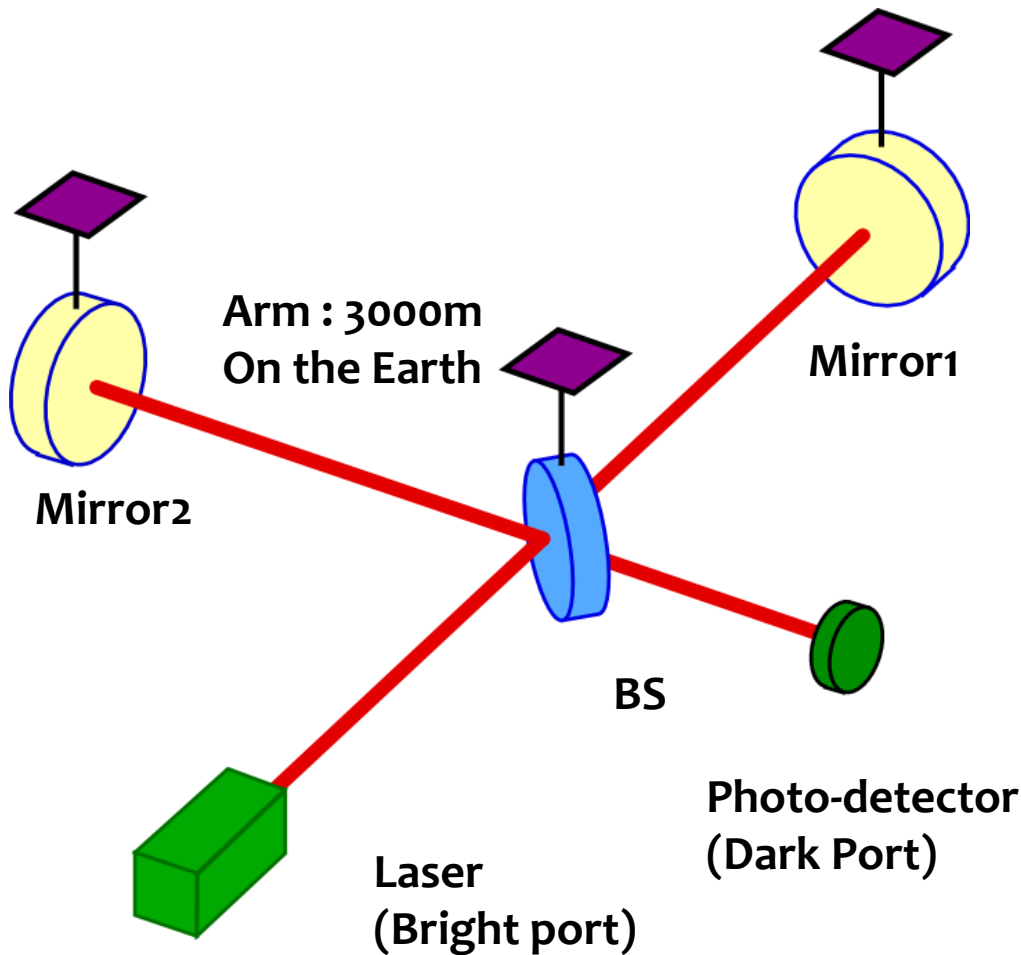
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Astronomy and Gravity Division
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**Various Noise sources
in
GWDS**

Michelson Interferometer (MI)

■ GW signal Extraction using Michelson Interferometer

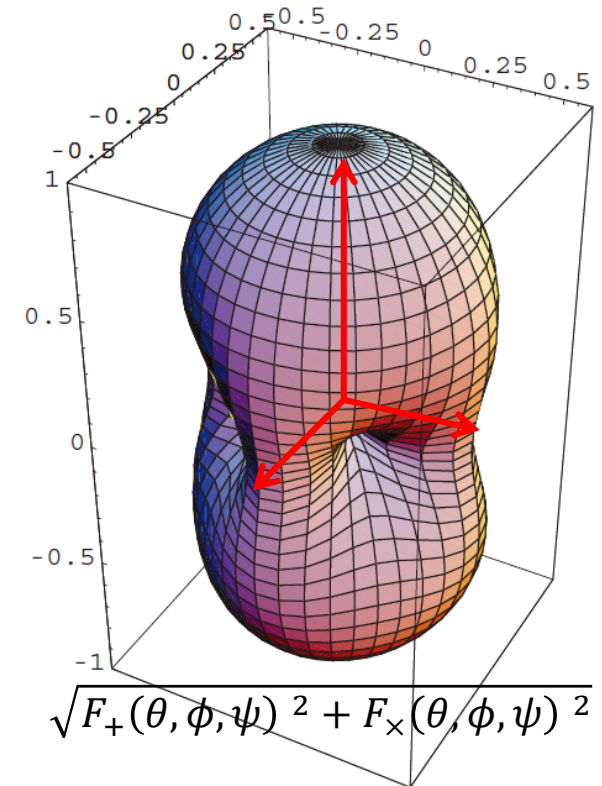
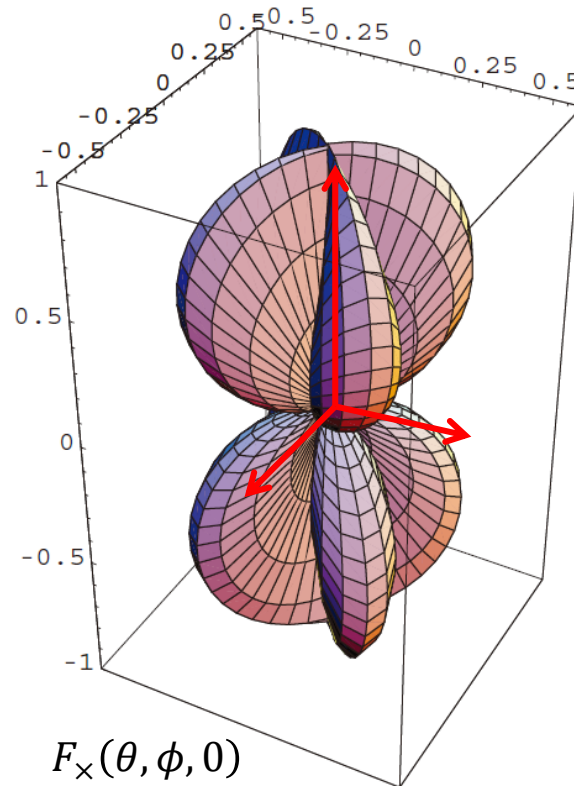
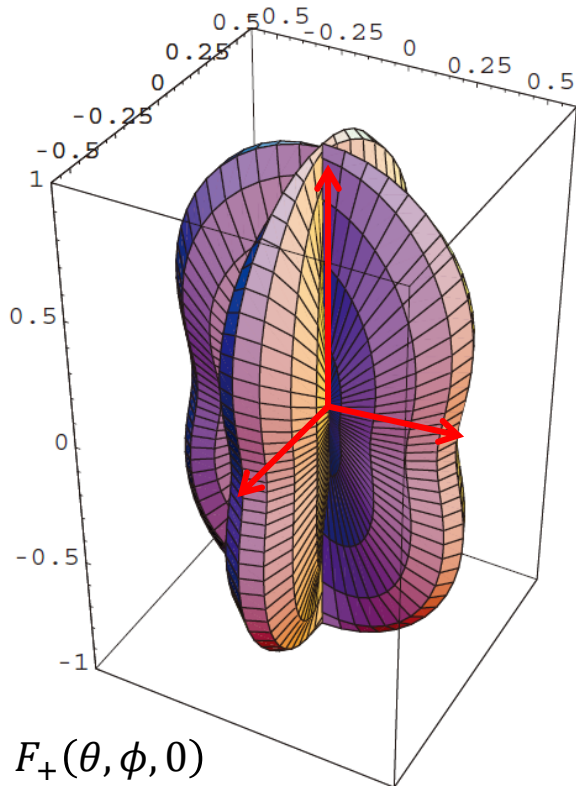
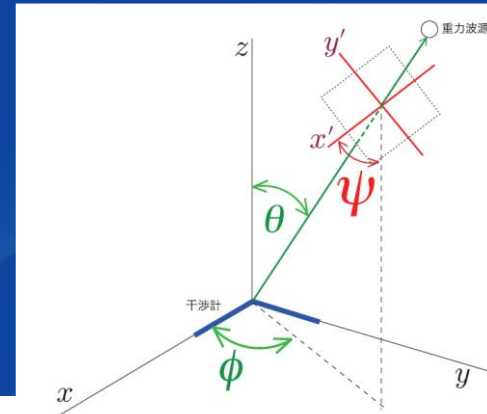


- Free falling mirrors and BS are simulated as pendulums on the Earth over the pendulum resonance frequency.
- ~ 4 km used to be max length on the Earth because of gravity direction difference. (~10km is planned in ET)

Antenna Pattern of MI

MI type GW detector has a antenna pattern

- Assuming MI arms are aligned along X, Y axis.
- MI has max sensitivity for GWs propagating along Z axis.
- MI has no sensitivity for GWs propagating along 45, 135, 225, 315 degree directions.
- Observed signal is $h_{obs} = F_+(\theta, \phi, \psi)h_+ + F_\times(\theta, \phi, \psi)h_\times$



Antenna Pattern of MI

MI Antenna Pattern

$$h_{obs} = F_+ h_+ + F_\times h_\times$$

$$F_+ h_+ = -\frac{1 + \cos^2 \theta}{2} \cos 2\phi \cos 2\psi - \cos \theta \sin 2\phi \sin 2\psi$$

$$F_\times h_\times = \frac{1 + \cos^2 \theta}{2} \cos 2\phi \cos 2\psi - \cos \theta \sin 2\phi \cos 2\psi$$

All Sky average

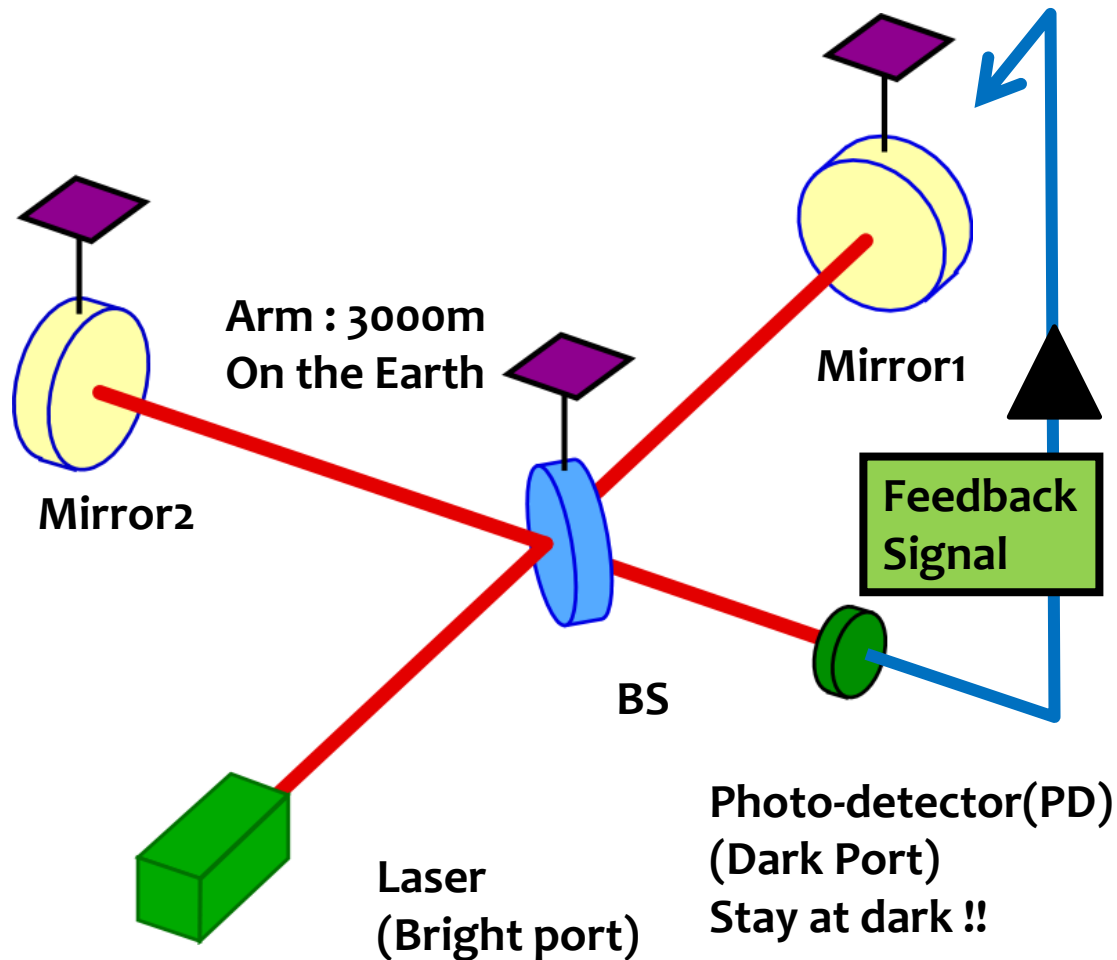
- SNR > 8 is used for express the GWD's detectability. However SNR strongly depends on GWD antenna pattern, GW radiation pattern and polarization .
- It is convenient to define a spherical GWD's detectable distance (that is effective observable distance = R_{eff})

$$\frac{4\pi}{3} R_{eff}^3 \equiv \frac{4\pi}{3} (\langle F_{rms} \rangle R)^3 \quad F_{rms} = \sqrt{\langle F_+ \rangle^2 + \langle F_\times \rangle^2} \quad F = \frac{h_{obs}}{h_+ + h_\times}$$

Sky average $F_{rms}^{pol, n-pol} = \frac{1}{\sqrt{5}}$ For one polarized GWs and no-polarized GWs

Michelson Interferometer (MI)

■ GW signal Extraction using Michelson Interferometer

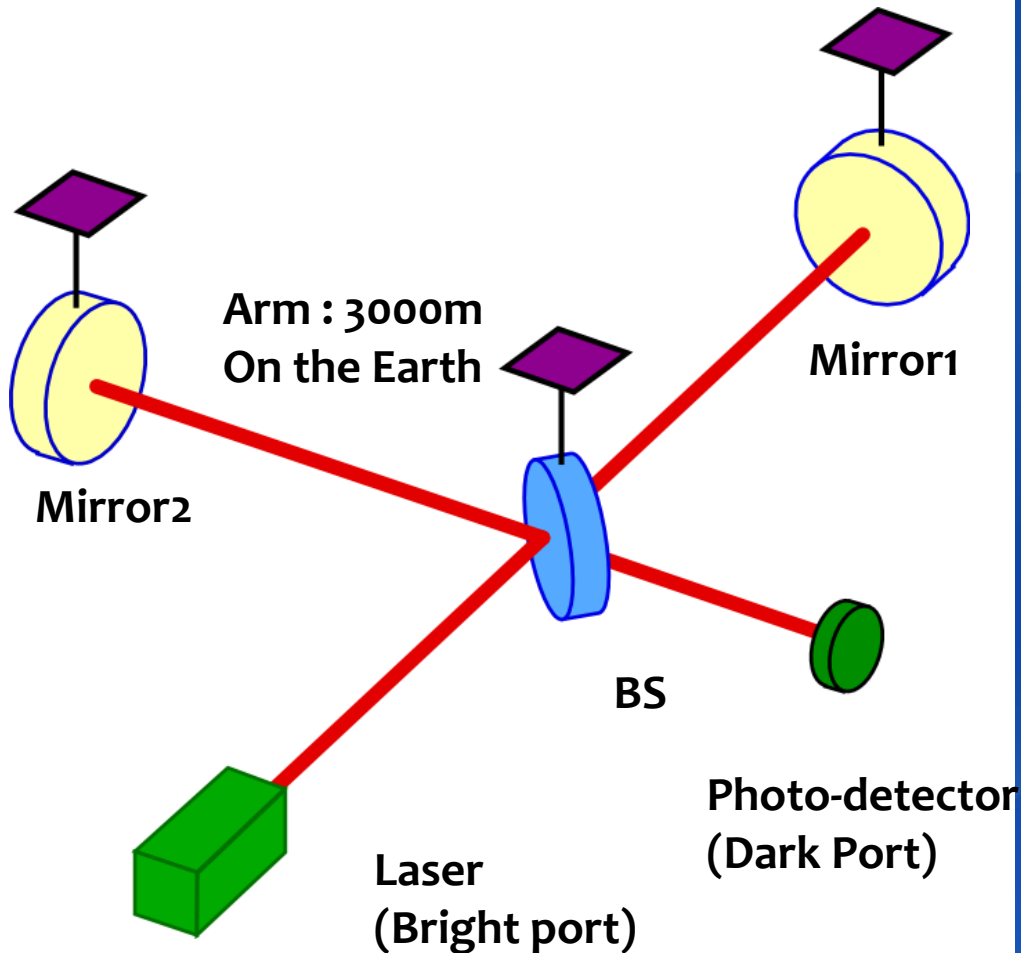


- Interfered fringe at PD was controlled to be dark fringe by adjusting the Mirror1 and/or 2 position according to PD signal.
- **Heterodyne or homodyne** technique are used for differential length signal extraction.
- GW signals are obtained in the differential feedback signal.

Quantum Noise in MI

- Shot noise and Radiation Pressure Noise -

■ GW signal Extraction using Michelson Interferometer



- MI sensitivity should be limited only by “**Shot Noise (SHN)**” and “**Radiation Pressure Noise (RPN)**” that are originated by the **uncertainty principle of photon**.
- Standard Quantum Limit (SQL) is same with the crossing point between SHN and RPN.
- Theoretically, **SQL can be beatable** because of the correlation between SHN and RPN.

First Interferometer Prototype

- “Wideband Laser-Interferometer gravitational-radiation experiment”, Robert. L. Forward, Phys. Rev. D 17 (1978)
- Arm length : 2m, He-Ne laser (35~65 mW) were used.

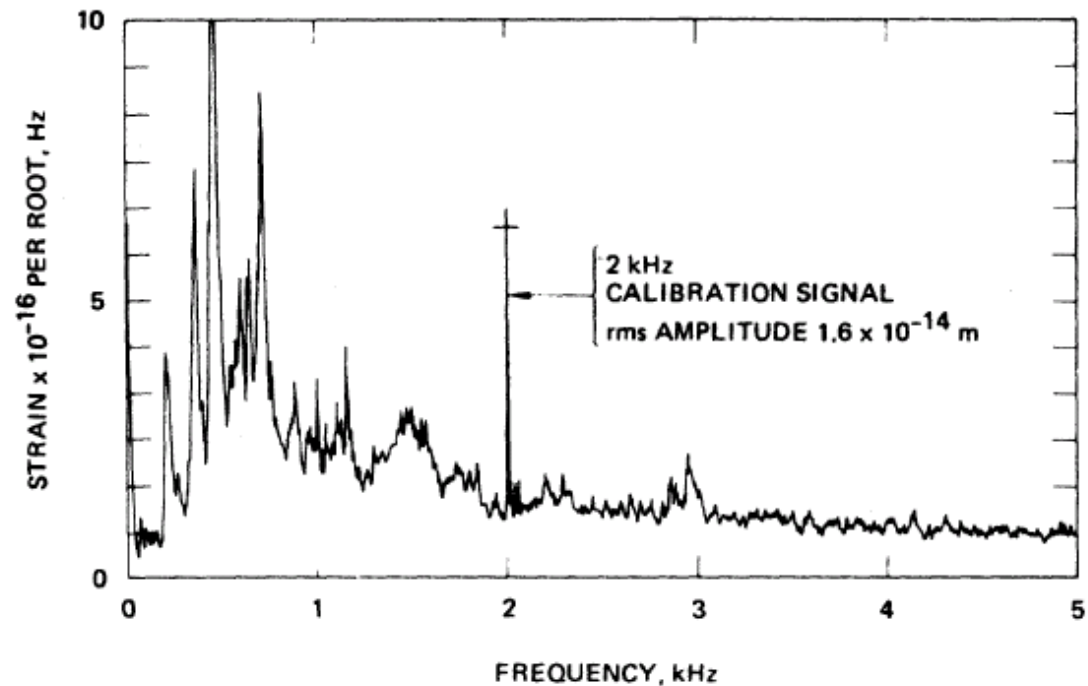
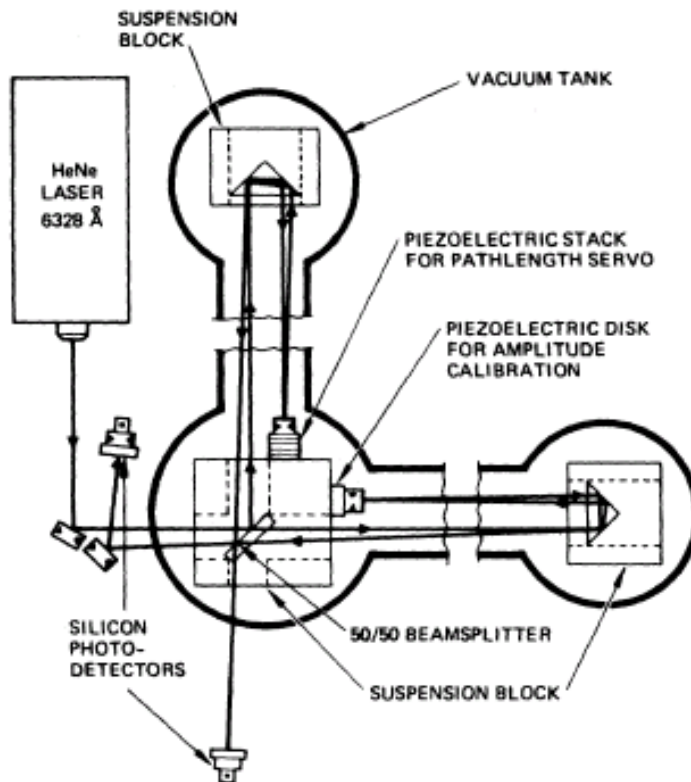


FIG. 14. Strain sensitivity of interferometer antenna.

Shot Noise

■ Michelson Interferometer

$$I_{PD}^{Ant} = \frac{I_{max} + I_{min}}{2} - \frac{I_{max} - I_{min}}{2} \cos(\phi_-)$$

$$\delta I_{PD}^{Ant} = -\frac{I_{max} - I_{min}}{2} \sin(\phi_-) \delta \phi_-$$

Shot Noise: $\langle \delta I_{shot}^2 \rangle = 2eI\Delta f$

S/N: $\frac{\delta I^2}{\langle \delta I_{shot}^2 \rangle} = \frac{1}{2eI\Delta f} \left(\frac{I_{max} - I_{min}}{2} \sin(\phi_-) \right)^2$

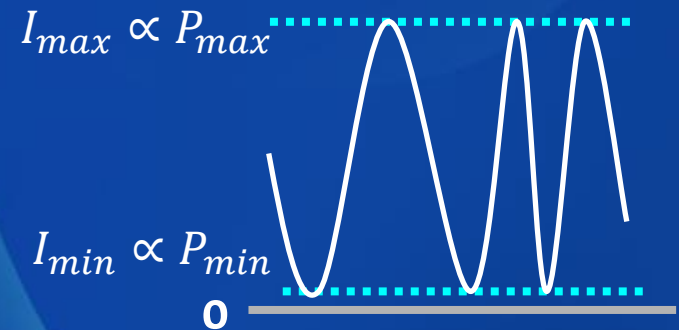
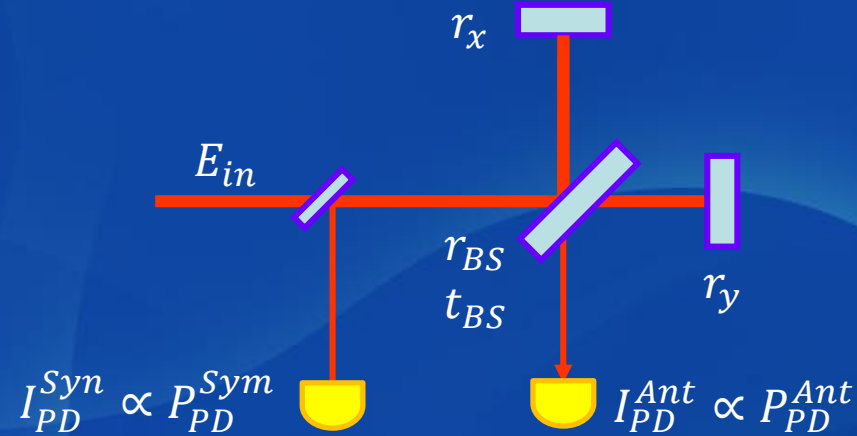
Obviously, $I_{min} = 0$ gives max S/N.

$$= \frac{I_{max}(1 - \cos(\phi_-))}{4e\Delta f} \delta \phi_-^2$$

$$\left[\frac{\delta I^2}{\langle \delta I_{shot}^2 \rangle} \right]_{max} = \frac{I_{max}}{2e\Delta f} \delta \phi_-^2$$

$$\delta \phi_- = \sqrt{\frac{2e\Delta f}{I_{max}}} \rightarrow \sqrt{\frac{2h\nu}{\eta P} \Delta f}$$

η photon \rightarrow electron
Efficiency of PD
 P Power at PD



So, equivalent phase noise :

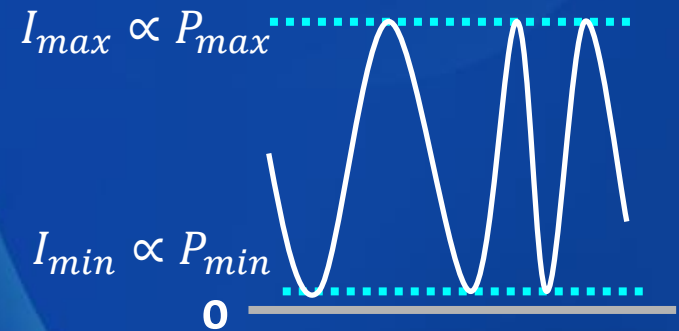
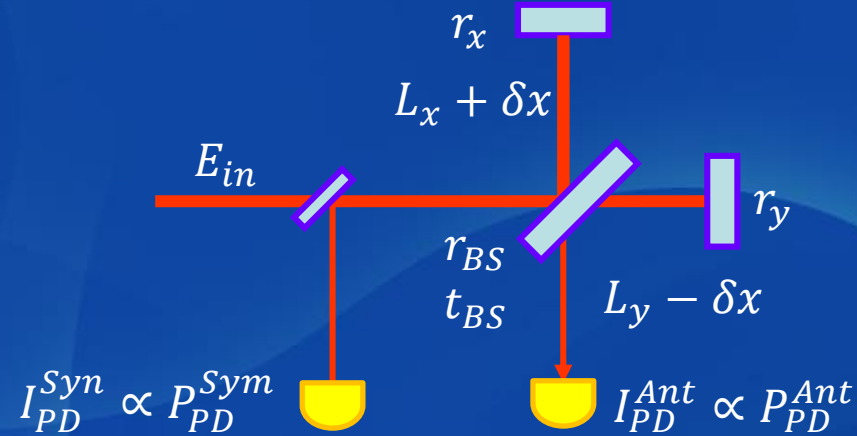
Shot Noise

■ Michelson Interferometer

$$\delta\phi_- = \sqrt{\frac{2e\Delta f}{I_{max}}} \rightarrow \sqrt{\frac{2h\nu}{\eta P} \Delta f}$$

$$\delta\phi_- \equiv \frac{2\pi(L_x - L_y)}{\lambda} = \frac{4\pi\delta x}{\lambda}$$

δx : mirror differential motion part



Shot Noise

- Assuming 75km arm length of MI, $h \sim 10^{-22}$ strain due to GWs, and 1 μm laser wave length, we need the following phase sensitivity,

$$\delta\phi_{GW} = 4.4 \times 10^{-11} [\text{rad}]$$

$P \sim 100 \text{ kW}$ required from $\delta\phi_{-} = \sqrt{\frac{2h\nu}{\eta P} \Delta f}$

$$\Delta f = 1 [\text{kHz}], \quad \eta = 1,$$
$$\nu = \frac{c}{\lambda}, \quad \lambda = 1064 [\text{nm}]$$

$P \sim 800 \text{ kW}$ is targeted in advanced GWDs
(Power Recycled Fabry-Perot MI
with Resonant Sideband Extraction Technique).

Radiation Pressure Noise

Classical Image

One photon gives

$$F_r = \frac{2h\nu}{c}$$

N photon gives

$$F_r = N \frac{2h\nu}{c}$$

N photons have fluctuation of

$$\sqrt{N} \left(= \sqrt{\frac{2P}{h\nu}} \right)$$

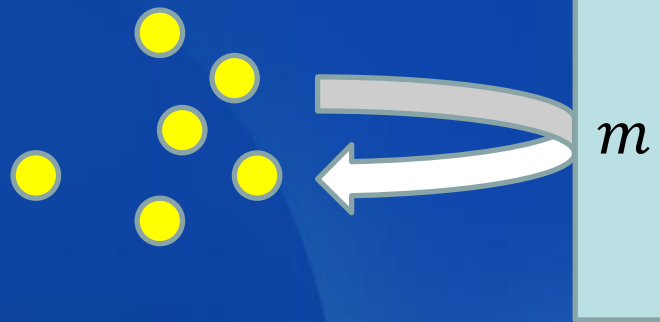
$$\delta F_r = \sqrt{2h\nu P} \frac{2}{c}$$

Equation of Motion
& Fourier TF

$$\delta x_r = \frac{2\sqrt{2h\nu P}}{m c \omega^2}$$

Photons

- Frequency : ν
- Total Power : P



Suspended
Mirror

Gravity

No Laser Power

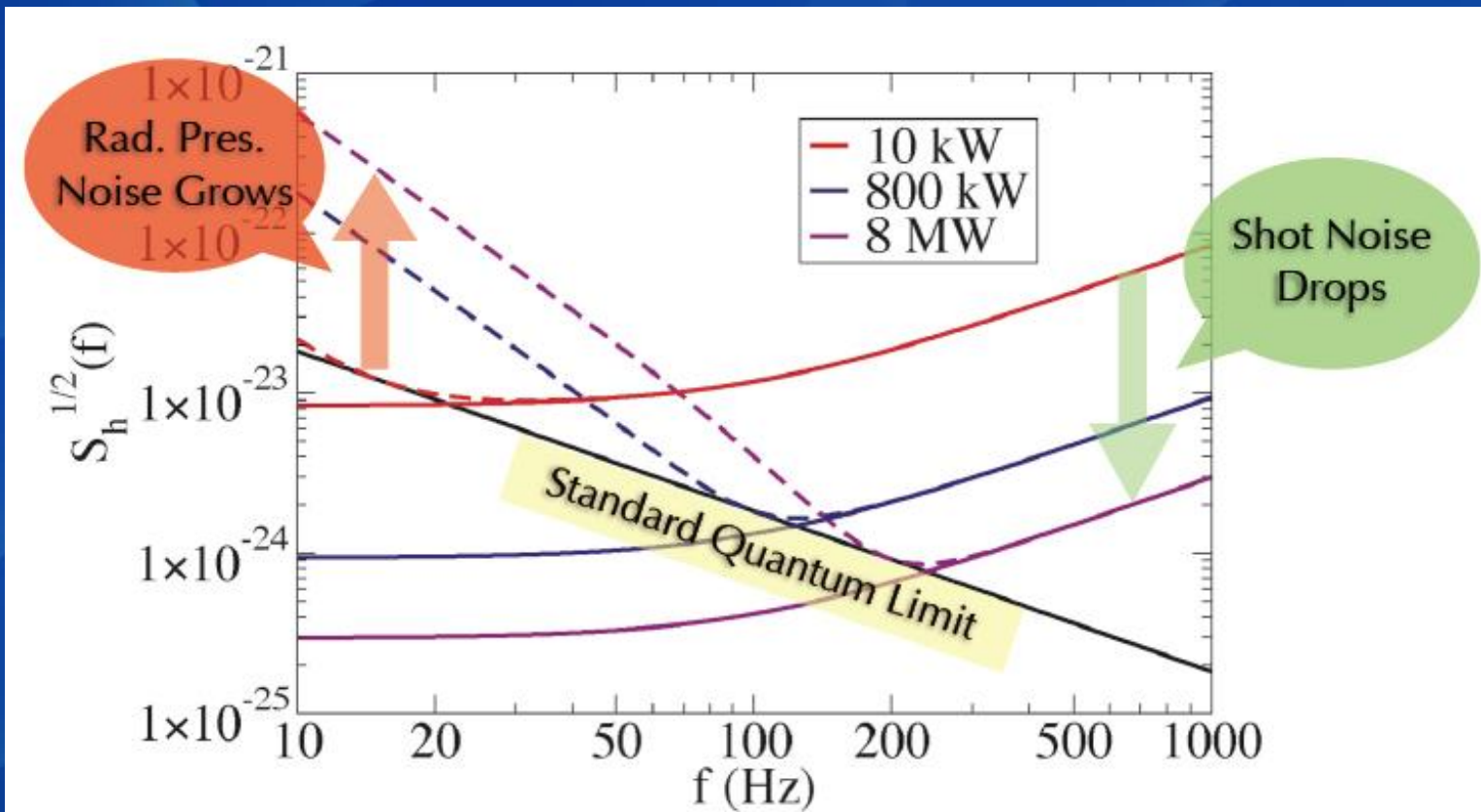
m

High Laser Power

Quantum Noise in MI

- Shot Noise and Radiation Pressure Noise -

$$\delta x_{q,MI}^2 = \delta x_s^2 + \delta x_r^2 = \frac{\lambda^2}{16\pi^2} \frac{2h\nu}{P} + \frac{8h\nu P}{m^2 \omega^4 c^2} \geq \frac{2\lambda h\nu}{\pi m \omega^2 c} = \frac{4\hbar}{m\omega^2}$$



$L = 4 \text{ km}$, $M = 40 \text{ kg}$ and $\gamma = 100 \text{ Hz}$

γ : optical bandwidth of arm cavity

Quantum Noise in MI

- Heisenberg Uncertainty Principle -

Free Mass Case

Equation of Motion of mass (m kg)

$$\frac{dx}{dt} = \frac{p}{m}, \frac{dp}{dt} = 0$$

$$x(t) = \frac{p(0)}{m}t + x(0), p(t) = p(0)$$

Uncertainty Principle $\Delta x \Delta p \geq \frac{\hbar}{2}$

Uncertainty at time (τ) $\Delta x(\tau) = \frac{p(0)}{m}\tau + \Delta x(0)$

$$\langle \Delta x(\tau)^2 \rangle = \frac{\langle \Delta p(0)^2 \rangle}{m^2} \tau + \langle \Delta x(0)^2 \rangle$$

$$\geq \sqrt{\frac{\langle \Delta p(0)^2 \rangle}{m^2} \tau^2 \langle \Delta x(0)^2 \rangle} \geq \frac{\hbar \tau}{m}$$

Equal when $\Delta x(0) = \Delta x_{SQL} = \sqrt{\frac{\hbar \tau}{2m}}$

Oscillator Case

Equation of Motion of mass (m kg)

$$\frac{dx}{dt} = \frac{p}{m}, \frac{dp}{dt} = -m\omega_0^2 x$$

$$x(t) = x(0)\cos(\omega_0 t) + \frac{p(0)}{m\omega_0} \sin(\omega_0 t),$$

$$p(t) = p(0)\cos(\omega_0 t) - m\omega_0^2 x(0)\sin(\omega_0 t)$$

Energy using Uncertainty Principle

$$E = \frac{p^2}{2m} + \frac{m\omega_0^2 x^2}{2} \geq \frac{(\Delta p)^2}{2m} + \frac{m\omega_0^2 (\Delta x)^2}{2}$$
$$\geq \frac{\hbar \omega_0}{m}$$

Equal when $\Delta x = \Delta x_{SQL} = \sqrt{\frac{\hbar}{2m\omega}}$

Quantum Noise in FPMI

- Shot Noise and Radiation Pressure Noise -

■ Shot Noise

$$h_{shot} = \sqrt{\frac{\hbar \lambda f_{cut}}{2LP_{cavity}}} \sqrt{1 + (f/f_{cut})^2}$$

f_{cut} : Cut off Frequency of Interferometer

P_{cavity} : Laser power inside one FP arm cavity

λ : Laser wave length L : FP cavity arm length m : Mirror Mass

$$= 2.5 \times 10^{-24} \left(\frac{3\text{km}}{L}\right) \left(\frac{0.597\text{MW}}{P_{cavity}}\right)^{\frac{1}{2}} \left(\frac{f_{cut}}{200\text{Hz}}\right)^{\frac{1}{2}} \left(\frac{\lambda}{1064\text{nm}}\right)^{\frac{1}{2}} \sqrt{1 + (f/f_{cut})^2} [1/\sqrt{\text{Hz}}]$$

■ Radiation Pressure Noise

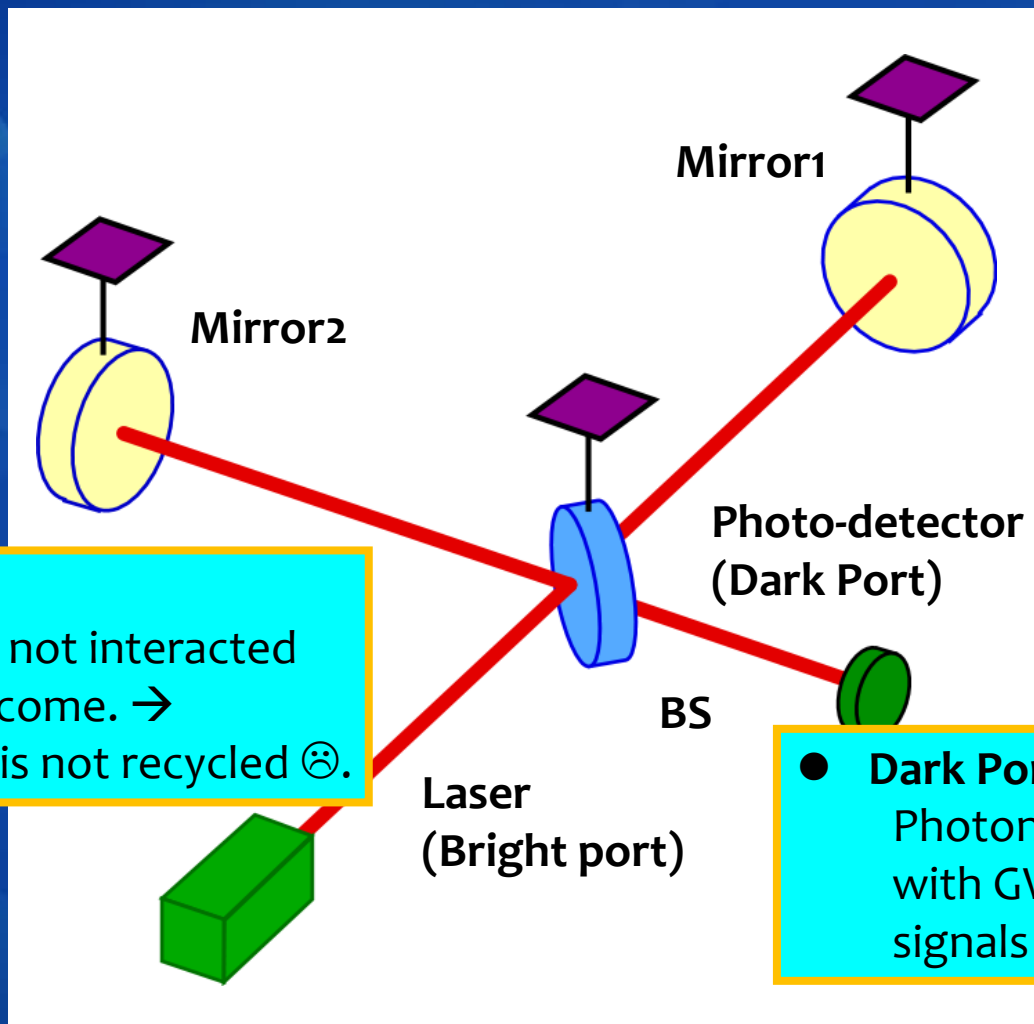
$$h_{radiation} = \frac{4}{Lm\omega^2} \sqrt{\frac{2\pi\hbar P_{cavity}}{\lambda L f_{cut}}} \sqrt{\frac{1}{1 + (f/f_{cut})^2}}$$
$$= 1.6 \times 10^{-22} \left(\frac{3\text{km}}{L}\right) \left(\frac{30\text{kg}}{m}\right) \left(\frac{P_{cavity}}{0.597\text{MW}}\right)^{\frac{1}{2}} \left(\frac{200\text{Hz}}{f_{cut}}\right)^{\frac{1}{2}} \left(\frac{1064\text{nm}}{\lambda}\right)^{\frac{1}{2}} \left(\frac{10\text{Hz}}{f}\right)^2 \frac{1}{\sqrt{1 + (f/f_{cut})^2}} [1/\sqrt{\text{Hz}}]$$

■ Standard Quantum Limit

$$h_{SQL} = \frac{1}{L\omega} \sqrt{\frac{8\hbar}{m}} = 2.8 \times 10^{-24} \left(\frac{3\text{km}}{L}\right) \left(\frac{30\text{kg}}{m}\right)^{\frac{1}{2}} \left(\frac{100\text{Hz}}{f}\right) [1/\sqrt{\text{Hz}}]$$

Michelson Interferometer (MI)

■ GW signals and Dark/Bright Port in the dark fringe locking



- **Bright Port**

Photon that is not interacted with GWs will come. → Discarded if it is not recycled ☹️.

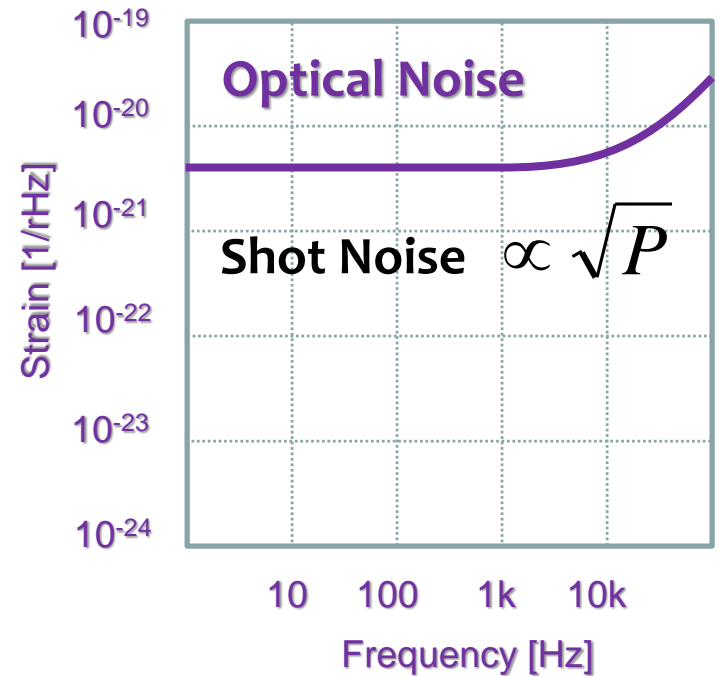
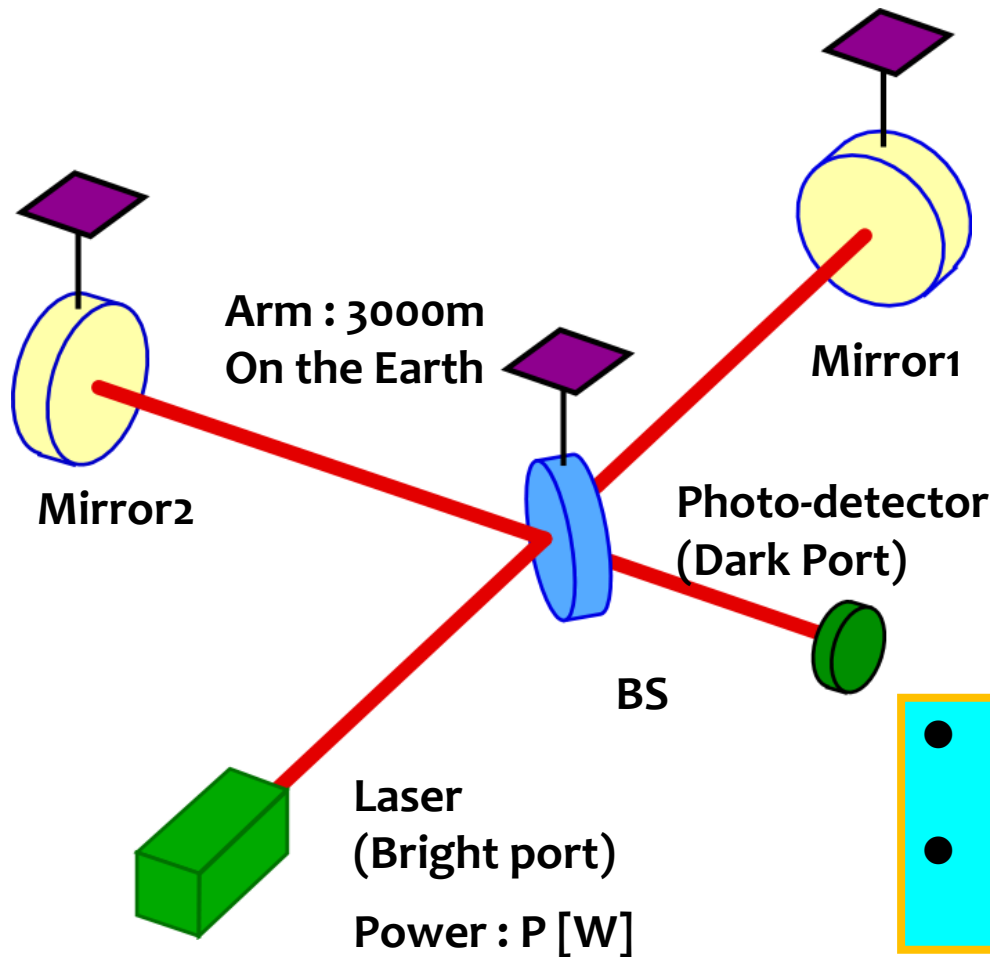
- **Dark Port**

Photon that is interacted with GWs will come. → GW signals 😊!!

Michelson Interferometer

(MI)

■ Michelson Interferometer

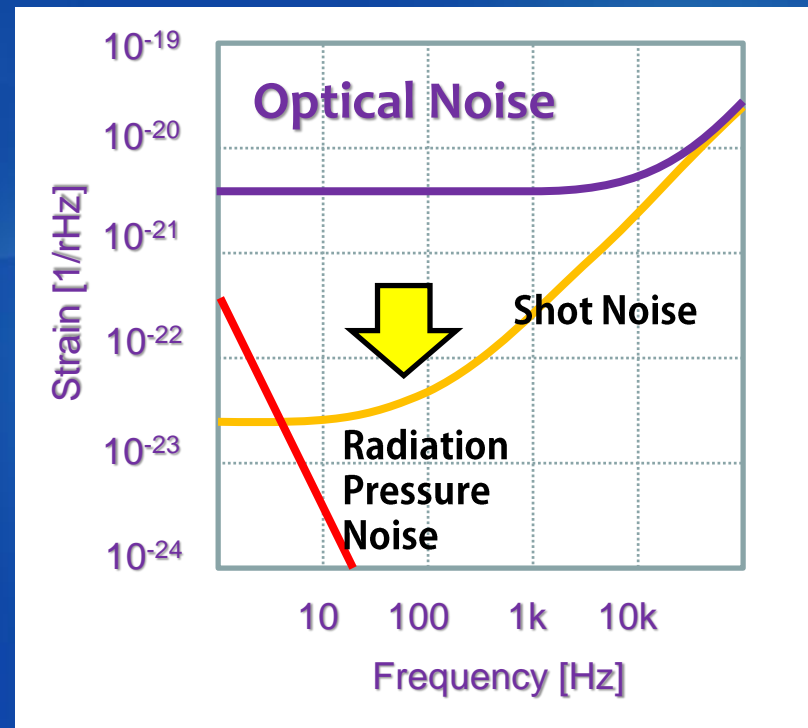
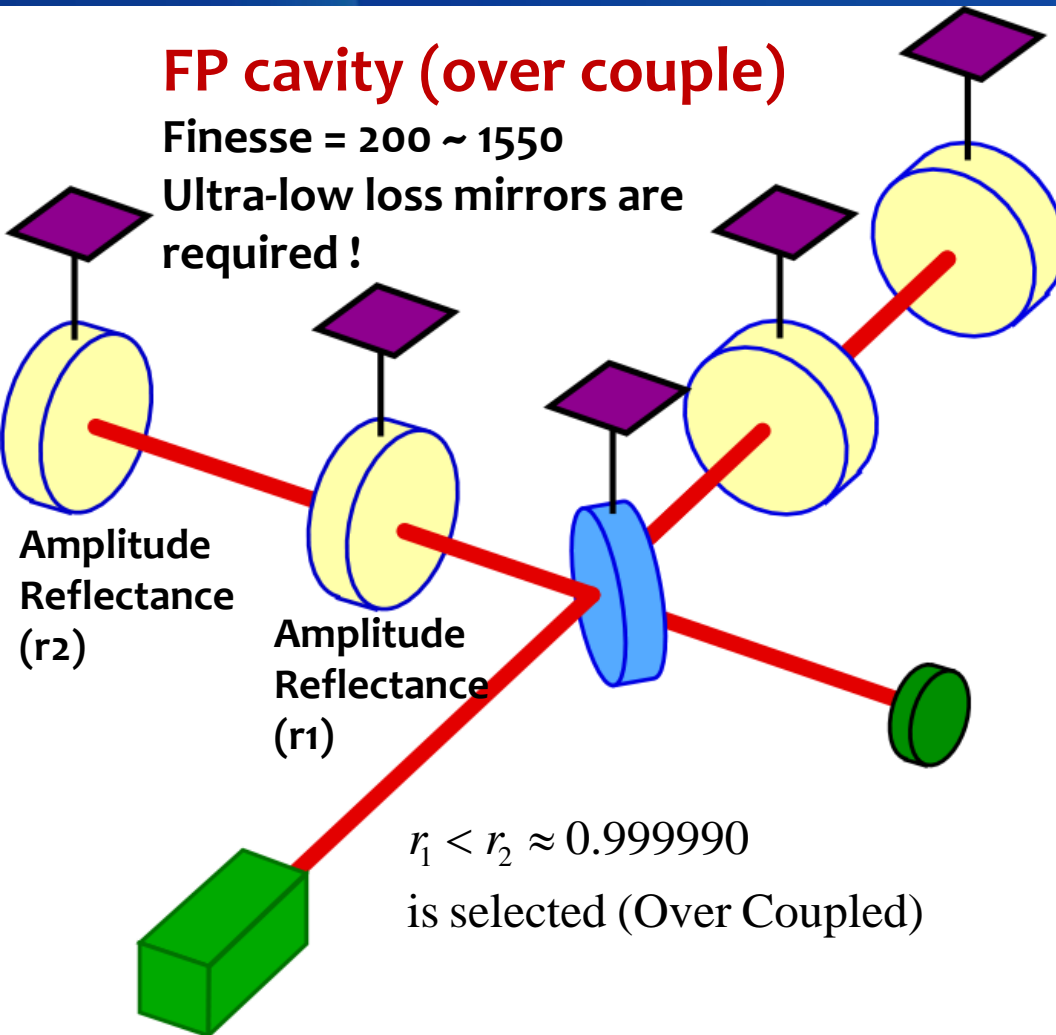


- Problem 1
Short light path \rightarrow Less GW Effects
- Solution
Insert Fabry-Perot Cavity for multi path

FP Michelson Interferometer

- FPMI -

■ Michelson Interferometer



- Finesse : Multi Reflection parameter

$$\mathcal{F} = \frac{\pi \sqrt{r_1 r_2}}{1 - r_1 r_2} \quad N = \frac{2\mathcal{F}}{\pi}$$

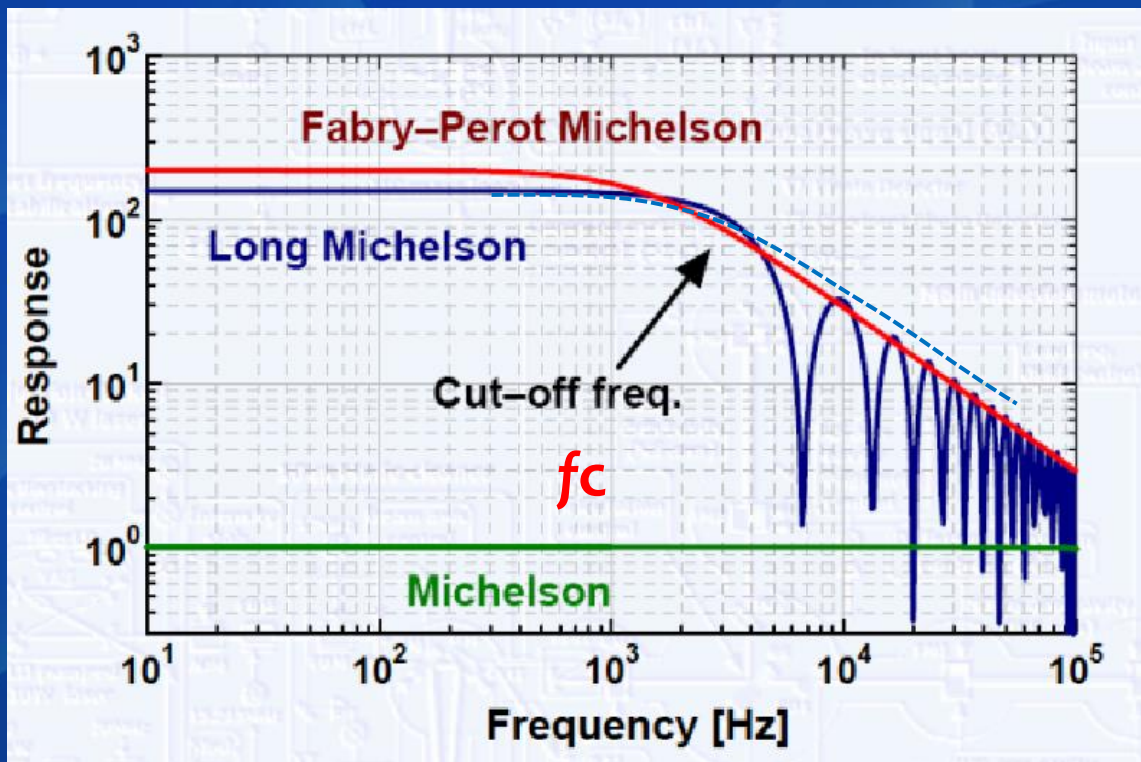
- Effective multi reflection number :

FP Michelson Interferometer

- Fabry-Perot Cavity Effect -

■ Fabry-Perot Cavity Response for GWs

- GW Effect is proportional to traveling path length below frequency (f_c) that corresponds to optical storage time.
- While, GW effect cancellation will be dominant above f_c .

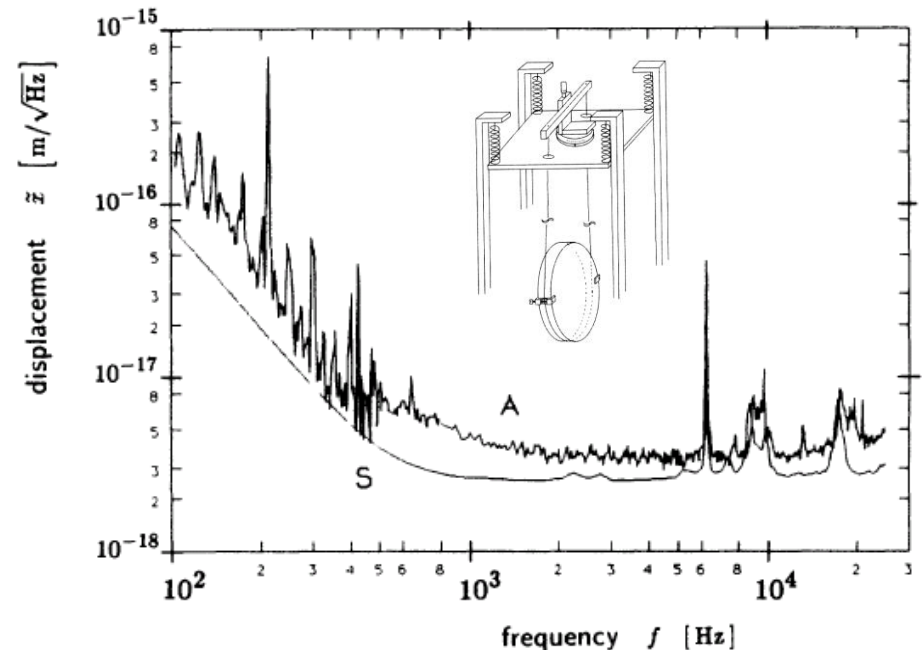
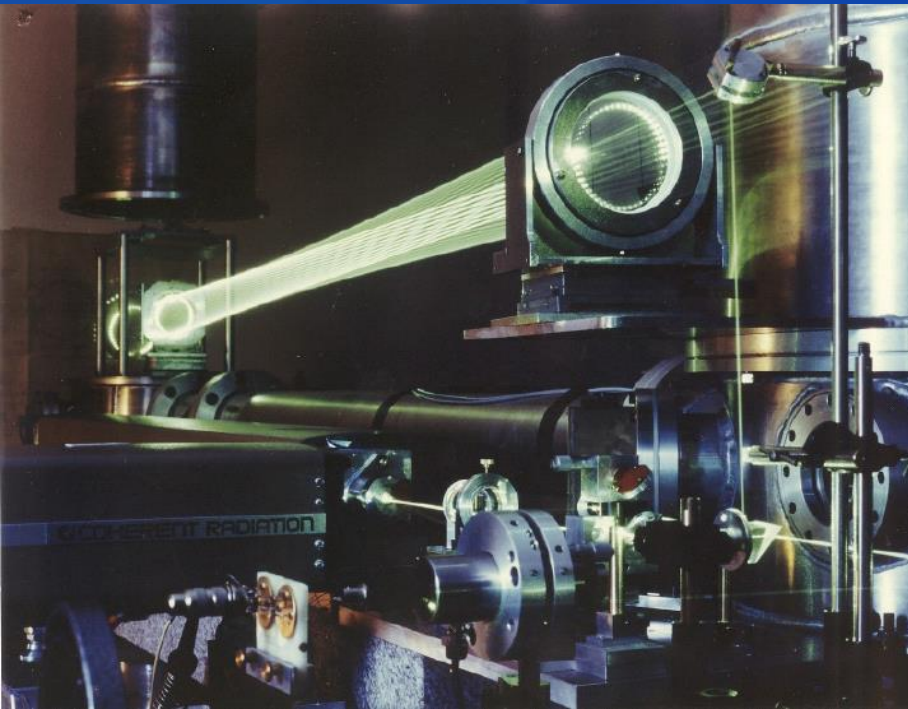


$$f_c = \frac{c}{4LF}$$

Delay-Line Laser Interferometer

- “Noise behavior of the **Garching** 30-meter prototype gravitational-wave detector”, D. Shoemaker et al., Phys. Rev. D 38 (1988).
- Arm length : 30m @ **MPQ**, Ar ion laser were used. 100 reflections bw arm mirrors (total 3000m light path).
- Mirrors are suspended like a pendulum for seismic noise Isolation

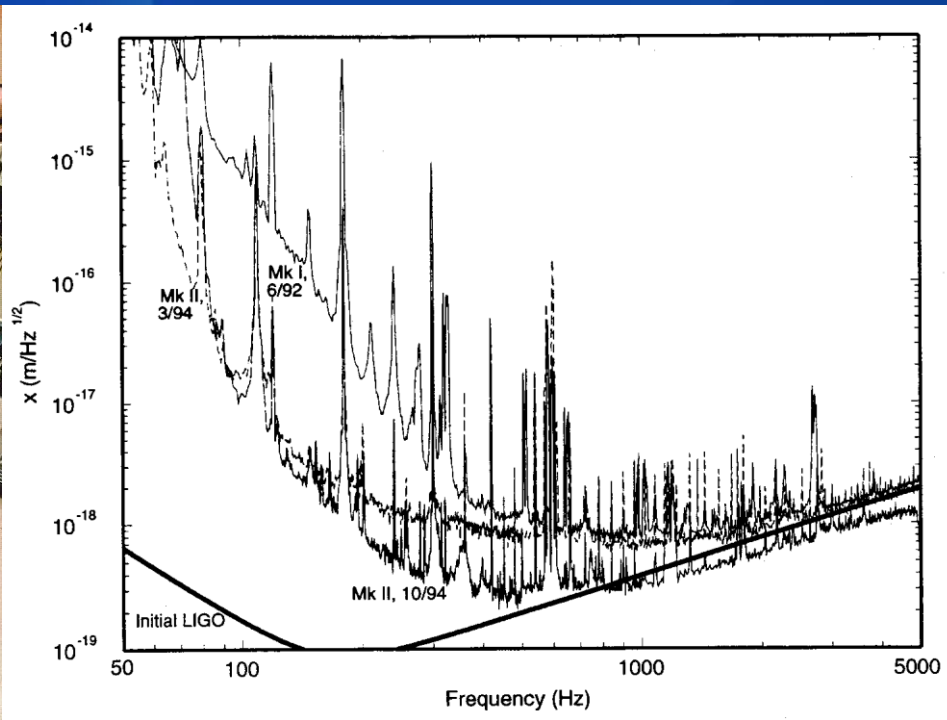
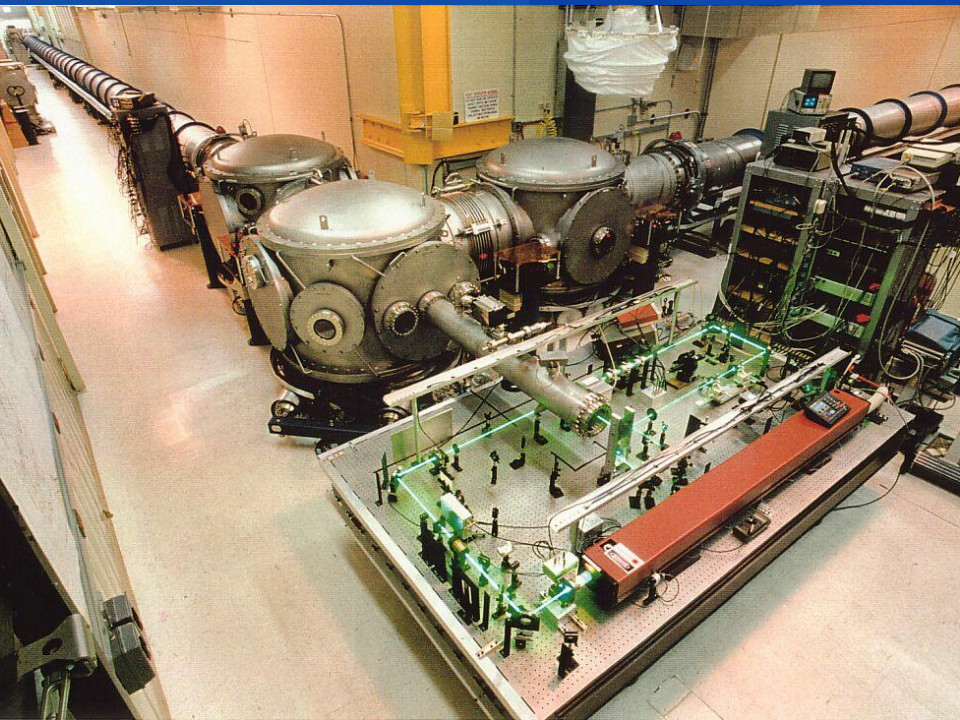
$$h \sim 5 \times 10^{-17} [m/\sqrt{Hz}]$$



Fabry-Perot Laser Interferometer

- Phys. Lett. A 218 (1996) 157. (LIGO was funded in 1992 !)
- Arm length : 40m @ Caltech, Ar ion laser were used. Fabry-Perot cavities were set in both arms.
- Mirrors are suspended like a pendulum for seismic noise Isolation

$$h \sim 3 \times 10^{-19} [m/\sqrt{Hz}]$$



Japanese Laser Interferometer

2017
~2018

2015

2010

2002

1995
(Ph.D)

1991
(M,D)



KAGRA (3km, Underground, Cryogenic)

KAGRA Started in 2010

Thermal Noise Reduction Verification in CLIO

Cryogenic IFO Demonstration using CLIK and CLIO

TAMA 300 Development

Underground IFO (LISM)

TAMA LISM Observation

Low Loss High quality mirror Development

FP and Delay-Line Comparison

3m FP type Proto-Type

LISM FP type in Tokyo

Delay-Line Type in ISAS

Michelson Interferometer (MI)

GW signals and Dark/Bright Port in the dark fringe locking

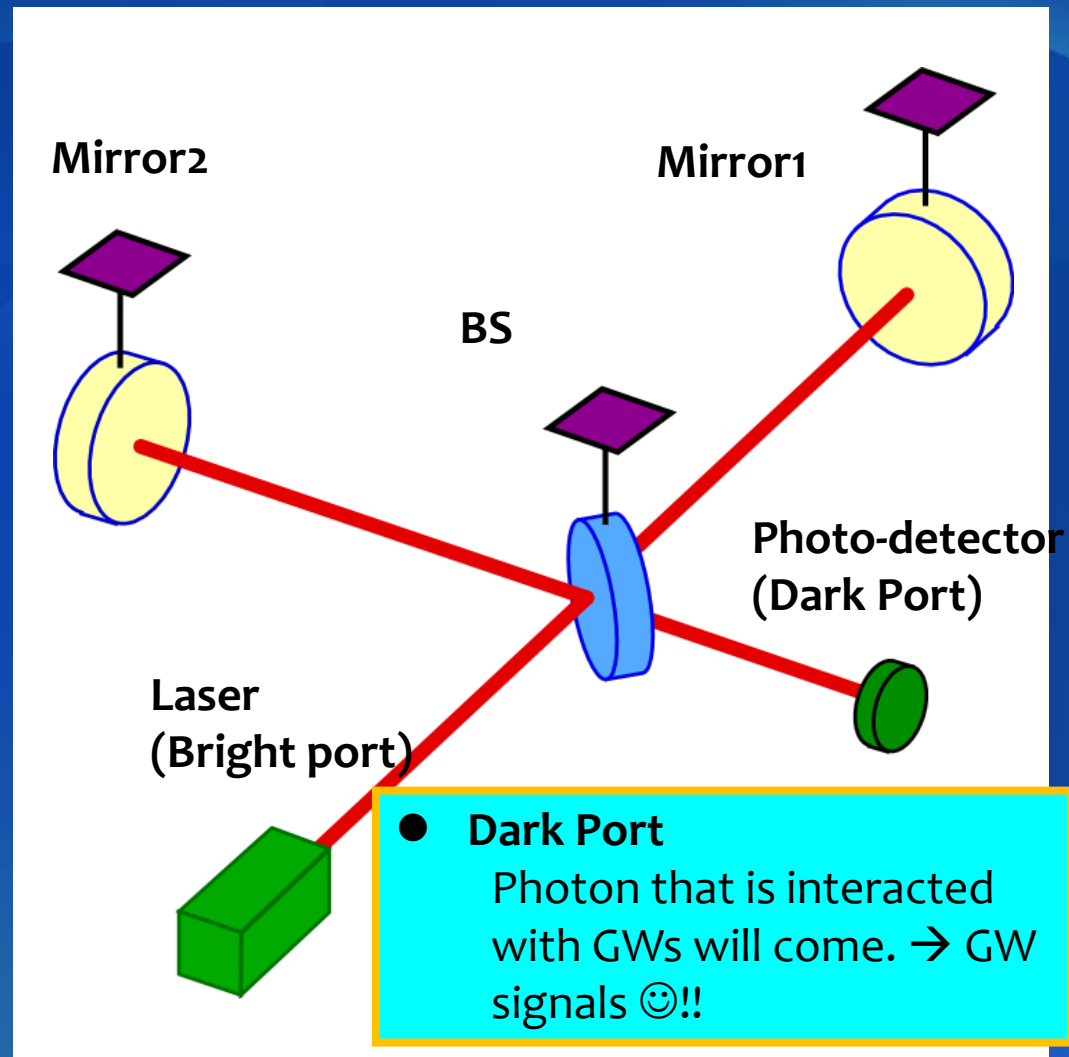
● Bright Port

Photon that is not interacted with GWs will come. →

Discarded if it is not recycled ☹.

Bright fringe can be regarded as reflected laser power from a “Compound” mirror whose actual entity is MI !!

Let's make a FP cavity with a new mirror in front of a laser and “compound” mirror to enhance the effective injected laser power to MI!!



FP Michelson Interferometer

-Recycled MI-

■ GW signals and Dark/Bright Port in the dark fringe locking

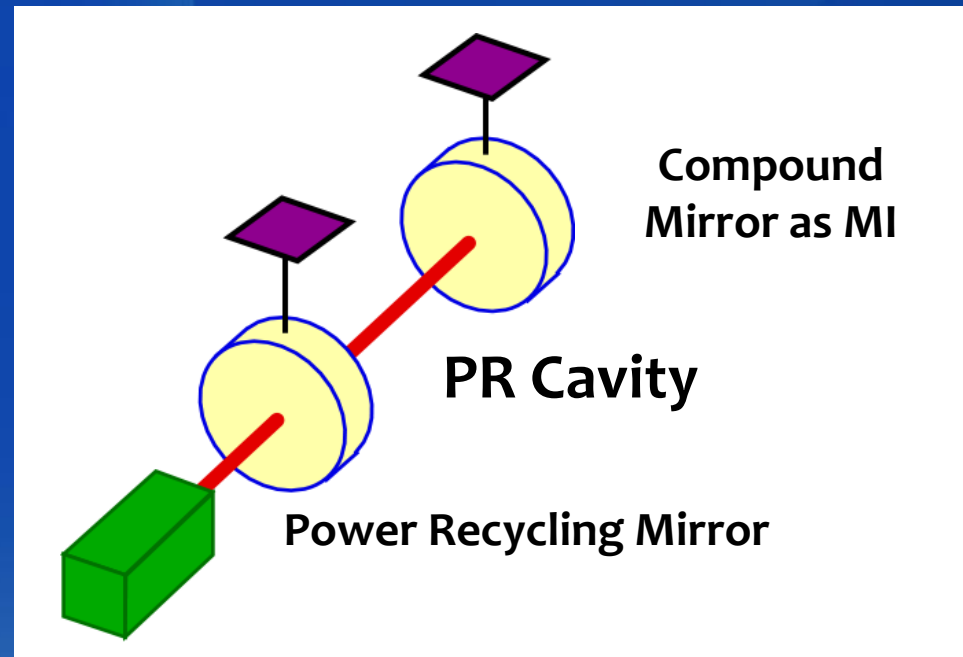
● Bright Port

Photon that is not interacted with GWs will come. → **Discarded if it is not recycled** 😞.

Bright fringe can be regarded as reflected laser power **from a “Compound” mirror** whose actual entity is MI 😊 !!

Let's make a FP cavity with a **new mirror in front of a laser** and **“compound” mirror** to enhance the effective injected laser power to MI.

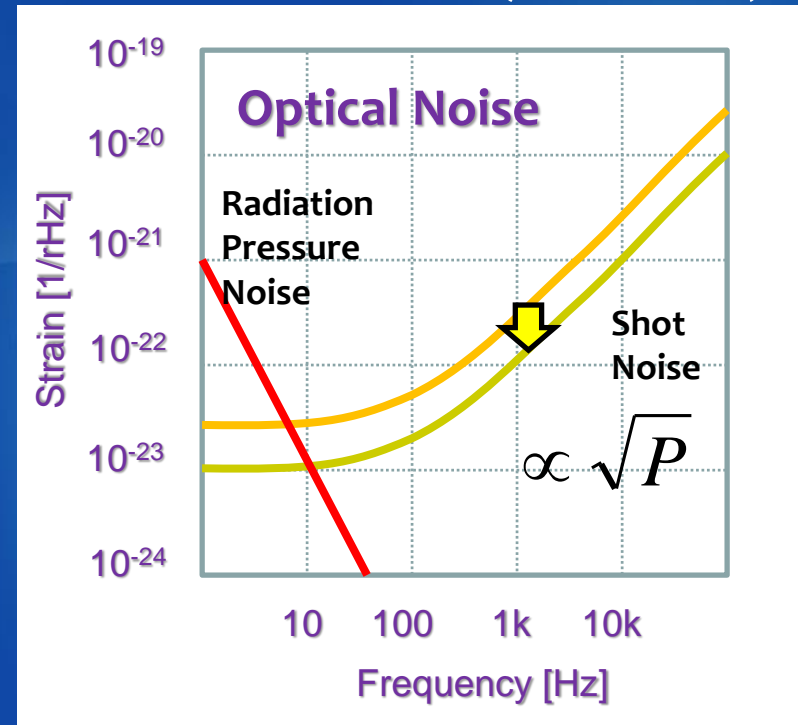
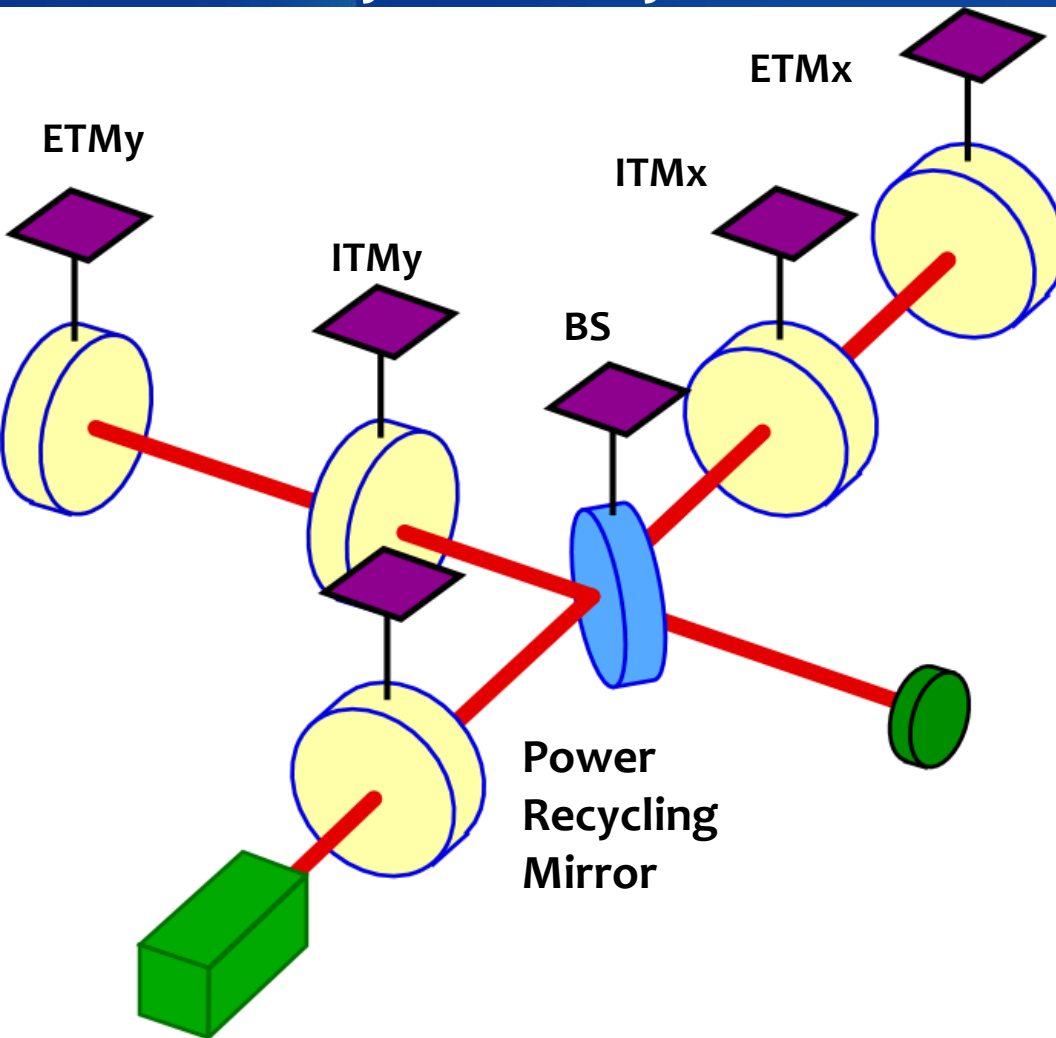
- Enhancement of effective laser power is defined as Recycling Gain.
- 10 – 50 gain is practical because reflectance of the Compound mirror cannot be so high.



Power Recycled FPMI

- PR-FPMI -

Power Recycled Fabry-Perot Michelson Interferometer (PR-FPMI)



Better Sensitivity Quest

- What is GW signal proportional to ? -

■ What is GW signal proportional to ??

!! Photon number stored in a FP cavity !!

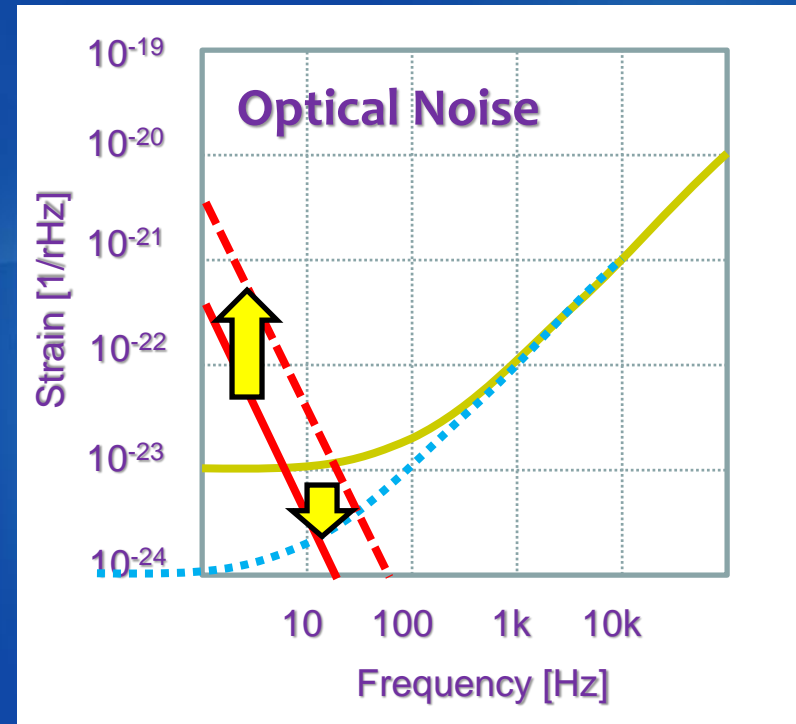
Arm length x Finesse x Input Power x Recycling Gain

All parameters reach best effort, except Finesse. However, high finesse just enhanced the sensitivity at low frequency that is dominated by other noise such as seismic, gravity gradient and RP noise, because the FP cavity cancelling effect that was referred in the previous page.

Is there any method that can individually set the storage time of photons that are not interacted with GWs (go to PRM) and that are interacted (go to dark port)?

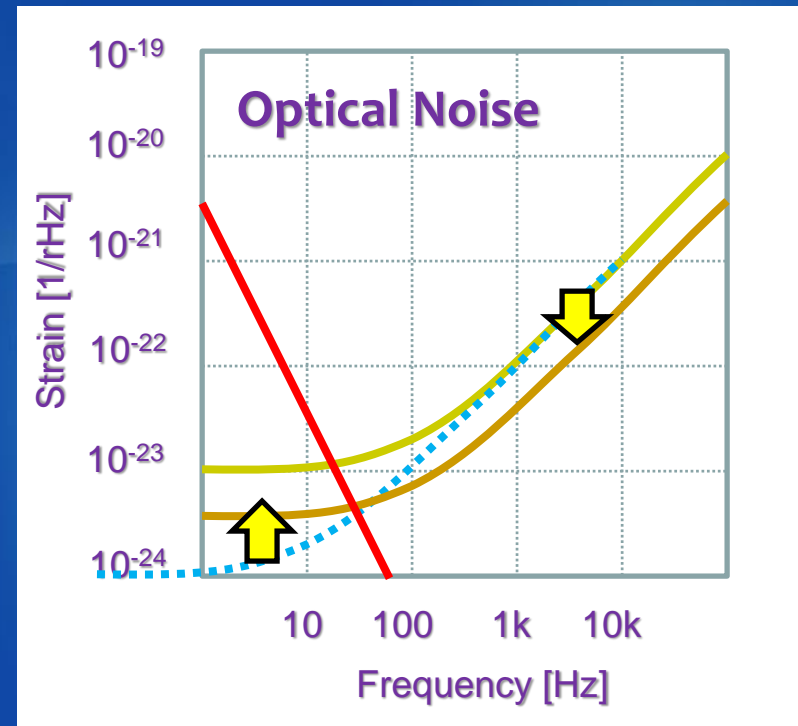
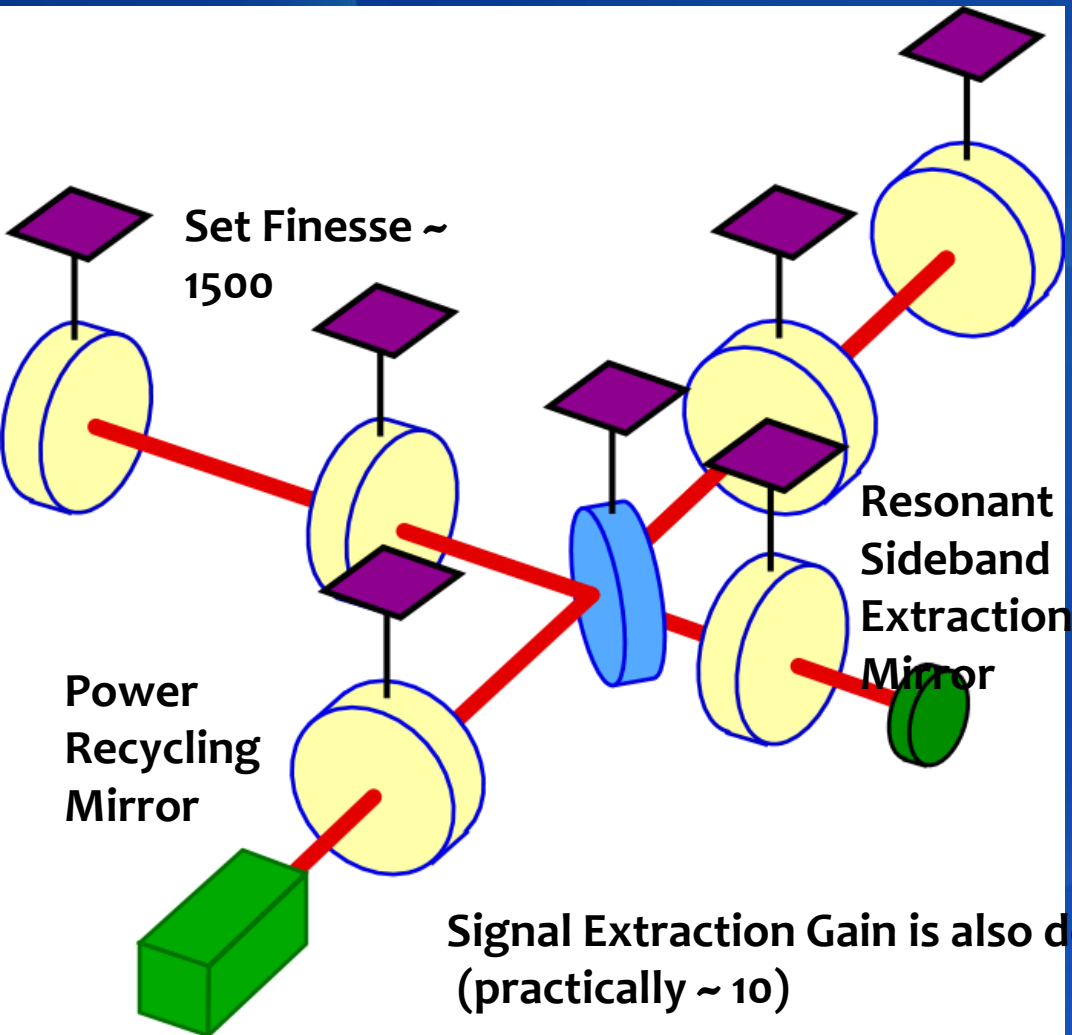
Answer :

Resonant Sideband Extraction (RSE) Mirror !!



PR-FPMI with RSE technique

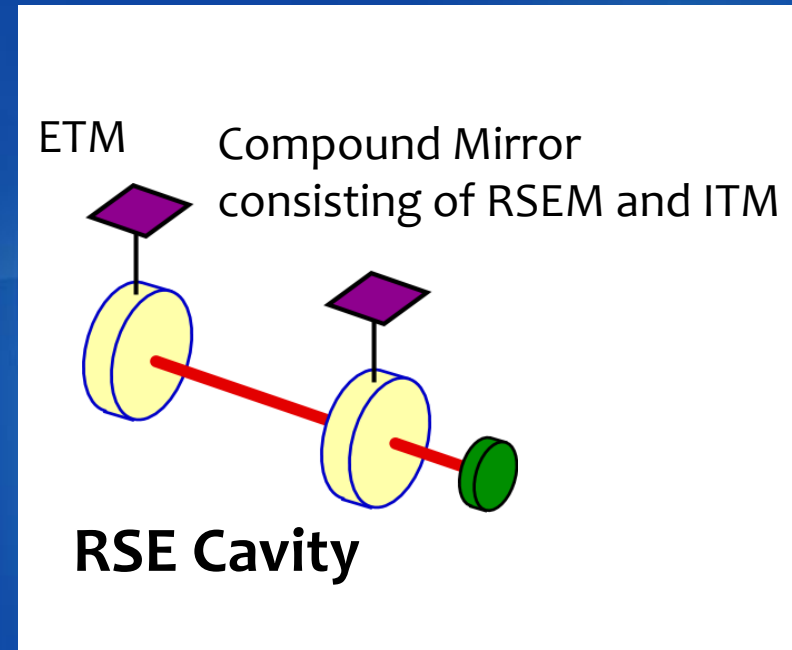
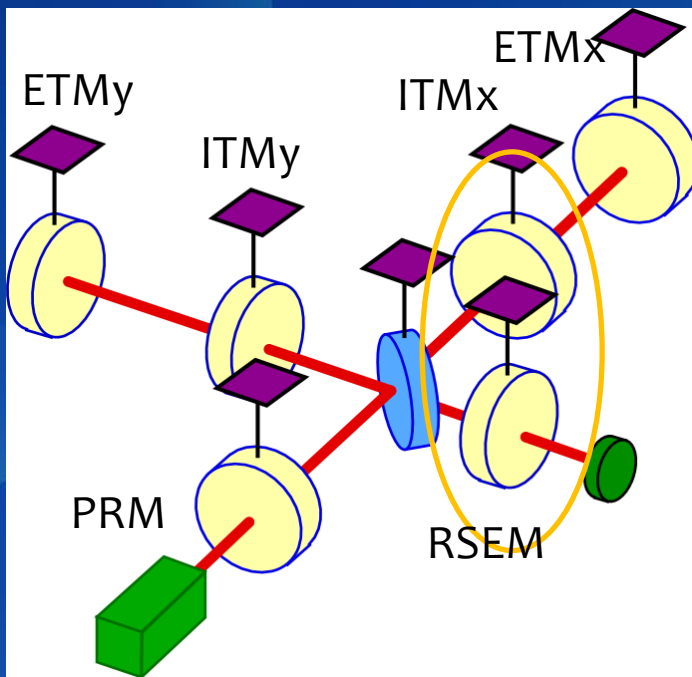
PR-FPMI With Resonant Sideband Extraction



This is one of example of sensitivity using RSE technique.

RFPMI with RSE technique

RFPMI With Resonant Sideband Extraction

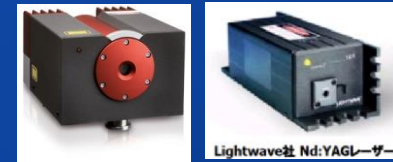


- **GW sidebands (photons that are interacted with GWs)** are generated in the FP cavity. For these photons, reflectance of ETM and a compound mirror, which consists of RSEM and ITM, decide the optical storage time !
- **RSE Cavity optical storage time can be set freely (longer and shorter than arm FP cavity storage time) by RSE mirror reflectance and position.**
- On the other hand, optical storage time of photons that are NOT interacted with GWs is decided by FP arm cavity itself.

Laser

- as a ruler for precise length measurement -

- Invention of principle of a laser in 1953 by Towns and Schawlow is epochmaking.
- Development of laser realized interferometric GWD idea by R.Weiss (MIT), instead of Resonant Type GWD, at that time.
- **Ar+ gas laser(514nm)** was used for 80' GWD development.
 - Unstable
 - High energy → mirror burn-in due to hydro-carbon in vacuum
- **Nd³⁺:YAG laser (1064nm)** as a solid state laser (NPRO style) by Byer made revolution in 90'
 - Narrow line width
 - Easy handling
 - Stable and High Efficiency (100 times better)
 - IR region → avoid burn-in
 - Easy power-up by amplifier
- After 2010', a **fiber laser** will be main !?



Laser

- requirements -

- Requirement for 1st and 2nd generation GWD (default : 1064nm)
 - Single mode (TEM₀₀) → should be additionally stabilized
 - Low Frequency noise (dv/v) → should be additionally stabilized
 - Low Amplitude noise (dP/P) → should be additionally stabilized
 - Low transverse mode ($M_2 \sim 1$)
 - Stable and tunable of the laser frequency (PZT on a crystal, Heater and Outer EOM)
 - ~200W CW power
 - Optics for the laser can be high quality (low loss, low absorption, low scattering, high damage threshold)
 - Because GWD is a “phase sensor”, a short wavelength laser seems to be preferable. However, optics cannot meet GWD quality ☹.
 - Two applicants
 - Solid state seed laser + Bulk Amps (MOPA)
 - Solid state seed laser + Fiber Amps (MOFA)

Laser

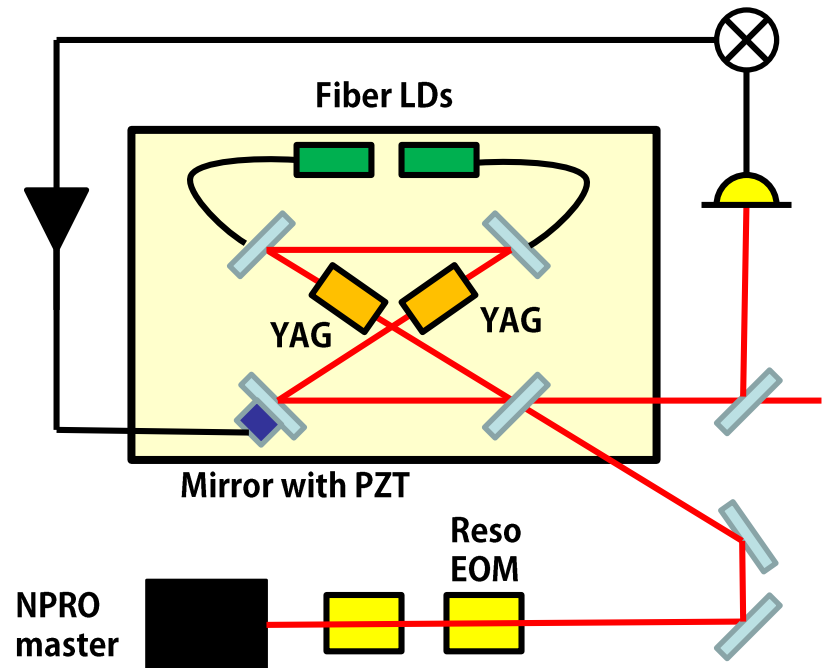
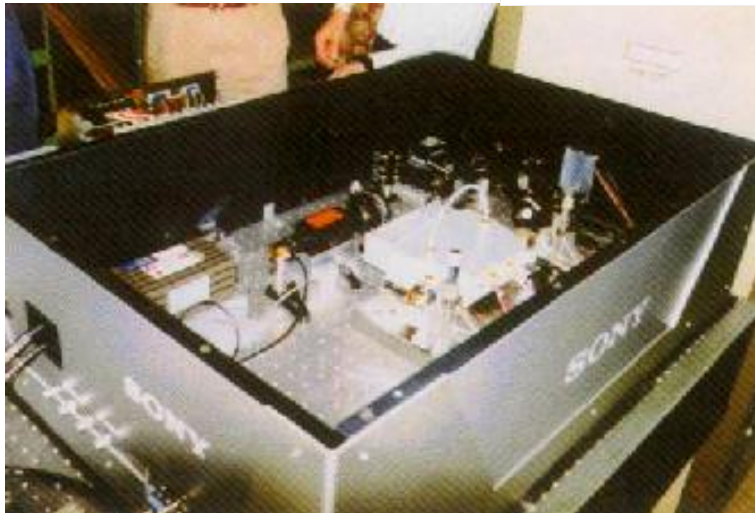
- High Power laser development -

■ Injection Locking Laser

- Laser oscillation of a slave laser consisting of bow tie FP cavity that includes two YAG crystal inside is synchronized with a master NPRO laser.
- Frequency noise is dominated by a master laser, while amplitude noise is dominated by the slave laser.

In 2000, SONY produced
10W, 1064nm laser

SONY



Laser

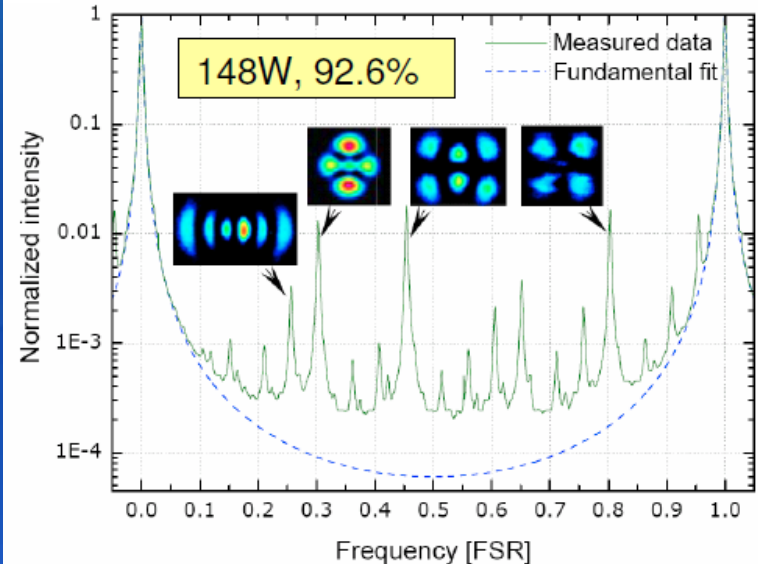
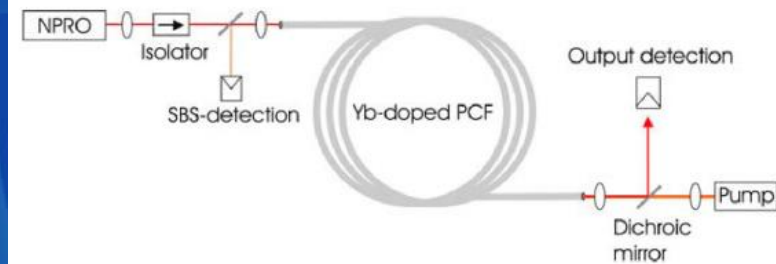
- MOPA or MOFA -

■ Requirement for 1st and 2nd generation GWD

- Solid state seed laser + Bulk Amps (MOPA)
- Solid state seed laser + Fiber Amps (MOFA)



Fiber Laser Idea



FP cavity Mirrors

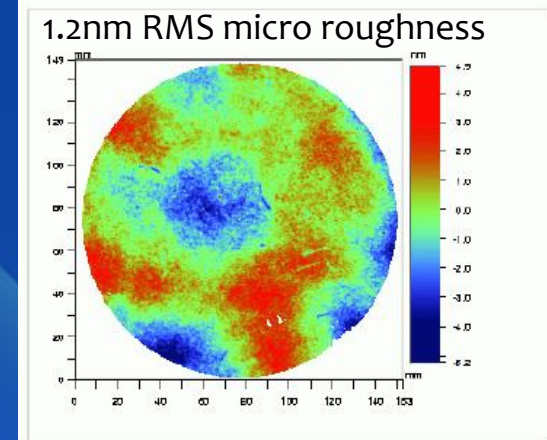
as “free falling mass”, “photon handler”, “thermal noise origin”

■ **Size Requirement** : Large is better to reduce SQL, thermal noise

- ~ 50kg is practical limit. Diameter becomes 35cm ~ 22cm.
- practical means substrate production, polishing, coating.

■ **Optical Requirement** : Low loss (scattering and absorption) and homogeneity

- Multi-layered optical coating is inevitable for 1064nm on substrate
- Smooth surface (polishing) is required
- Loss spoils shot noise, recycling gain, squeezing effect and increases scattered light noise.
- Less than ~ 50 ppm loss for 30cm diameter is required.
- ~ 2km Radius Of Curvature (ROC) required



FP cavity Mirrors

as “free falling mass”, “photon handler”, “thermal noise origin”

■ Substrate Requirement : Low loss (absorption and mechanical) and homogeneity

- Absorption results in heat lensing → spoils GWD optical performance.
- For substrate (SiO₂ or Sapphire, Silicon in 3rd GWD) : $\phi < 10^{-8}$
- For optical coating (Ta₂O₅/SiO₂) : $\phi < 10^{-4}$ (mainly comes from Ta₂O₅)
 - Thermal noise due to optical coating limits the 2nd GWD sensitivity !!

Subjects	Requirement
Substrate	SiO ₂ : Adv-LIGO, Adv-VIRGO, GEOHF Al ₂ O ₃ : KAGRA
Size	D:35cm ~ 22 cm t : ~15cm
Substrate Loss	< 0.1ppm (SiO ₂), <20ppm(Al ₂ O ₃)
Coating Loss	< 40 ppm (< 0.5ppm for absorption)

Subjects	Requirement
Surface Micro Roughness	< 0.1 nm rms
Surface Waviness	< $\lambda / 500$
Coating Mechanical loss	$\phi < 10^{-4}$
Substrate loss	$\phi < 10^{-8}$
ROC	~ 3km
ROC error	~ 10m

FP cavity Mirrors

- how to make or what company should we order to ? -

■ Mirror manufacturer are strictly limited in the World

□ Polish and Coating

- ✓ LMA Lyon in France
- ✓ CSIRO in Australia
- ✓ REO
- ✓ ATF



□ Polish

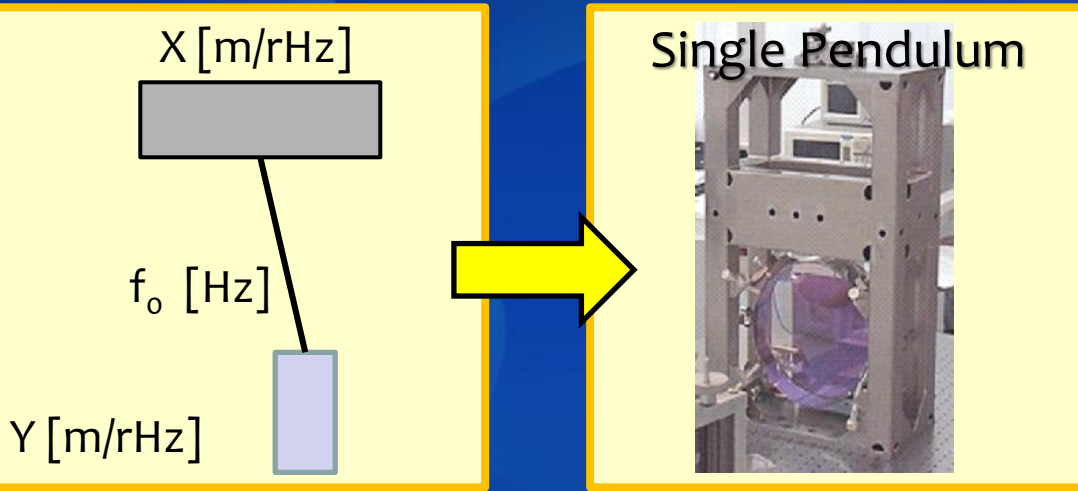
- ✓ ZYGO in USA

■ In Japan...

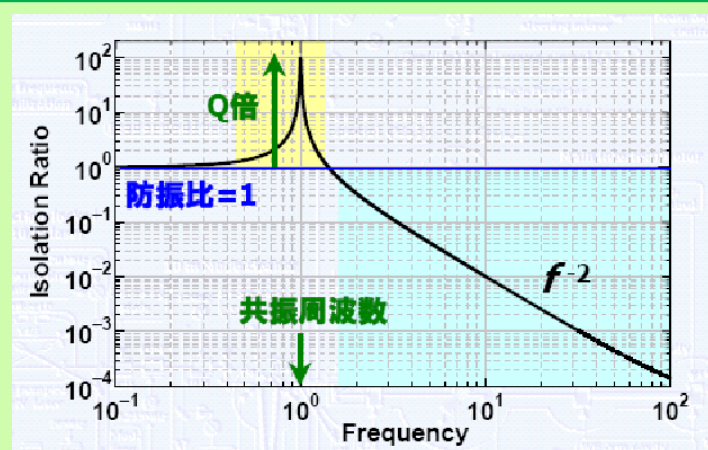
- Japan Aviation Electronics (not now)
- Sigma KOKI (small optics ?)
- Showa Koki
- Tokai Kogaku
- Canon (sapphire ??)

Inevitable Seismic Noise Introduction

■ Mirrors are suspended like a pendulum on the Earth as free mass



- A mirror is inevitably excited by seismic noise through the pendulum transfer function (TF).
- TF has isolation effect for seismic noise !



Transfer Function (TF)

$$\frac{Y}{X} = \left(\frac{f_0}{f} \right)^2$$

$$\frac{Y}{X} = \left(\frac{f_0}{f} \right)^{2N}$$

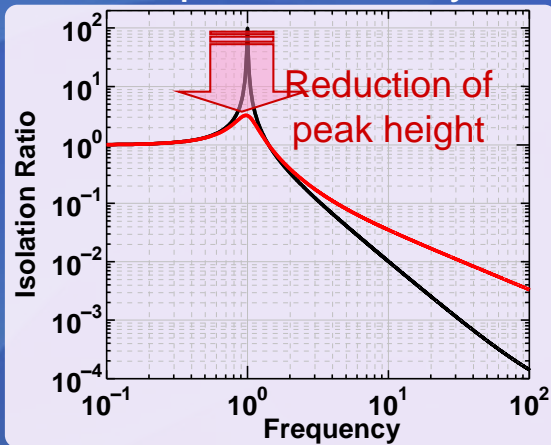
- For more isolation...
 - Multi-stage
 - Lower pendulum resonance frequency
- Trade-off : Many resonance mode (excited by Q) damping is necessary.

Better Seismic Noise Isolation

- For more isolation...
 - Multi-stage
 - Lower pendulum resonance frequency

Damping

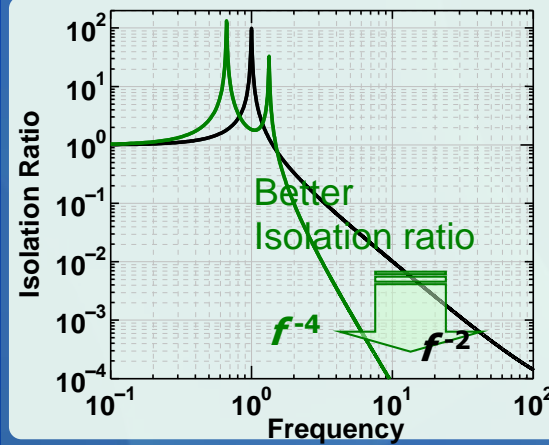
Reduce Q-value
→ improve stability



Degraded isolation

Multi stage

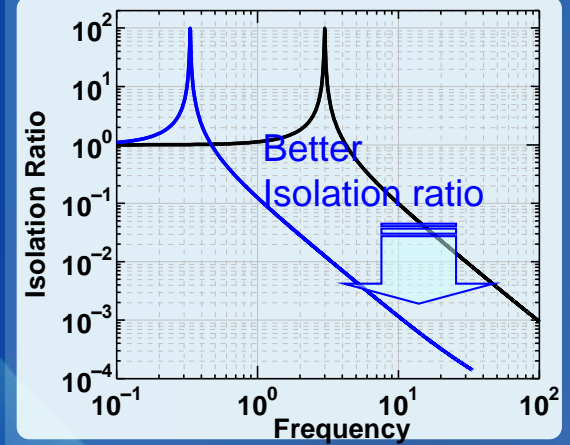
More steep reduction
→ Better isolation



Resonant peaks

Low reso. freq.

Low-freq. cut-off
→ Better isolation



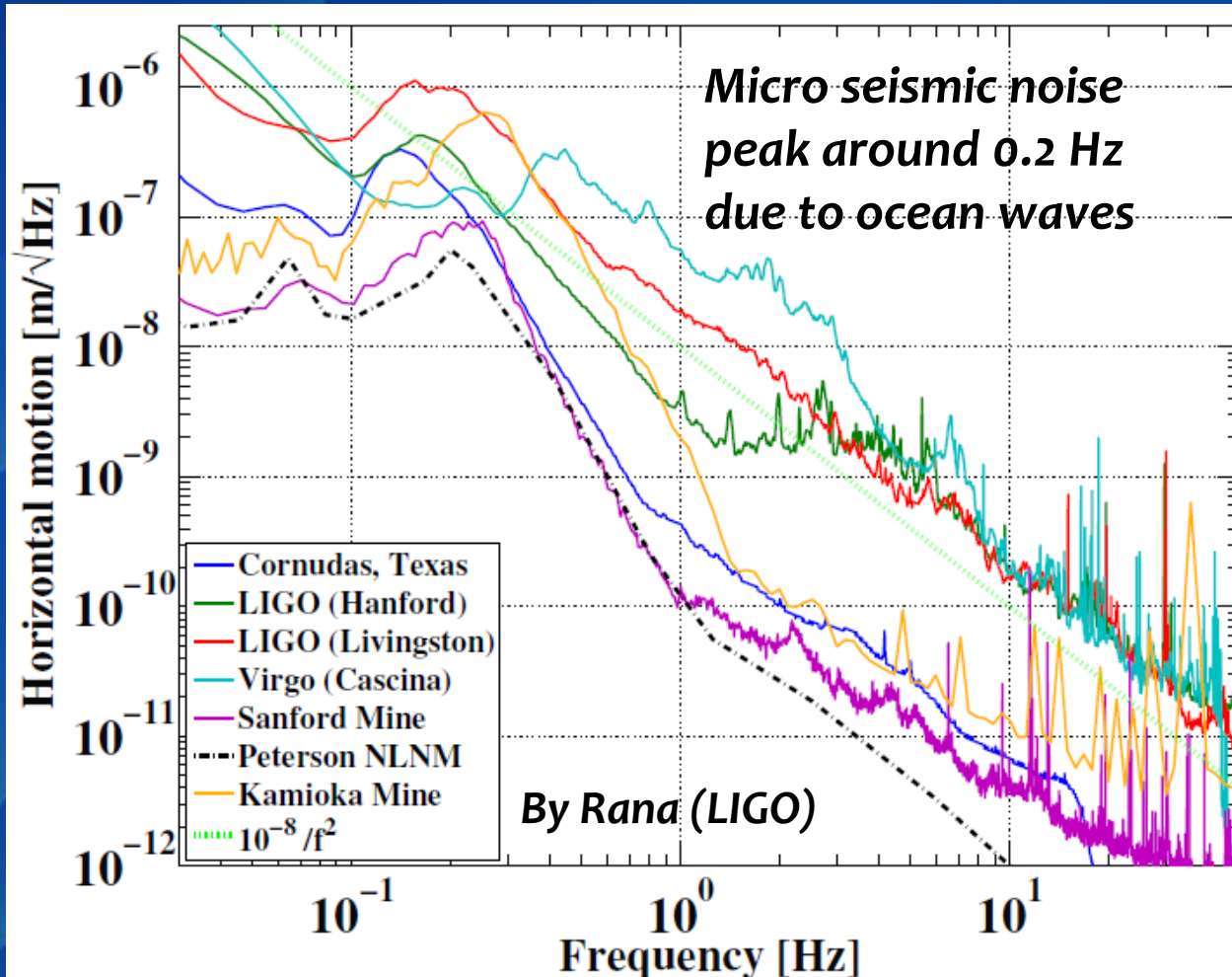
Drift by environment

Ando's (U Tokyo) View Graph

Seismic Noise Variation on the Earth

- 0.1 Hz ~ 100 Hz -

■ Seismic Noise ($f > 0.1\text{Hz}$) that can be isolated by technology



**Seismic Noise Model
Above 1 Hz**

$$f_{seis} = A \times \left(\frac{1}{f} \right)^2 [\text{m}/\sqrt{\text{Hz}}]$$

$A \approx 10^{-7}$ city area

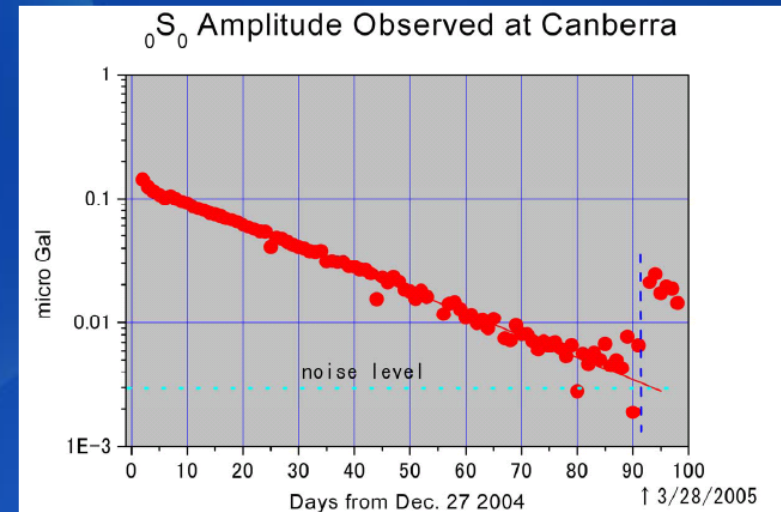
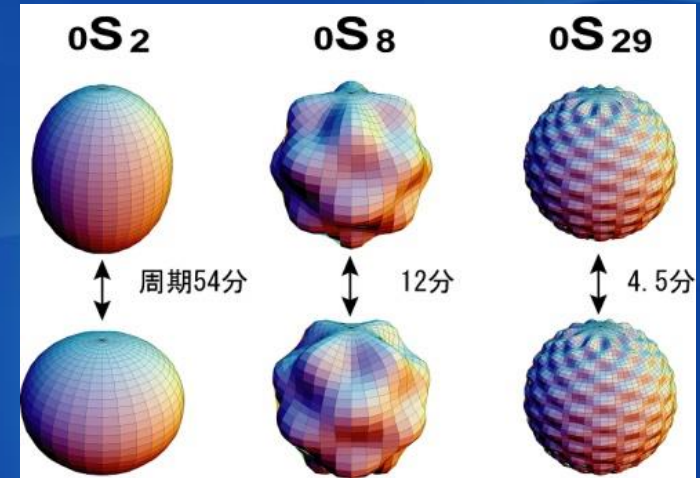
$A \approx 10^{-9}$ underground

Seismic Noise Variation on the Earth

- Low Frequency Range (< 0.1 Hz) -

■ Seismic Noise ($f < 0.1$ Hz) that cannot be isolated by technology

- Tidal motion (~ 12 hours)
- Local trend (\sim months)
- Underground water (~ 1 month)
- Snow fall (~ 1 month)
- Rain, Wind, Air Pressure (\sim months)
- Free Oscillations of the Earth (~ 10 minutes)
- Permanent step motion due to large earthquake far away (permanent)



Seismic Noise Isolation

-Active and Passive Isolation and Resonance Damping -

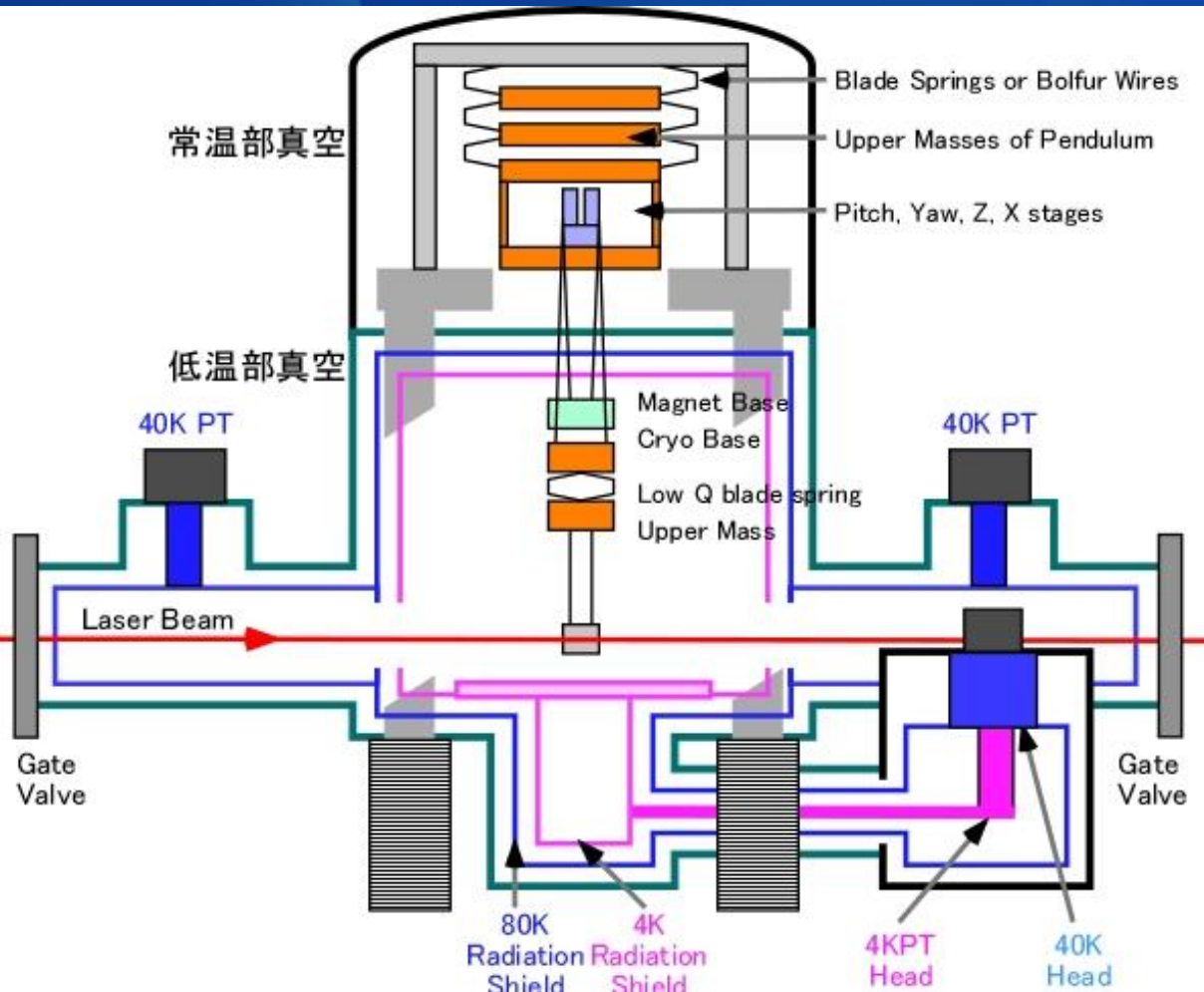
■ Passive Isolation and damping (Simple)

- Metal wire pendulum (~ 1 Hz resonance Frequency) for Horizontal
- Classical blade spring (~ 5 Hz resonance Frequency) for Vertical
- Eddy Current Damping (no active damping)

Seismic Noise Isolation

-Active and Passive Isolation and Resonance Damping -

■ Passive Isolation and damping Example in CLIO (japan)



- 6 stages seismic noise isolation pendulum in CLIO
- 4 stages blade spring, 2 stages wire pendulums
- Orange stages (Copper Plate) motion are damped by NdFe magnets those are set in themselves and magnet Base.
- No active damping.

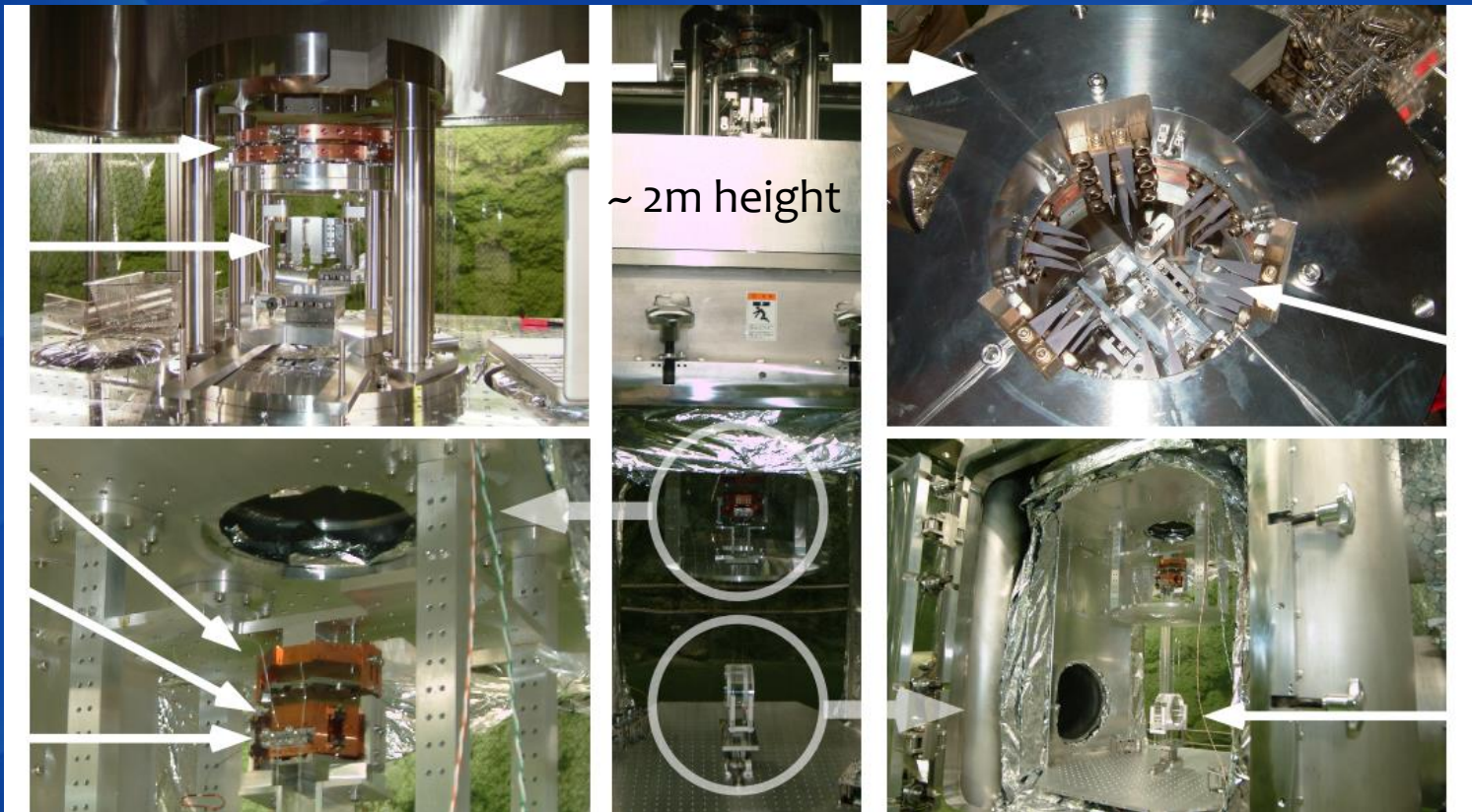
Seismic Noise Isolation

-Active and Passive Isolation and Resonance Damping -

■ Passive Isolation and damping Example in CLIO (japan)

Motorized mirror alignment stage

3 stage blade springs



magnet base

Cryo base

Upper Stage

~ 2m height

Upper masses and magnet base

Sapphire Mirror

Seismic Noise Isolation

-Active and Passive Isolation and Resonance Damping -

Passive Isolation and Active damping in VIRGO Super Attenuator

- Inverted Pendulum (IP) (~ 30 MHz resonance Frequency) for Horizontal
- Geometrical Anti Spring (GAS) (~ 200 MHz resonance Frequency) for Vertical
- Active damping using LVDT sensors and “Digital” feedback to magnet-coil actuators
- Recoil-mass introduction not to introduce actuating point noise

Gravity point is set upper area

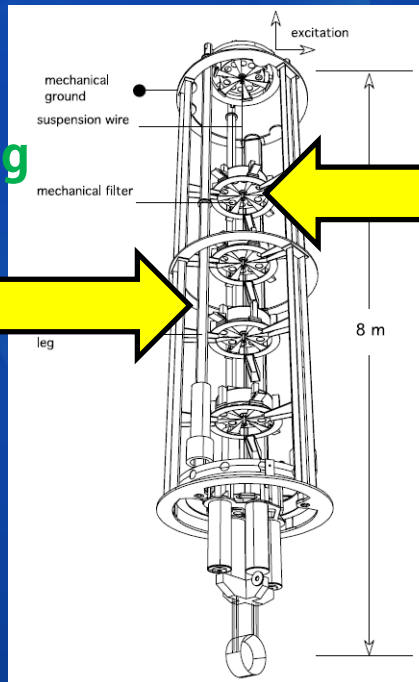
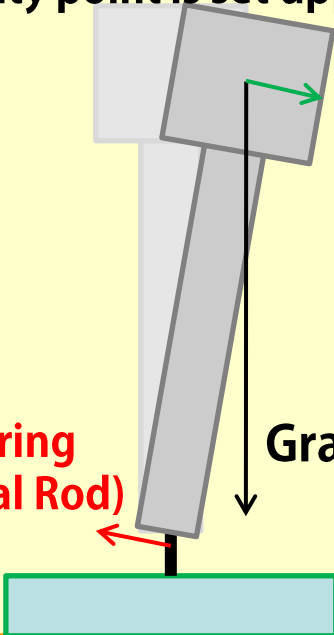
IP

Anti-Spring

30MHz
Resonance
Frequency

Spring
(Metal Rod)

Gravity



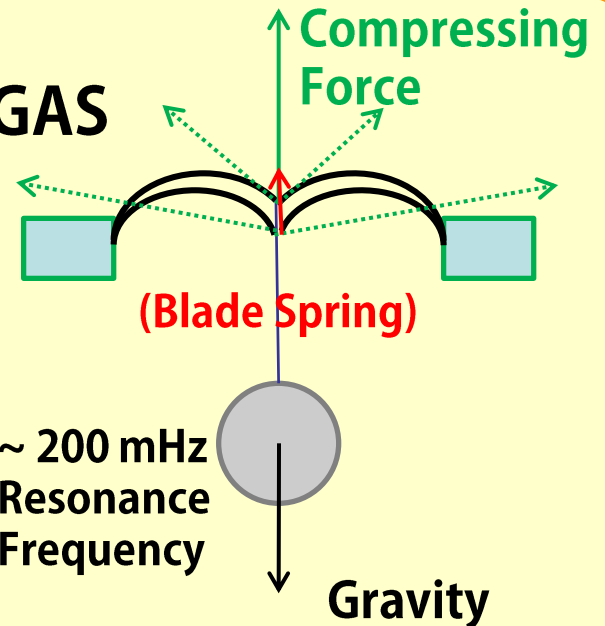
GAS

Compressing
Force

(Blade Spring)

~ 200 MHz
Resonance
Frequency

Gravity



Seismic Noise Isolation

-Active and Passive Isolation and Resonance Damping -

- Principle to obtain low resonance frequency :

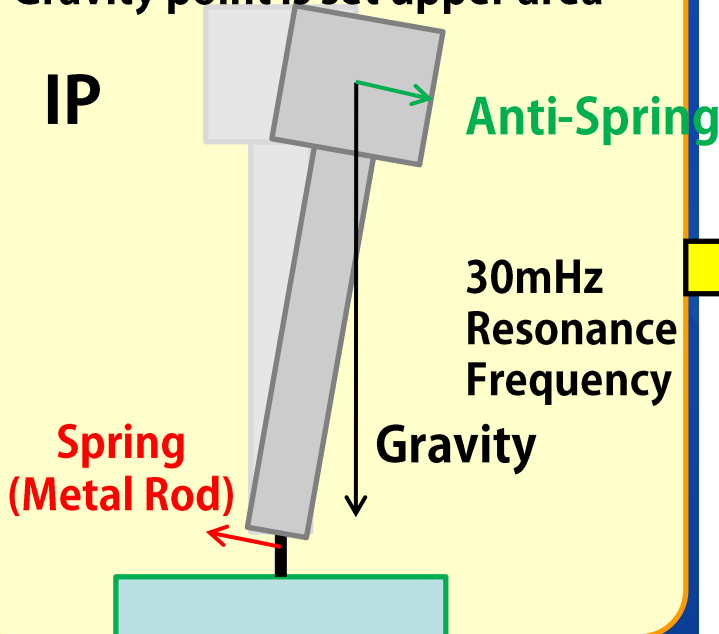
Set the residual spring constant to be small with combination of spring and anti-spring.

- Problem and Trade-off :

1. Spring constant sensitive to temperature → Balanced position change.
2. Internal buckling results in instability.
3. Isolation effect lost at ~100 times resonance frequency because of percussive effect.

Gravity point is set upper area

IP

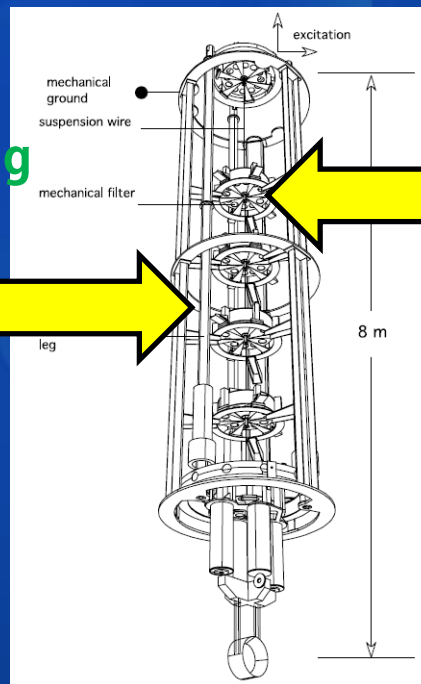


Anti-Spring

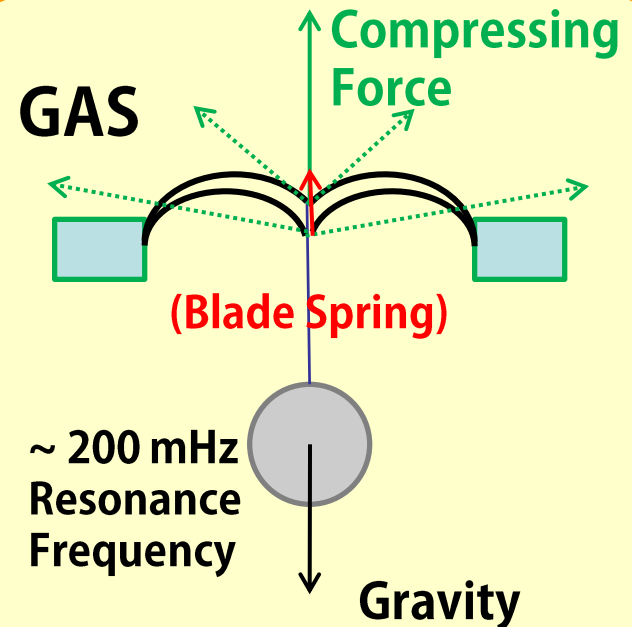
30mHz
Resonance
Frequency

Spring
(Metal Rod)

Gravity



GAS



(Blade Spring)

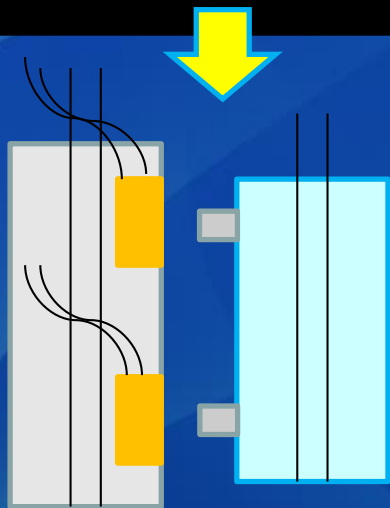
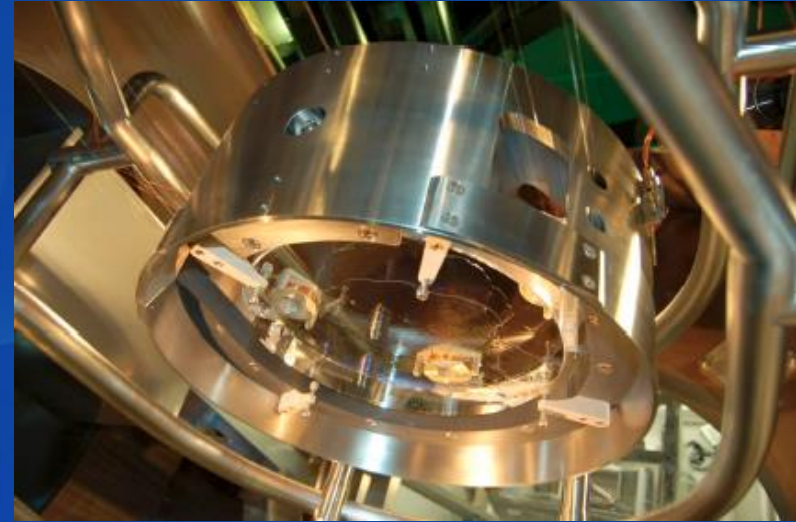
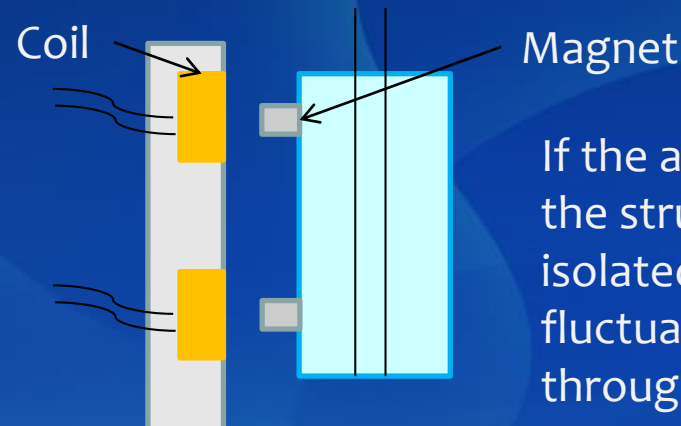
~ 200 mHz
Resonance
Frequency

Gravity

Seismic Noise Isolation

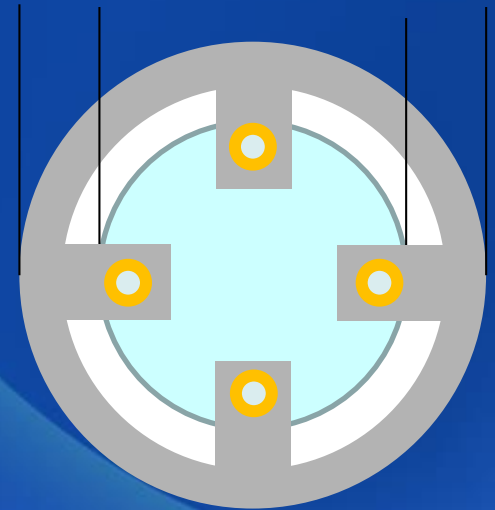
-Active and Passive Isolation and Resonance Damping -

■ Recoil-Mass Introduction



For this remedy, the actuators are also isolated like a pendulum.

←(Counter recoils mass arrangement)→



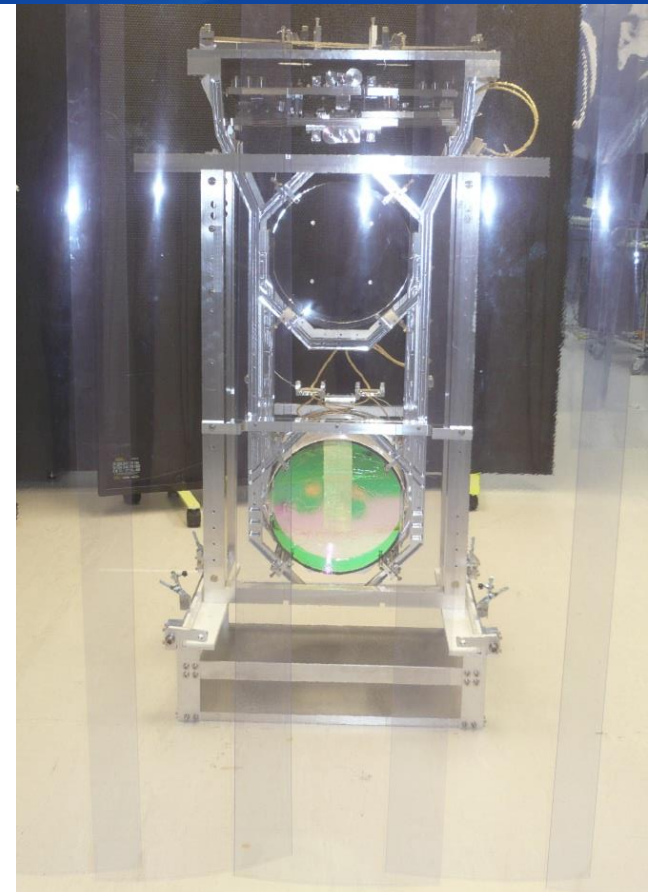
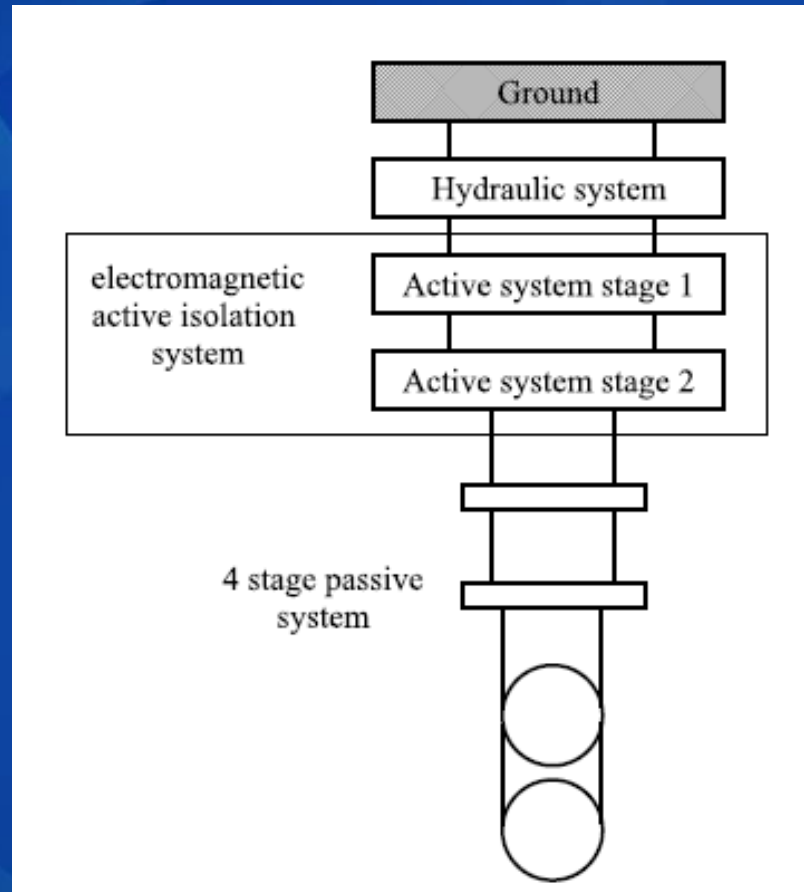
Seismic Noise Isolation

-Active and Passive Isolation and Resonance Damping -

■ Active Isolation introduction and Active damping in GEO and Advanced-LIGO

- HEPI Low frequency active isolation using **accelerometer**.

- Electromagnetic active isolation system.

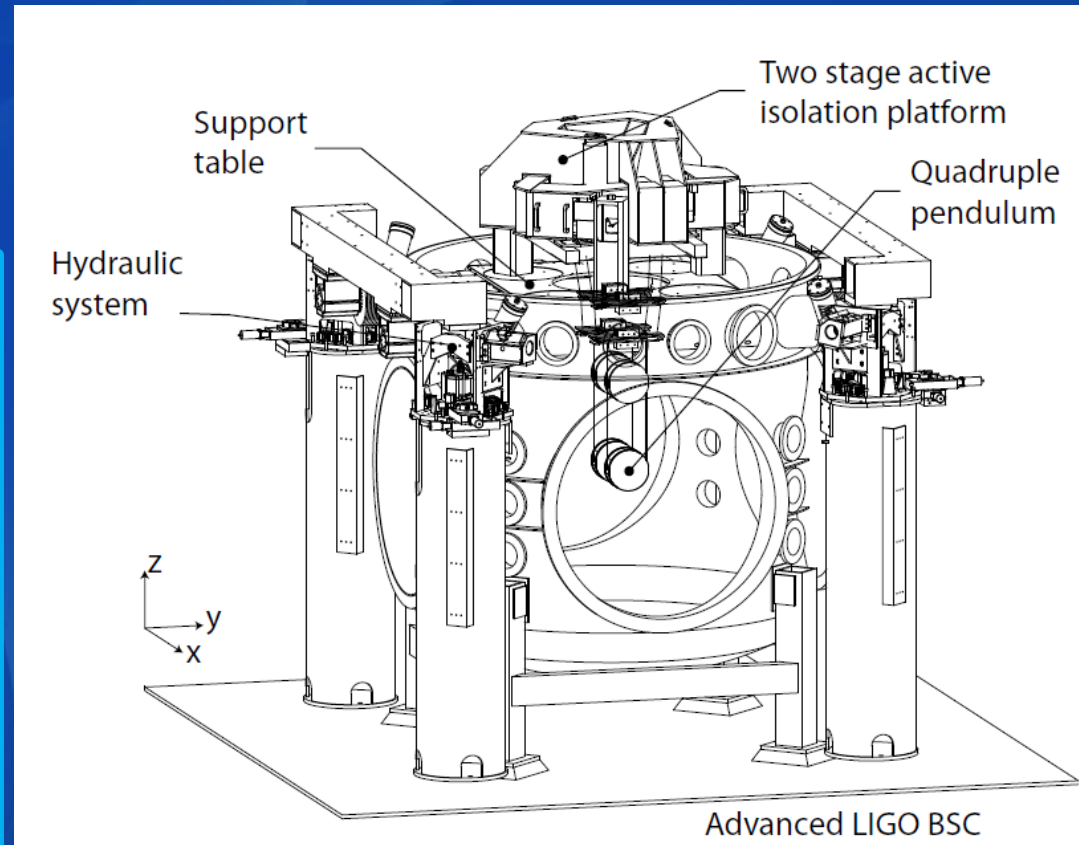
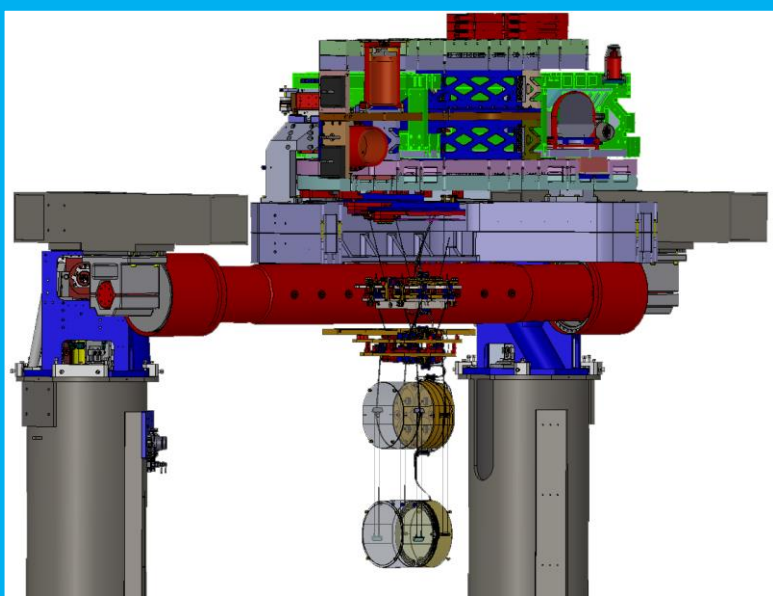


Seismic Noise Isolation

-Active and Passive Isolation and Resonance Damping -

■ Active Isolation introduction and Active damping in GEO and Advanced-LIGO

- 4 stage passive isolation using classical blade springs and wire suspensions.
- Recoil-mass introduction not to introduce actuating point noise

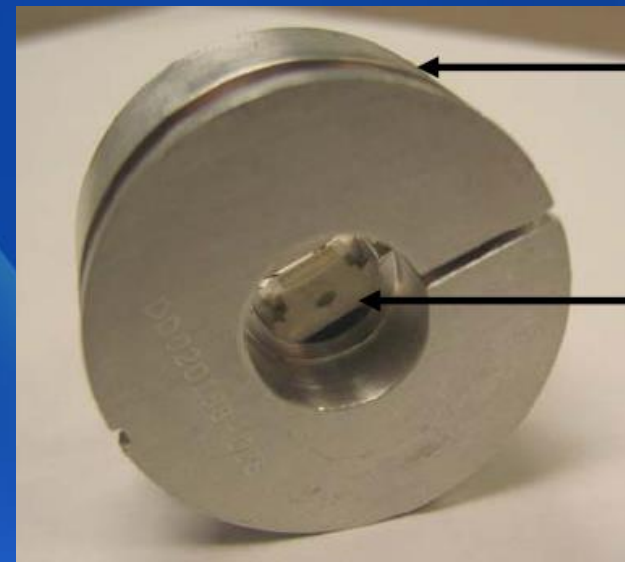
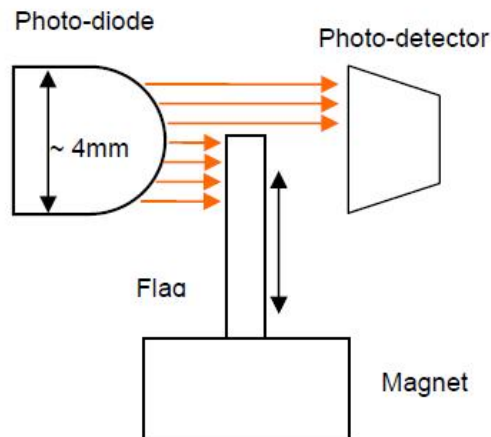
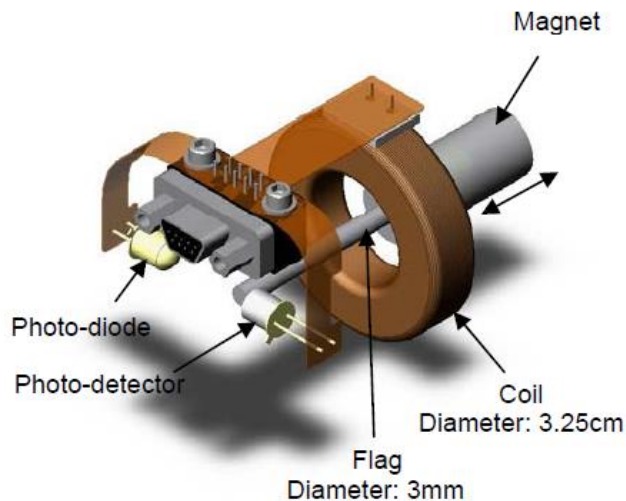
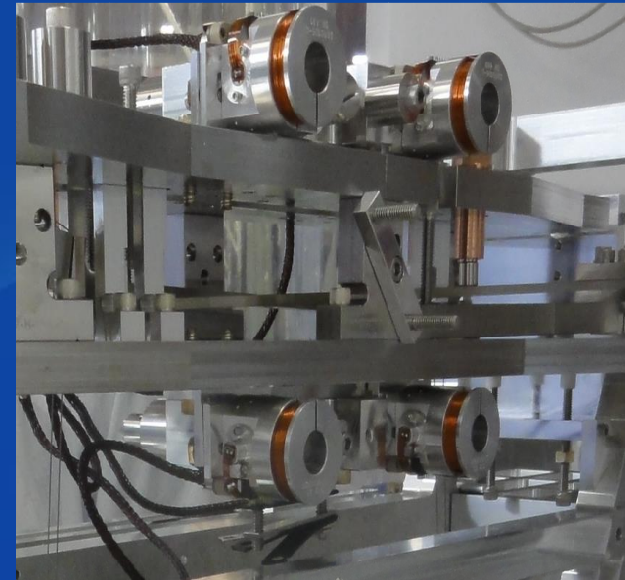


Mirror Actuators

- Magnet Coil Actuator with optical shadow sensor-

■ Mirror and Upper Masses Motion Sensors and Actuators

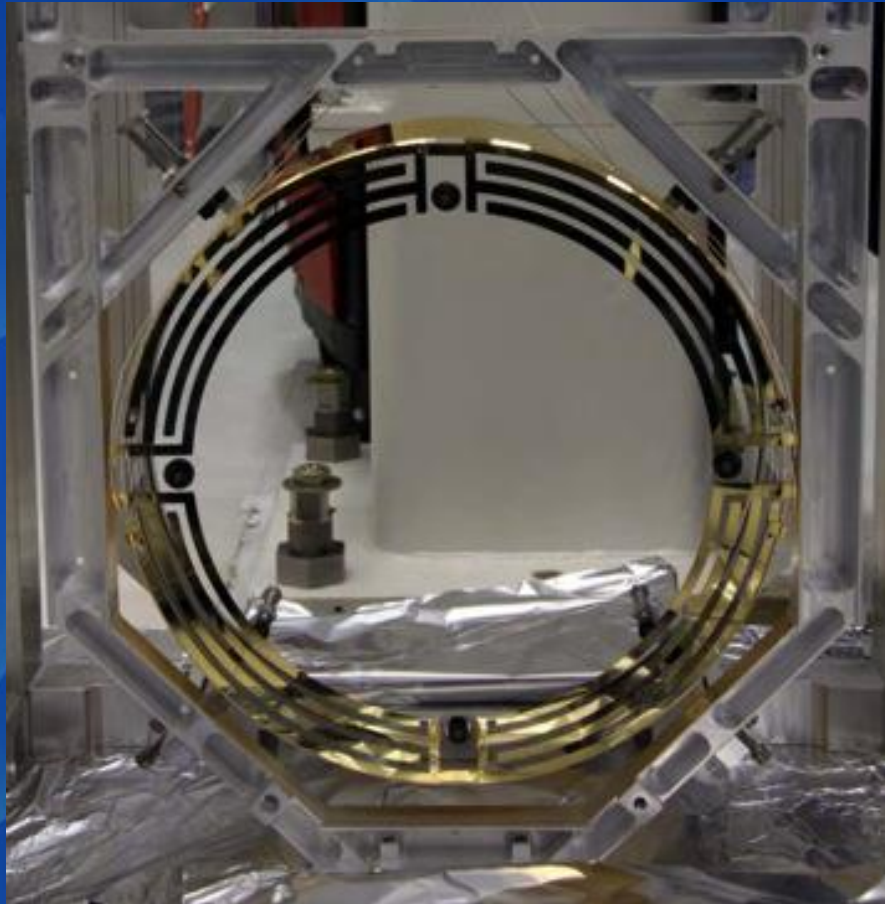
- OSEM : a set of shadow sensor and magnet-coil actuator in one holder
- Magnet are glued on the back of mirrors and OSEMs are set in the recoil-mass for mirror motion sensing and actuation
- Force is linear in \sim mm range.
- Electrical noise coupling is large.



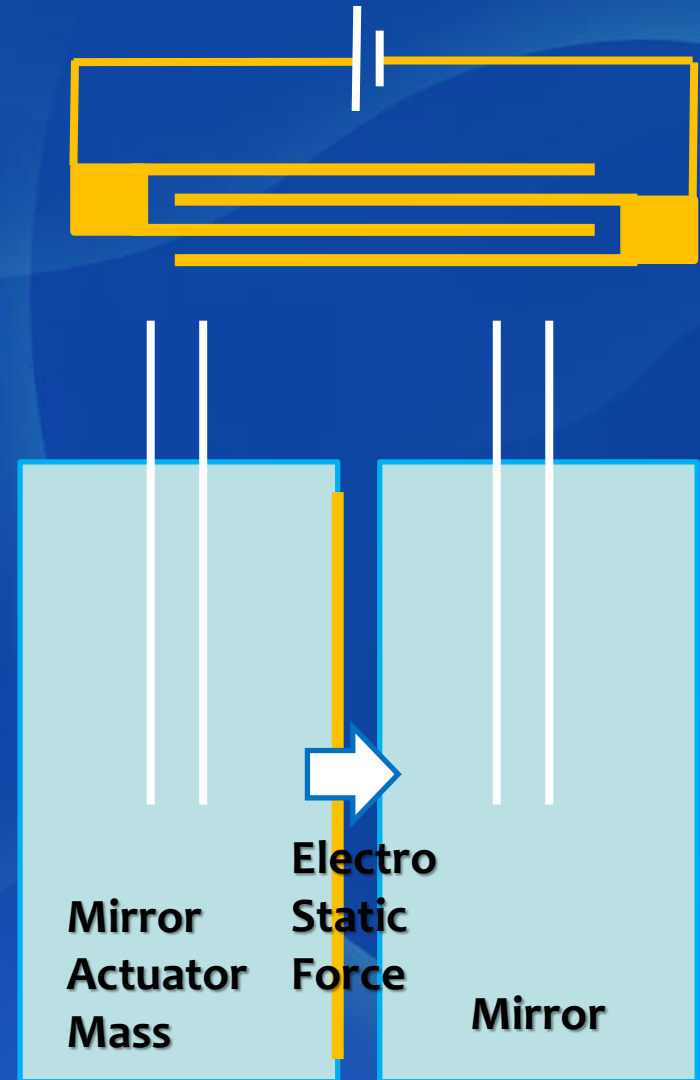
Mirror Actuators

- Electro Static Actuator -

- Low Noise !
- Non linear force, Naive Treatment (distance control is strict : ~ sub mm)



High Voltage

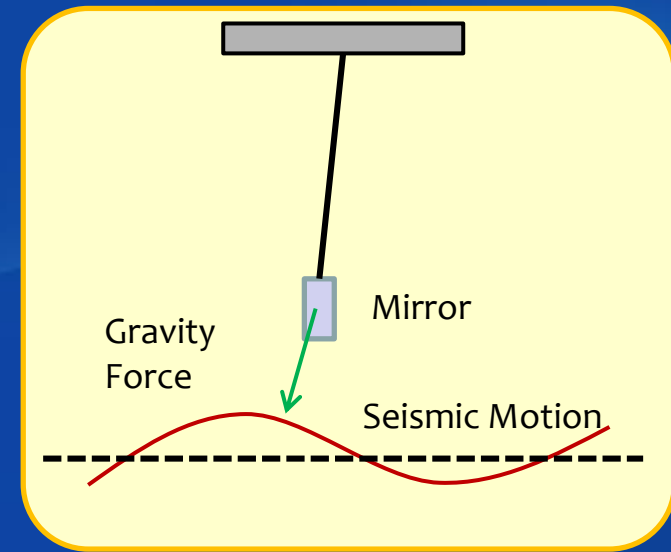


Gravity Gradient Noise (GGN)

- gravity force fluctuation from the ground due to seismic noise -

■ What is GGN ?

- Gravitational coupling due to seismic density fluctuations
- Cannot be shielded ☹️
- Expected noise model is proportional to f^{-4}
- Limiting noise sources for the 3rd generation GWDs
- A few solutions to minimize GGN are
 - Go underground
 - Subtraction using some sensors



$$GGN(f^{-4}) = TF(f)^{-2} \times Seismic(f)^{-2}$$

Gravity Gradient Noise (GGN)

- gravity force fluctuation from the ground due to seismic noise -

■ GGN estimation

P.R.Saulson et al PRD 1984

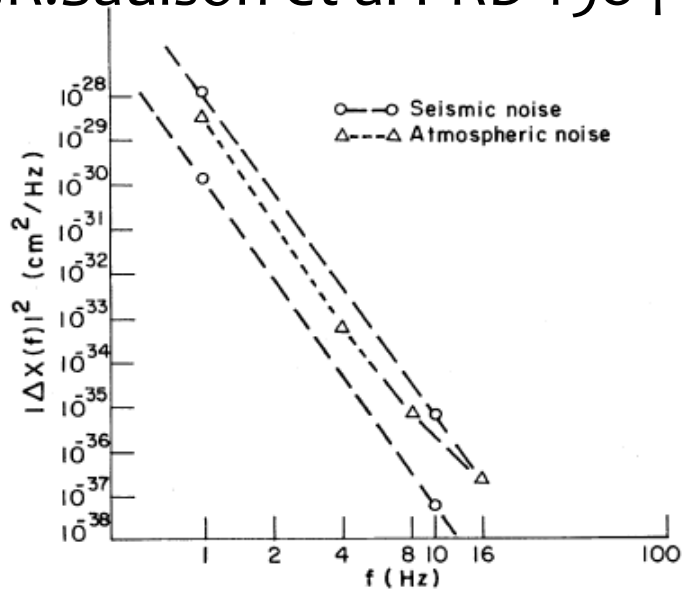
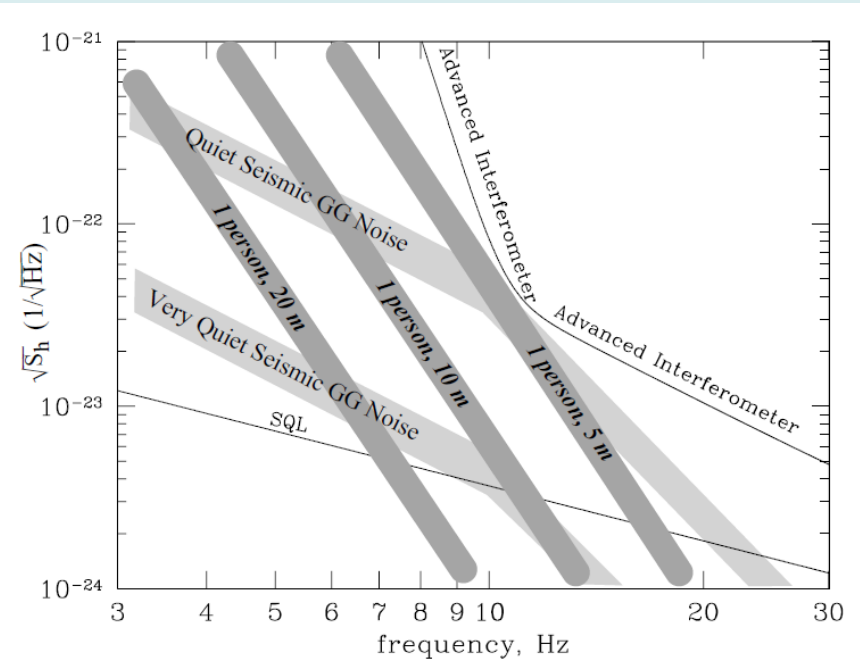


FIG. 2. Interferometer path-length-difference power spectrum, due to random gravitational forces from seismic noise and from atmospheric noise. Two estimates of the seismic noise are shown, corresponding to an average site and a quiet site.

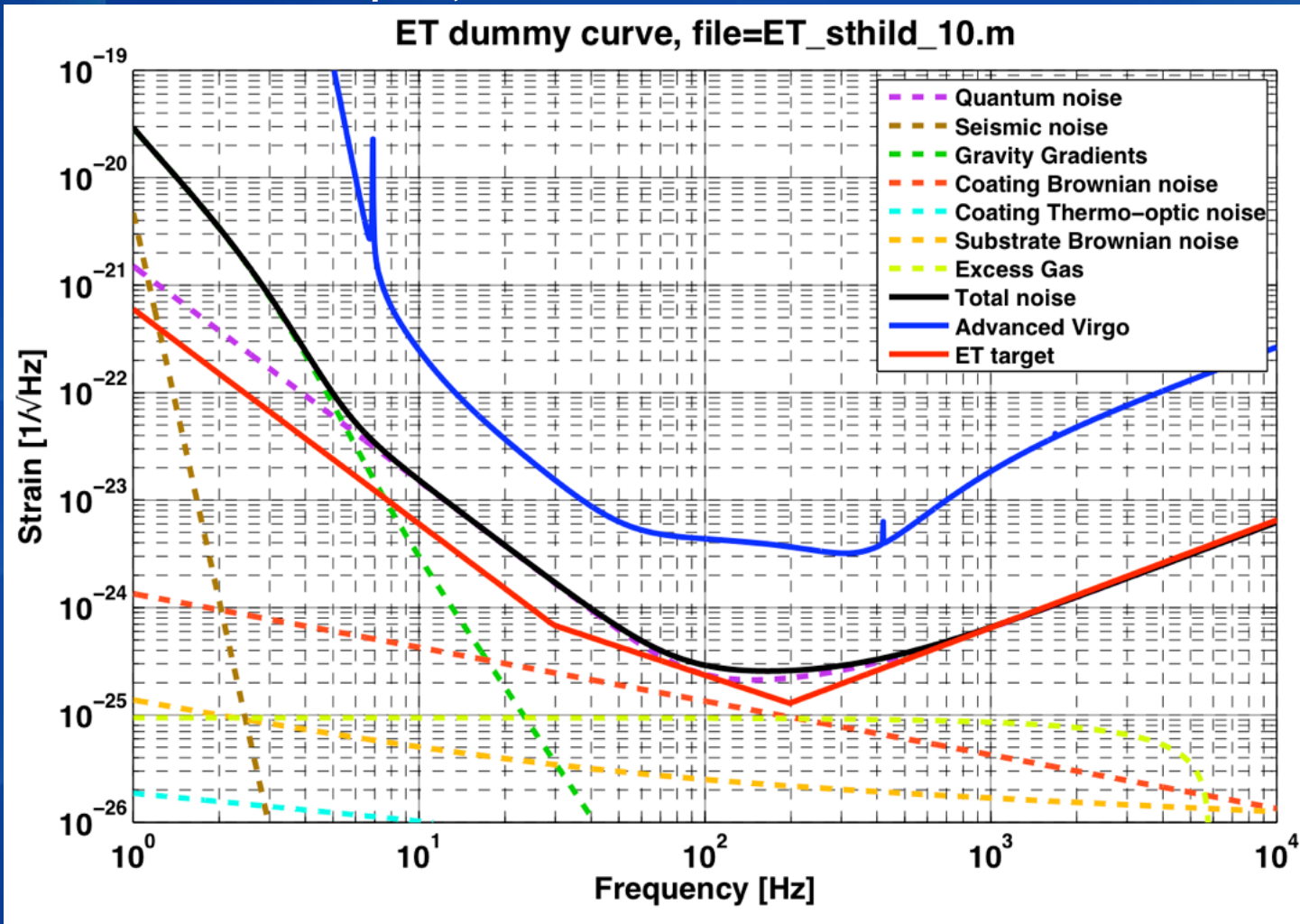
Kip Thorne et al PRD 1999



Gravity Gradient Noise (GGN)

- gravity force fluctuation from the ground due to seismic noise -

■ GGN estimation in ET project



Sound Noise

- Even the main optics are housed in vacuum tanks, we noticed that sounds deteriorated the sensitivity.
- Input optics (laser and its injection optics) and output optics (PDs) isolation from sounds were effective.
- The transfer function from sounds to sensitivity might depend on each GWD environment, so it might be difficult to derive some universal laws.
- Anyway, sounds shield is important.

Residual Gas Noise

- Residual gas introduces refraction index fluctuation and it results in path length fluctuation as noise in GWD.
- Requirement : 10^{-7} [Pa] (for KAGRA) assuming safety factor of 10 for ~ totally 6~8km in length and 1.2~0.8m in diameter volume (~8000 m³).
- Theoretical Model of “Residual Gas Noise”

$$x_{vac} = \sqrt{\frac{L}{4\pi} \frac{8\sqrt{2}}{\sqrt{\pi}} \frac{(n_0 - 1)^2}{(A_0/V_0)u_0\sqrt{\lambda L}} \frac{p}{p_0} \left(\frac{T_0}{T}\right)^{\frac{3}{2}}} \quad [\text{m}/\sqrt{\text{Hz}}]$$

L : Cavity length, λ : Laser wave length, n_0 : Reflective index,
 u_0 : Particle mean velocity, V_0 : 1 mol volume, A_0 : Avogadro number,
 T : Temperature, T_0 : 273.15 [K], p : Pressure, p_0 : 1 [atom],

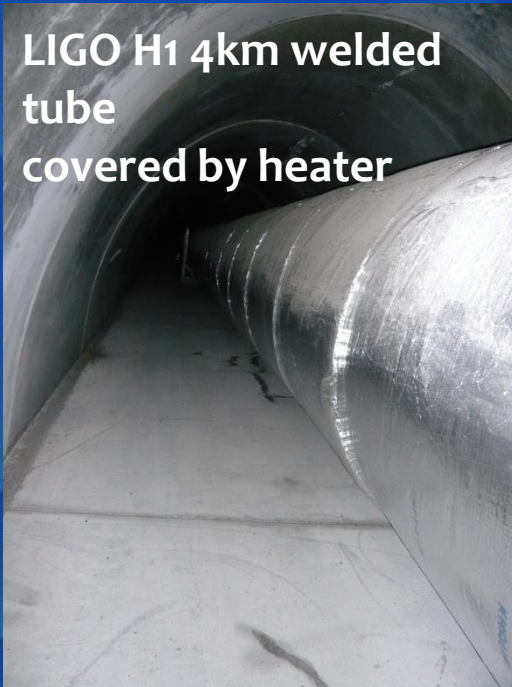
Keep 10^{-7} [Pa] in ~ 8000 m³

■ How to obtain 10^{-7} [Pa]

□ LIGO, VIRGO

- 1 : Surface of vacuum tube has no special treatment, except normal polishing.
- 2 : 4km~3km arm tube are connected with welding.
- 3 : Rough evacuation by TMPs, then evacuation with Baking.
- 4 : maintain with Cold Trap using liquid Nitrogen and Ion pump.

LIGO H1 4km welded tube covered by heater



LIGO H1 Mirror Tank in Hanford



VIRGO

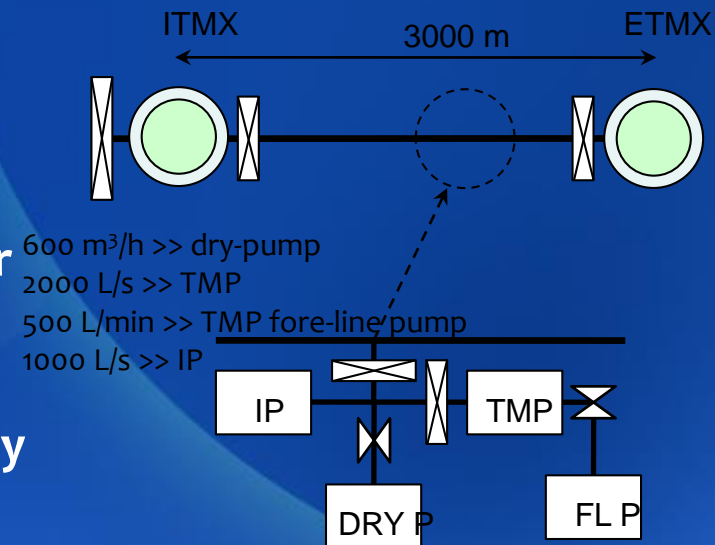
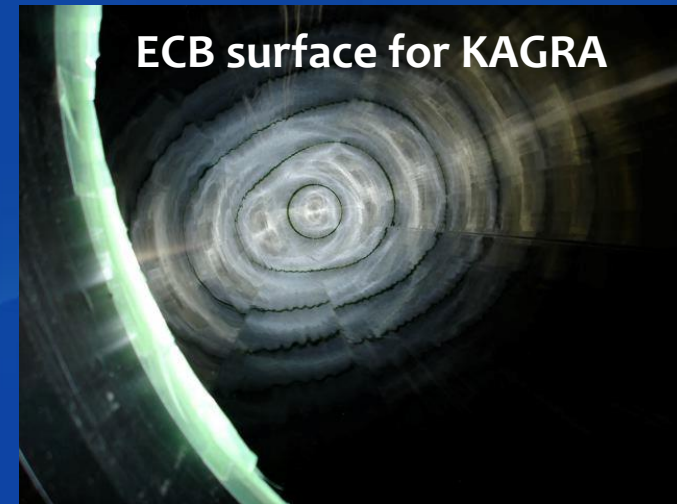


Keep 10^{-7} [Pa] in ~ 7000 m³

■ How to obtain 10^{-7} [Pa]

□ KAGRA

1. Because of tunnel, welding and LN₂ drop off
2. Surface passivation by electro-polish followed by baking”
 - Outgassing rate: 10^{-8} Pa m³ m⁻² s⁻¹, or lower surface roughness
 - R_{max} 3 μm, Ra 0.5 μm
3. Mirror finish by Electro-Chemical Buffing (tubes in the mid 800-m region)”
 - Surface roughness; R_{max} 0.2 μm, Ra 0.03 μm
4. Flange connection with metal O-ring (silver plated)”
 - Erosion proof by humidity test
5. Rough Evacuation by TMPS and maintain by Ion-pumps



Laser has Noises

$$E(t) = [E_0 + \delta E(t)] e^{i\Omega t + i\phi(t)}$$

$\phi_N(t)$: Phase Noise

$$\nu_N(t) = \frac{1}{2\pi} \frac{d\phi_N(t)}{dt} : \text{Frequency Noise}$$

$$P(t) = |E(t)|^2 = P_0 + \delta P(t)$$

$\delta P(t)$: Intensity Noise

Laser Frequency Noise in MI

- If Interferometer is ideal (perfect balance in each arm length and power) : Laser Frequency noise don't spoil the GWD sensitivity.
- Laser Frequency Noise couples with imperfection of the interferometer with a factor of **arm length difference and Finesse** (→ **common Mode Rejection Ratio(CMRR)**).

$$E_{APD}(t) = \frac{E_0}{2} e^{\left\{i\Omega\left(t - \frac{2L_x}{c}\right) + i\phi_N\left(t - \frac{2L_x}{c}\right)\right\}} - \frac{E_0}{2} e^{\left\{i\Omega\left(t - \frac{2L_y}{c}\right) + i\phi_N\left(t - \frac{2L_y}{c}\right)\right\}}$$



$$P_{APD}(t) = \frac{P_0}{2} \left\{ 1 - \cos \left(\frac{2\Omega(L_x - L_y)}{c} + \underbrace{\phi_N\left(t - \frac{2L_x}{c}\right) - \phi_N\left(t - \frac{2L_y}{c}\right)}_{\delta\phi_N} \right) \right\}$$

$$\delta\phi_N \equiv \phi_N\left(t - \frac{2L_x}{c}\right) - \phi_N\left(t - \frac{2L_y}{c}\right) = \frac{d\phi_N}{dt} \times \frac{2(L_x - L_y)}{c} = 4\pi\nu_N(t) \frac{\Delta L}{c}$$

$$\Delta L \equiv L_x - L_y \quad \nu_N(t) = \frac{1}{2\pi} \frac{d\phi_N(t)}{dt}$$

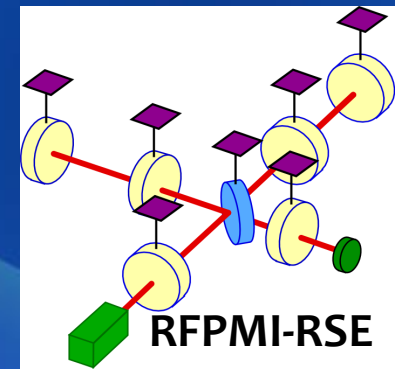
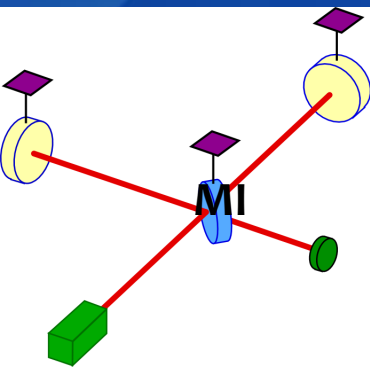
Laser Frequency Noise

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- Laser Frequency Noise couples with imperfection of the interferometer with a factor of **arm length difference and Finesse** (→ **common Mode Rejection Ratio(CMRR)**).

$$\delta\phi_N^{MI} = 4\pi \frac{\nu_N \Delta L}{c} = 4\pi \left(\frac{L}{\lambda}\right) \times CMRR \times \left(\frac{\nu_N}{\nu}\right) [\text{m}/\sqrt{\text{Hz}}]$$

$$CMRR = \frac{\Delta L}{L} + \frac{\Delta\mathcal{F}}{\mathcal{F}} \approx \frac{1}{300} \rightarrow \delta\nu < 10^{-8} [\text{Hz}/\sqrt{\text{Hz}}] \quad \frac{\Delta L}{L} \sim 10^{-2}$$

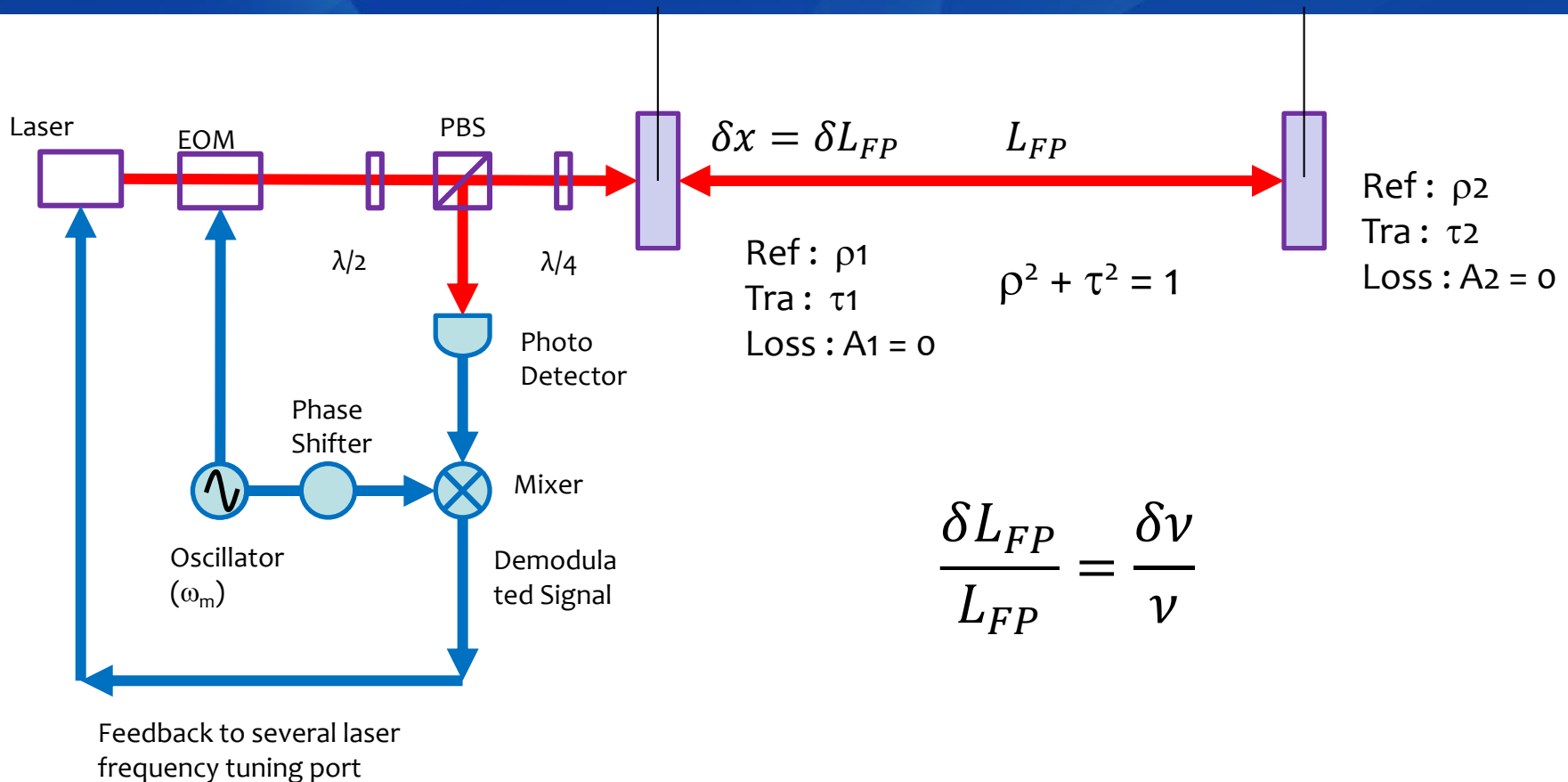
L : Cavity length, ΔL : Cavity length difference,
 \mathcal{F} : Finesse, $\Delta\mathcal{F}$: Finesse Difference,
 ν : Laser frequency, ν_N : Laser Frequency Noise



Laser Frequency Noise Stabilization

- Concept -

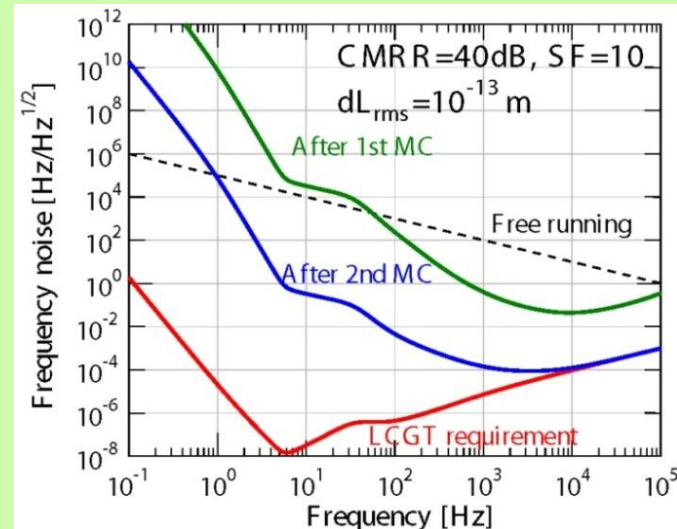
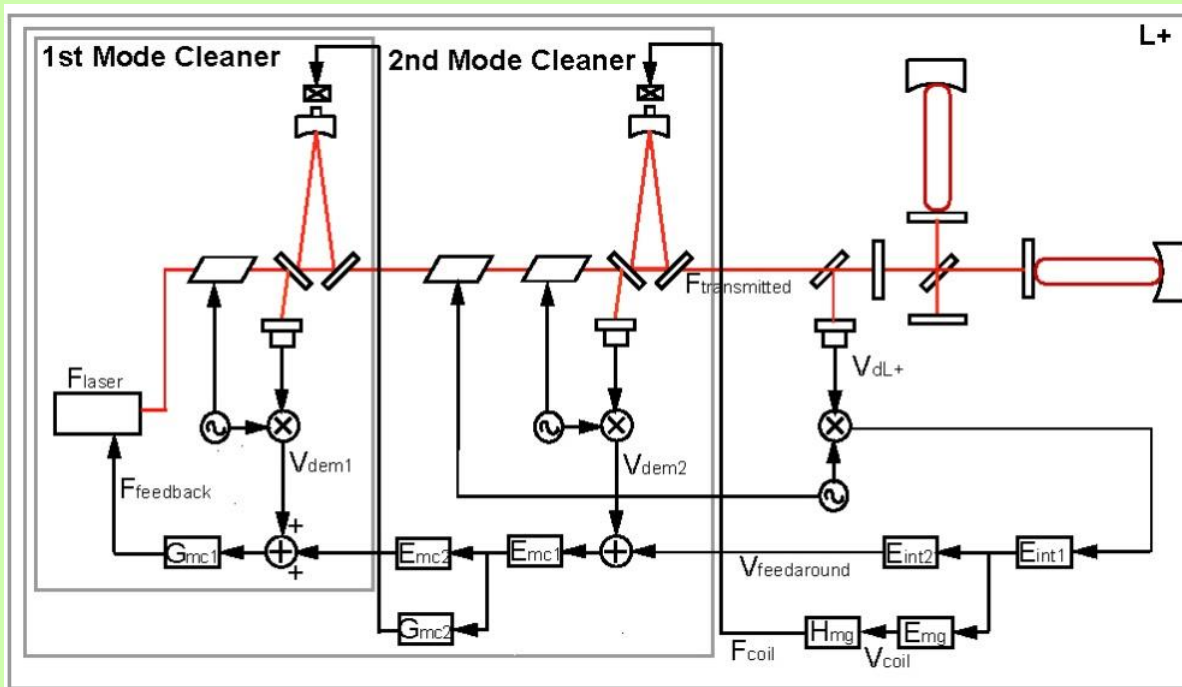
- Original laser frequency noise ($\sim 1\text{kHz}$ line width) is not enough at all for GWD.
 - **100 Hz/rHz @ 100Hz original noise should be 10^{-8} Hz/rHz !!**
- 4~3 km arm FP cavity whose mirrors are well isolated from seismic noise can be a good frequency reference.



Laser Frequency Noise Stabilization

- Concept -

- Because it is difficult to stabilize the original F-noise to FP level, two FP cavities are prepared for pre-stabilization.
 1. Pre-Mode Cleaner FP ($\sim 1\text{m}$): Rigid triangle or bow tie cavity
 2. Mode Cleaner FP ($\sim 30\text{m}$): Moderately isolated triangle cavity
 3. FP arm Cavity ($\sim 3\text{km}$): of course, mirrors are isolated by multi-stage pendulums



Laser Intensity Noise

$$P(t, \phi) = |E(t, \phi)|^2 = \{P_0 + \delta P(t)\} \frac{1 - \cos(\phi)}{2}$$

ϕ : Differential Phase of MI

The ratio between “Intensity Noise” and “Intensity Change due to MI phase change” shows Intensity noise contribution.

Intensity Noise

$$\delta P(t) \frac{1 - \cos(\phi)}{2}$$



$$\delta P(t) \frac{1 - \cos(\phi)}{2} = dP = P_0 \frac{\sin(\phi)}{2} d\phi$$



$$d\phi = \frac{\delta P(t)(1 - \cos(\phi))}{P_0 \sin(\phi)} \rightarrow \frac{\delta P(t) \phi}{P_0} \frac{1}{2} \text{ assuming } (\phi \ll 1) \quad d\phi \rightarrow 0, \text{ if } \phi \rightarrow 0$$

Intensity Change due to MI Phase change

$$\frac{dP}{d\phi} \approx \frac{d}{d\phi} \left(P_0 \frac{1 - \cos(\phi)}{2} \right) = P_0 \frac{\sin(\phi)}{2}$$



Laser Intensity Noise

■ Laser Intensity Noise also couples with imperfection of the interferometer

$$x_{intensity} = x_{RMS} \frac{\delta P}{P} \text{ [m}/\sqrt{\text{Hz}}] \quad \text{For MI} \quad x_{RMS} \equiv \frac{\phi}{2}$$

$$x_{intensity} = k_{BS} x_{RMS} \frac{\delta P}{P} \text{ [m}/\sqrt{\text{Hz}}] \quad \text{For FPMI}$$

$$\frac{\delta P}{P} < 10^{-8} \quad k_{BS} \sim \frac{1}{\mathcal{F}}$$

P : laser power, δP : Laser power fluctuation,

$x_{RMS} \left(= \frac{x_{original}}{1 + G} \right)$: Mirror Fluctuation under feedback control,

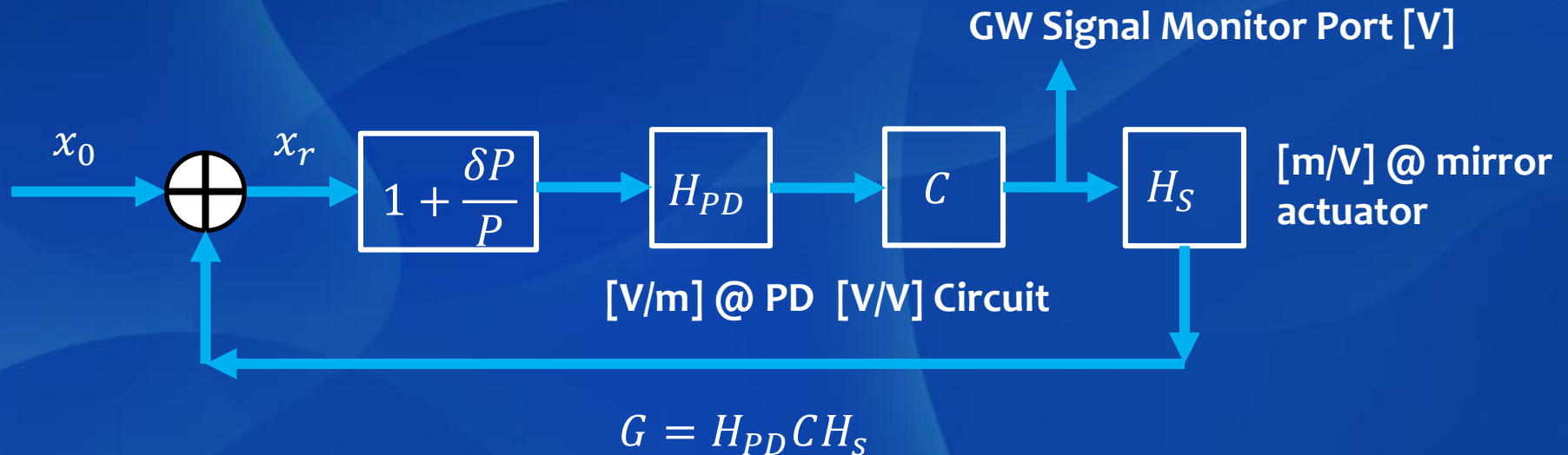
$x_{original}$: Original mirror fluctuation before control before control,

G : Loop gain,

k_{BS} : coupling ratio of l_- signal with L_- signal

Laser Intensity Noise

- Intensity Noise contribution in GW signal monitor port (MI case)



$$\text{GW Signal Monitor Port} \cong \frac{G}{1 + G} \frac{1}{H_S} \left(x_0 + \frac{x_0}{1 + G} \frac{\delta P}{P} \right)$$

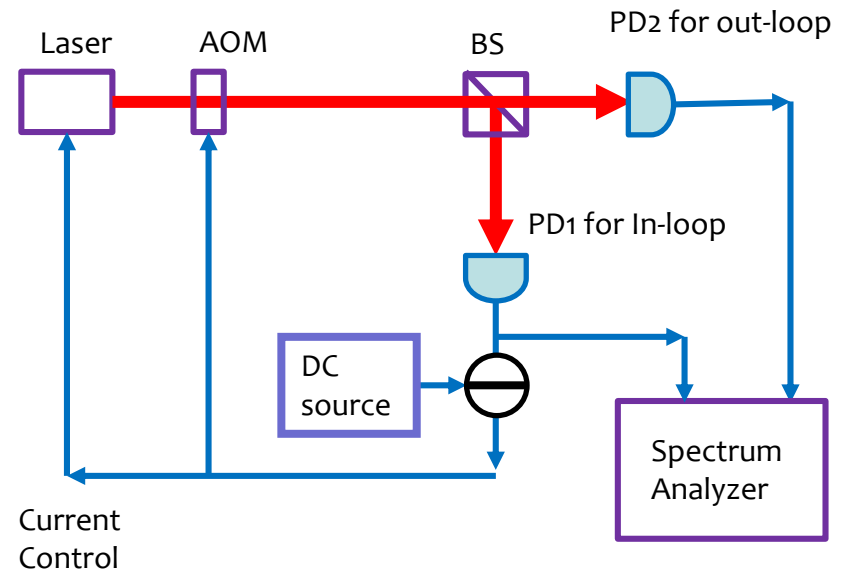
Laser Amplitude Noise Reduction

■ Stabilization concept is simple

- Split a beam in two portions and detect them by different PDs.
- One PD output compared with stabilized DC voltage, and the residual is feedback to laser current control port or AOM on the beam axis.
- The intensity from the other PD is used for intensity stabilization evaluation.

■ Annoying Results for 10 years

- The intensity noise from the PD that is used for stabilization is of course well stabilized according to control loop gain.
- **While, the intensity noise from the other PD was not stabilized as well as in loop one ???**

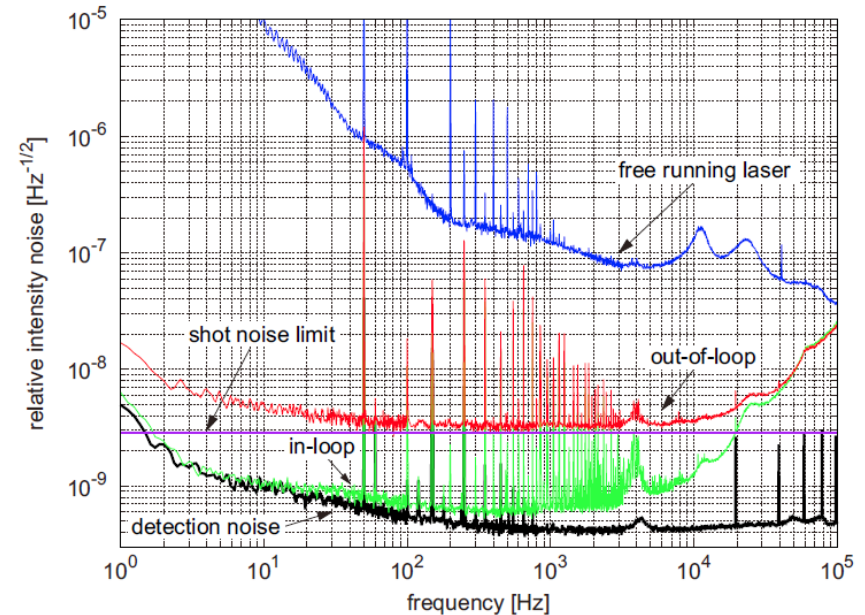
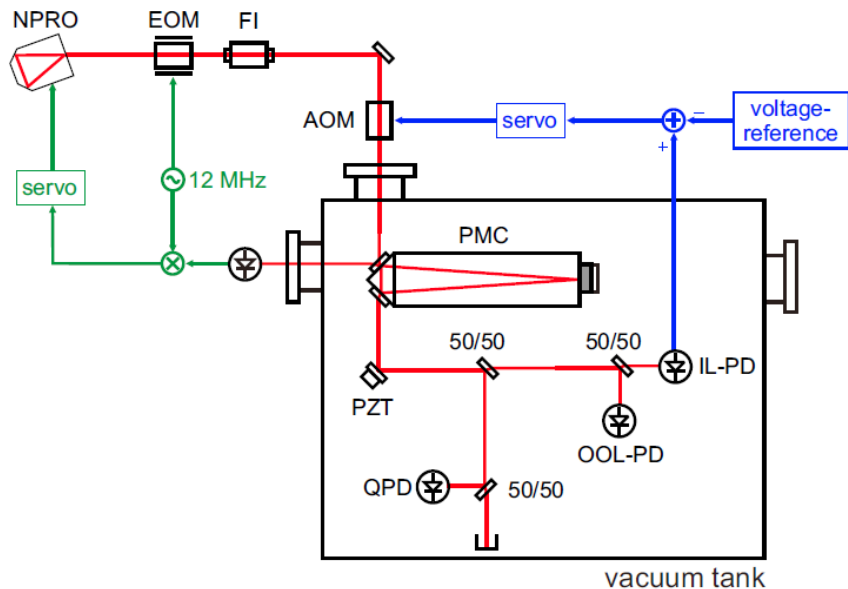


Laser Amplitude Noise Reduction

A mystery has been solved by a Mode Cleaner.

- F.Seifert found that intensity stabilized level discrepancy between in and out loop by using a beam after a mode cleaner.
- This implies that beam jitter noise including higher mode mixing affects the discrepancy.

F.Seifert et al MPQ(AEI) and Univ. Hanover



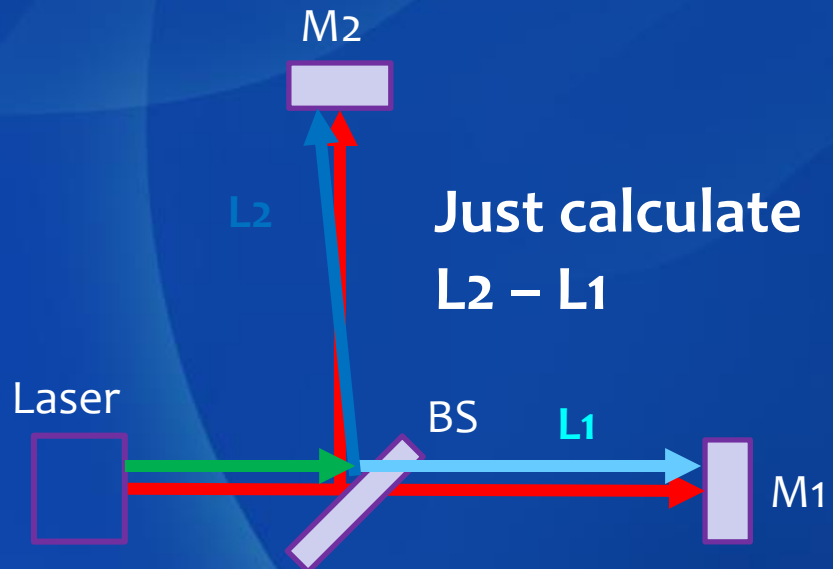
Beam Jitter Noise

- (Beam lateral shift originated from higher order transverse mode in a laser and/or seismic noise) and (the fluctuation in yaw/pitch motion of a beam splitter) couple and generate Beam Jitter Noise.

$$x_{jitter} = 2\sqrt{2}\delta\alpha \delta x \text{ [m}/\sqrt{\text{Hz}}]$$

$\delta\alpha$: Lateral Shift of Beam

δx : BS Angular Fluctuation



- Reduction for Beam Jitter Noise
 - Isolated Mode Cleaner to reduce transverse mode and seismic noise reduction.
 - BS seismic noise isolation.

Scattered Light Noise

- Mirror surface could not be perfect flat and have a few defects in the coating films.
- Substrate has also imperfection of crystal structure and impurity.
- These produce scattered light toward the vacuum tube and small fraction of them decouples with the main beam.

→ Scattered light Noise

If there is no scattered light from the mirror, we cannot monitor the spots on mirrors as the bottom picture ☹

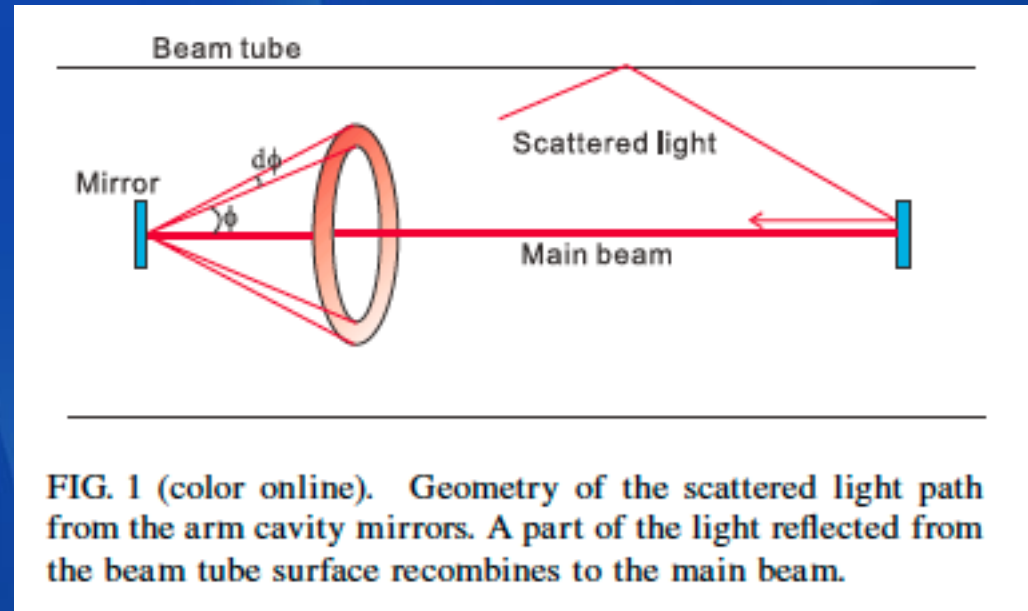
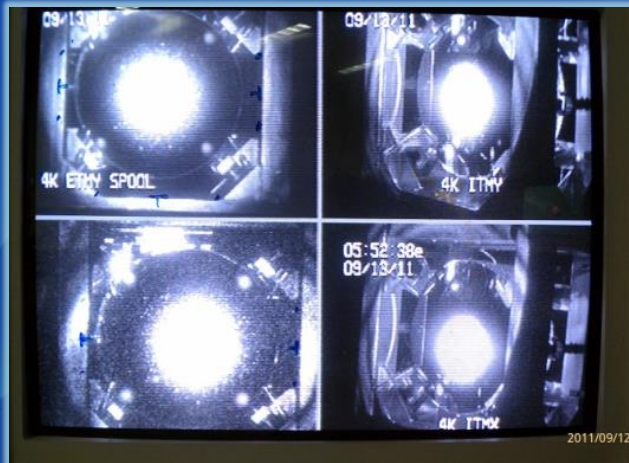


FIG. 1 (color online). Geometry of the scattered light path from the arm cavity mirrors. A part of the light reflected from the beam tube surface recombines to the main beam.

Scattered Light Noise

Scattered light noise model

- Two types
 - Non-up-converted (can be quantitatively estimated)
 - Up-converted (should not appear !! It reveals big motion of components)
- Decoupling process is so complicated because of many multi path reflection that theoretical prediction is based on simple models.
- Assuming Non-up-converted Scattered Light... ($x_{seis} < \lambda/2$)

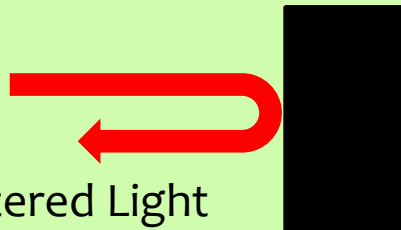
$$h_{GW} = k \sqrt{P_{ifo}} \phi_{GW}$$

ϕ_{GW} : GW signal phase change,

P_{ifo} : Power in IFO,

We define $\kappa \equiv kL \sqrt{P_{ifo}}$

Reflector of Scattered Light (SL)



Scattered Light
Reflection

$$x_{sctr} = A(f) x_{seis} \quad [m/\sqrt{Hz}]$$

x_{seis} : SL Reflector Fluctuation,

$A(f)$: Excitation factor of Reflector,

● Phase Change of SL : $\phi_{sctr} = 2\pi \frac{2x_{seis}}{\lambda}$ λ : wave length,

● Scattered Light Noise : $d_{sctr} = Lk \sqrt{P_{recom}} \phi_{sctr} = \kappa \sqrt{\frac{P_{recom}}{P_{ifo}}} \phi_{sctr}$ L : Arm length,

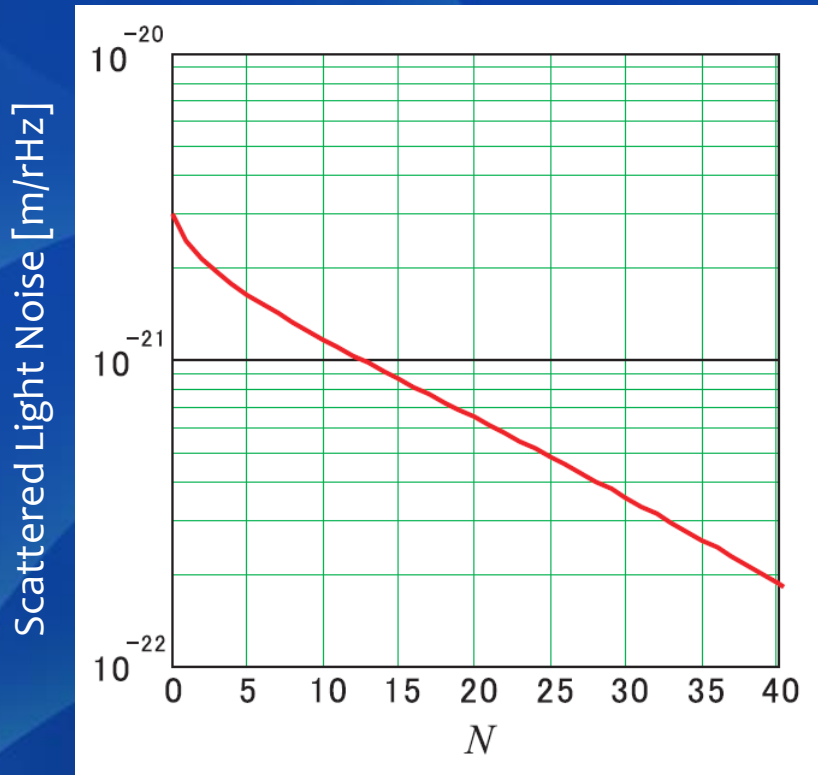
Scattered Light Noise Reduction

■ For narrow angle SL



$$N = \frac{h L}{d R}$$

h: Baffle height
d: baffle interval
L: Arm length
R: Duct radius

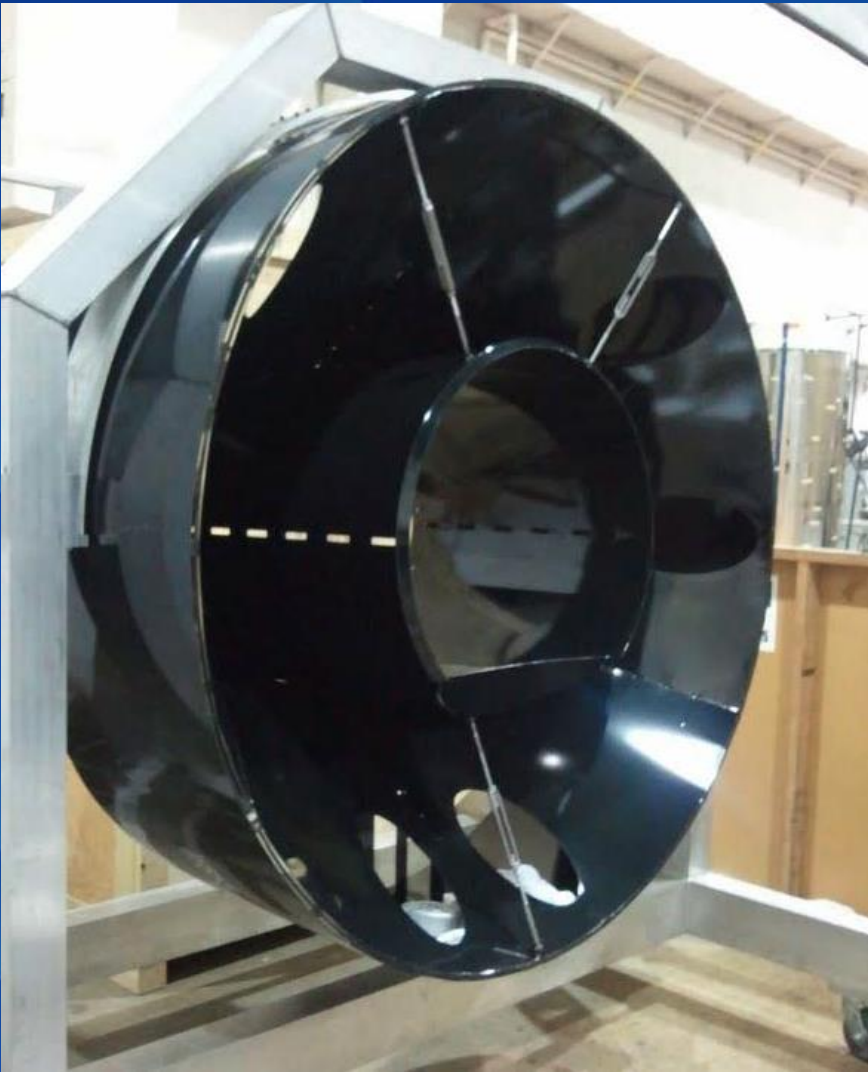


	LIGO	VIRGO	KAGRA
R [m]	1.2	1.2	0.8
H [cm]	6	10	5
Angle [d]	55	45	50 ??
Coating	Oxide	AR	DLC
number	222	18	250

Effective Baffle Numbers in one arm (KAGRA)

Scattered Light Noise Reduction

- For wide angle scattered light



Thermal Noise

- Final barrier to reach targeted quantum noise level -

Thermal Noise

- Fluctuation of a harmonic oscillator (HO) induced by thermal bath.

Motion Equation of a harmonic oscillator

$$m\ddot{x} + \gamma\dot{x} + kx = f_{th} \quad \text{Thermal Force}$$

- Fluctuation-dissipation theorem

$$\langle f_{th}^2 \rangle = 2k_B\gamma T \quad \text{White noise}$$

$$\gamma = m\omega_0\phi$$

Thermal Noise Spectrum $\propto \sqrt{\phi T}$ ϕ : Mechanical loss
 T : Temperature

- In order to reduce thermal noise, **reduce T and ϕ ($1/Q$). There are many kinds of origin of ϕ**

Thermal Noise Source in GWD

(1) HO \rightarrow Mirror



Possible substrate

- (1) SiO₂
- (2) Sapphire (20K)
- (3) Silicon (cryo)

(2) HO \rightarrow Pendulum (Final Stage)



Possible material

- (1) Piano Wire
- (2) SiO₂
- (3) Sapphire (20K)
- (4) Silicon (cryo)

Thermal Noise of Mirror

- structure damping model -

■ Internal structure damping Model

- Internal friction of material is the origin of energy loss
- In the case of mirror, Noise spectrum below resonance frequency is expressed...

$$x_{mir (structure)} = \frac{2}{L} \sqrt{\frac{4k_B T_m (1 - \sigma^2)}{\sqrt{\pi} Q_{mir} E_0 \omega_0 \omega}} [1/\sqrt{\text{Hz}}]$$

$$5.4 \times 10^{-25} \left(\frac{3\text{km}}{L} \right) \left(\frac{10^8}{Q_{mir}} \right)^{\frac{1}{2}} \left(\frac{T}{20\text{K}} \right)^{\frac{1}{2}} \left(\frac{3.5\text{cm}}{\omega_0} \right)^{\frac{1}{2}} \left(\frac{4 \times 10^{11}\text{Pa}}{E_0} \right)^{\frac{1}{2}} \left(\frac{100\text{Hz}}{f} \right)^{\frac{1}{2}}$$

Q_{mir} : Q of Mirror, T : Temperature, σ :Poisson Ratio,
 ω_0 : Beam spot size, f : Frequency, E_0 :Young's modulus

Thermal Noise

- structure damping model -

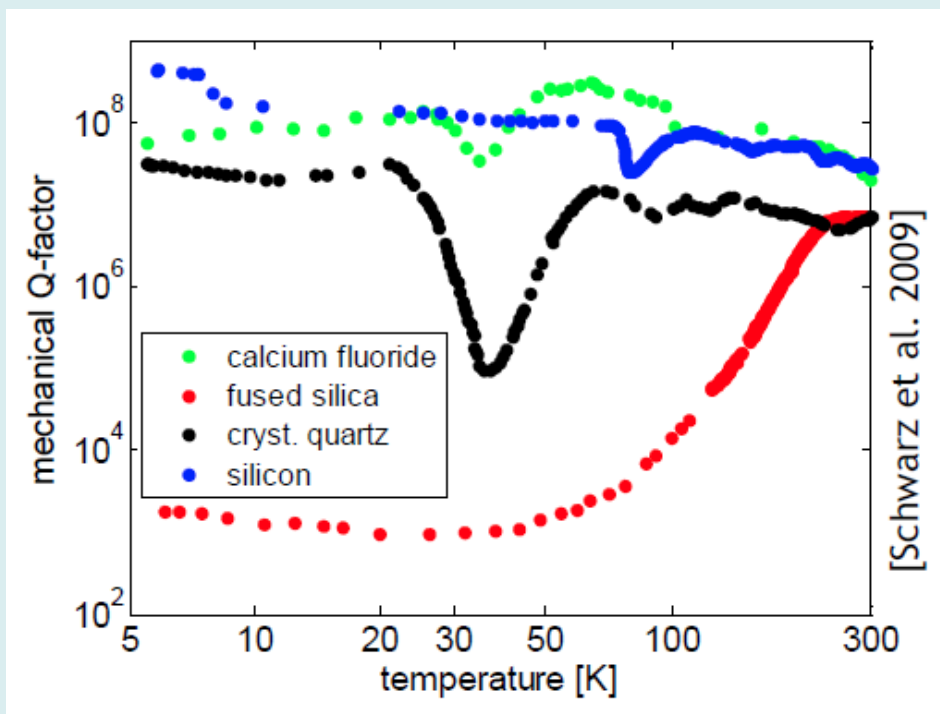
■ To Reduce mirror Thermal Noise ...

● Mechanical Q (1/φ) factor

- (1) SiO₂ : ~10⁷ (300K)
- (2) Sapphire : 3×10⁶ → 2×10⁸ (5K)
- (3) Silicon : 4×10⁷ → 4×10⁸ (6K)
- (4) CaF₂ : ~1×10⁷ (300K)

Generally, cryogenic temperature increase Q, except SiO₂.

● Mechanical Q temperature dependence



Thermal Noise of Mirror

-Thermo-elastic noise -

■ Thermo-elastic Noise (300K)

Loss originated from temperature gradient relaxation triggered by inhomogeneous elastic compression and expansion.

$$x_{mir (thermo)} = \frac{8\alpha T(1 + \sigma)}{\rho C L \omega} \sqrt{\frac{k_B \kappa}{\sqrt{\pi} \omega_0^3}} [1/\sqrt{\text{Hz}}]$$

$$= 8.8 \times 10^{-24} \left(\frac{3\text{km}}{L}\right) \left(\frac{\alpha}{5 \times 10^{-6} / \text{K}}\right) \left(\frac{T}{300 \text{ K}}\right) \left(\frac{\kappa}{40 \text{ W/m/K}}\right)^{\frac{1}{2}} \left(\frac{4 \text{ g/cm}^3}{\rho}\right) \left(\frac{3.0\text{cm}}{\omega_0}\right)^{\frac{3}{2}} \left(\frac{790 \text{ J/kg/K}}{C}\right)^{\frac{1}{4}} \left(\frac{100\text{Hz}}{f}\right)$$

α : Thermal expansion ratio, T : Temperature, σ :Poisson Ratio, κ :Thermal conductivity,
 ω_0 : Beam spot size, f : Frequency, E_0 :Young's modulus, C : Specific heat

Thermal Noise of Mirror

-Thermo-elastic noise -

■ Thermo-elastic Noise (20K)

Loss originated from temperature gradient relaxation triggered by inhomogeneous elastic compression and expansion.

$$x_{mir (thermo)} = \frac{8\alpha T(1 + \sigma)}{L} \sqrt{\frac{k_B}{\sqrt{\pi\kappa\rho C\omega}}} [1/\sqrt{\text{Hz}}]$$

$$= 9.4 \times 10^{-25} \left(\frac{3\text{km}}{L}\right) \left(\frac{\alpha}{5 \times 10^{-6} / \text{K}}\right) \left(\frac{T}{20 \text{K}}\right) \left(\frac{1.57 \times 10^4 \text{W/m/K}}{\kappa}\right)^{\frac{1}{4}} \left(\frac{4 \text{g/cm}^3}{\rho}\right)^{\frac{1}{4}} \left(\frac{3.0\text{cm}}{\omega_0}\right)^{\frac{3}{2}} \left(\frac{0.69 \text{J/kg/K}}{C}\right)^{\frac{1}{4}} \left(\frac{100\text{Hz}}{f}\right)^{\frac{1}{4}}$$

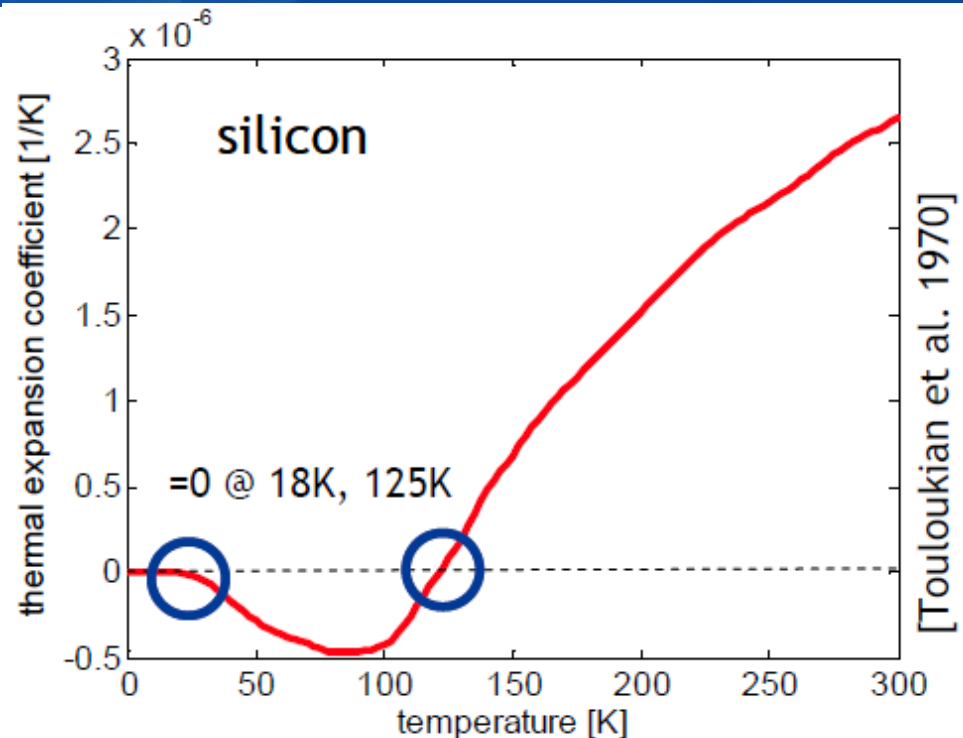
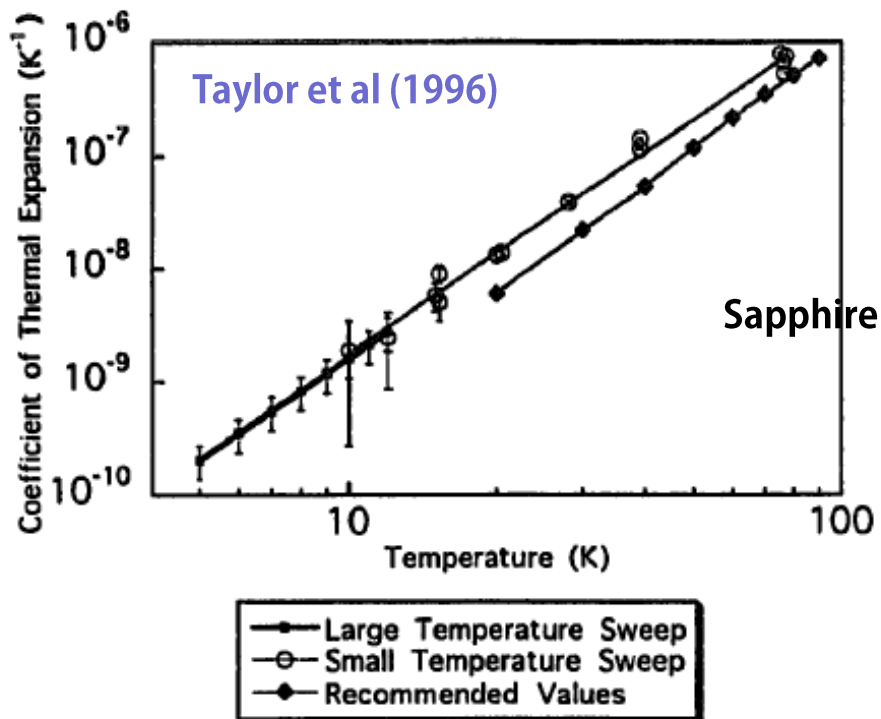
α : Thermal expansion ratio, T : Temperature, σ :Poisson Ratio, κ :Thermal conductivity,
 ω_0 : Beam spot size, f : Frequency, E_0 :Young's modulus, C : Specific heat

Thermal Noise

-Thermo-elastic noise -

Thermo-elastic Noise

- Thermo-Elastic noise reduction is effective at cryogenic temperature, because thermal expansion ratio (α) decreases and thermal conductivity (κ) increase in case of sapphire.



Thermal Noise of Mirror

- Brownian noise due to HR optical coating -

■ Brownian noise due to HR (Ta₂O₅/SiO₂ layered) optical coating

- This thermal noise limits the present GWD sensitivity.
- Ta₂O₅ is identified to be main mechanical loss source.

$$x_{coat(struct)} = \frac{4(1 + \sigma)}{\omega_0 L} \sqrt{\frac{2k_B T(1 + \sigma)(1 - 2\sigma)d_c \phi_c}{\pi \omega E_0}} \quad [1/\sqrt{\text{Hz}}]$$

$$= 1.0 \times 10^{-24} \left(\frac{3\text{km}}{L}\right) \left(\frac{\phi_c}{4 \times 10^{-4}}\right)^{\frac{1}{2}} \left(\frac{d_c}{5 \mu\text{m}}\right)^{\frac{1}{2}} \left(\frac{T}{20 \text{K}}\right)^{\frac{1}{2}} \left(\frac{3.5\text{cm}}{\omega_0}\right) \left(\frac{4 \times 10^{11}\text{Pa}}{E_0}\right)^{\frac{1}{2}} \left(\frac{100\text{Hz}}{f}\right)^{\frac{1}{2}}$$

ϕ_c : Loss of coating, d_c : Coating thickness, T : Temperature, σ : Poisson Ratio, κ : Thermal conductivity, ω_0 : Beam spot size, f : Frequency, E_0 : Young's modulus,

Thermal Noise of Mirror

- Brownian noise due to HR optical coating -

■ Brownian noise due to HR ($\text{Ta}_2\text{O}_5/\text{SiO}_2$ layered) optical coating

- This thermal noise limits the present GWD sensitivity.
- Ta_2O_5 is identified to be main mechanical loss source.

■ Many ideas to reduce loss of HR coating

- Reduce thickness of Ta_2O_5 , keeping same HR reflectivity
- Use Al_2O_3 instead of Ta_2O_5
- Dope Ti in Ta_2O_5 → Most possible solution
- Use not TEM₀₀ mode beam but LG₀₃ mode (doughnut mode)
→ **All of them results in 1.5 improvement (so small !!)**

■ Cryogenic mirror is a straightforward method ! But...

- There are many difficulties of cryogenic techniques.

Thermal Noise of Pendulum

- structure damping model -

■ Internal structure damping Model

- Internal friction of material is the origin of energy loss.
- Noise spectrum above resonance frequency.

$$x_{pend}^{struct} = \frac{4\omega_p \omega^{-\frac{5}{2}}}{L} \sqrt{\frac{k_B T}{m Q_p}} [1/\sqrt{\text{Hz}}]$$

$$= 1.9 \times 10^{-25} \left(\frac{3\text{km}}{L}\right) \left(\frac{9.4 \times 10^7}{Q_p}\right)^{\frac{1}{2}} \left(\frac{T}{20\text{ K}}\right)^{\frac{1}{2}} \left(\frac{30\text{ kg}}{m}\right)^{\frac{1}{2}} \left(\frac{f_p}{0.7\text{ Hz}}\right)^{\frac{1}{2}} \left(\frac{100\text{Hz}}{f}\right)^{\frac{5}{2}}$$

Q_p ($\sim 1/\phi_p$): Quality factor of pendulum, T : Temperature,
 f_p : Pendulum resonance frequency, f : Frequency,

Thermal Noise of Pendulum

- structure damping model -

■ To Reduce pendulum Thermal Noise ...

● Mechanical Q ($1/\phi$) factor

- (1) Metal : $\sim 10^6$
- (2) SiO₂ : $\sim 10^{7-8}$ (300K)
- (3) Sapphire : $1 \times 10^5 \rightarrow 1 \times 10^8$ (20K)
- (4) Silicon : ??

Generally, cryogenic temperature increase Q , except SiO₂.



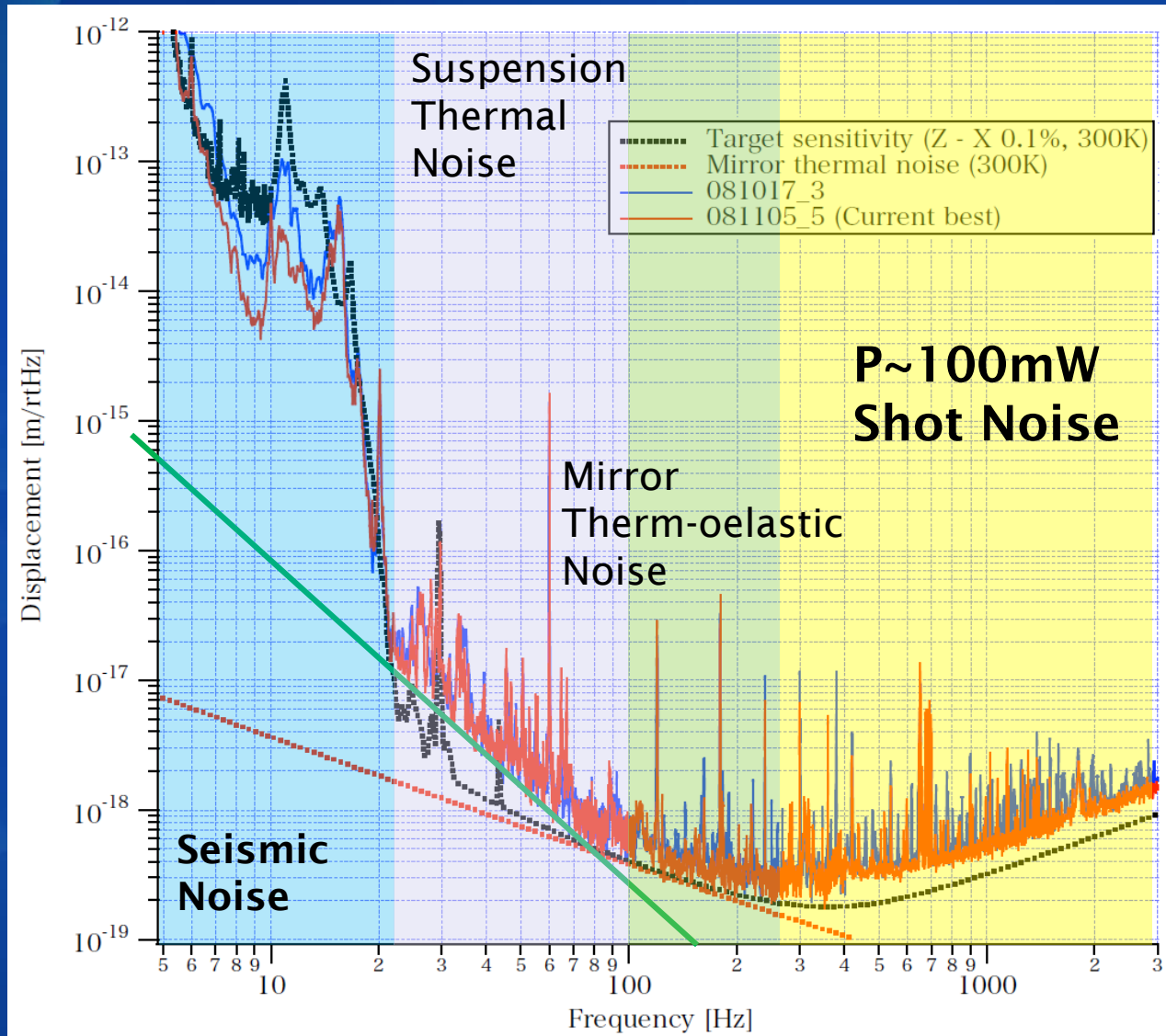
S.Reid et al @
GEO600

Hydroxy-Catalysis
Bonding not to
increase loss.

Actual Thermal Noise Measured in CLIO

Tungsten
Wire
Suspension
Thermal
Noise

Sapphire
Mirror
Thermo
Elastic
Noise



**P~100mW
Shot Noise**

**Seismic
Noise**

Summary

- PR-FPMI with RSE is the ultimate style to obtain the targeted strain sensitivity as GWD.
- Ultimate Sensitivity should be dominated only by “Quantum Noise”.
- There are many noise sources that should be suppressed and many techniques were investigated.