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A new control method for walking robots based on angular momentum

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Abstract

In this paper we present an efficient algorithm for controlling the angular momentum of walking robots through the manipulation of the zero moment point (ZMP). A remarkable feature of our control method is that the ZMP is considered an actuating signal of the controller. The proposed method can be applied in real time situations because it does not need an accurate tracking of joint angles. Its application to walking robots results in a smooth and soft motion. Experimental results, based on a theoretical explanation, verify the validity of the proposed method.

1. Introduction

In contrast to industrial robot manipulators, the interaction between the walking robots and the ground is complex. The concept of the zero moment point (ZMP) [1] is known to give good results in order to control this interaction. The ZMP represents the point at which the ground reaction force is applied. In many papers concerning walking robots, the ZMP trajectory is used as a reference for motion planning [2]. The leg motion and the reference trajectory of ZMP are prescribed. The upper body motion is generated such that the ZMP follows the prescribed trajectory [1,2]. When a stable walking pattern is designed, the joint angles are controlled to track the reference angles. In order to achieve stable motion, an accurate tracking control is needed. The ZMP deviates from the prescribed trajectory due to disturbances or tracking errors which influence the robot's stability. Therefore, an important problem is how to compensate for the deviation of ZMP due to unexpected disturbances or tracking errors [1,3,4].

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In contrast with previous research, we consider the ZMP to be an actuating signal of the controller and propose a feedback control law to control the angular momentum of walking robots. Angular momentum is a useful physical quantity for generating the gait of bipedal walking robots [5,6]. In order to manipulate the ZMP, we analyze the relationship between the foot pressure, ankle joint torque, and ZMP. Based on the ZMP manipulation method, the angular momentum control law is developed. In the controller, the angular momentum of the robot is used as the feedback signal to update the ZMP target position. Our method can be applied in real time situations because it does not require an accurate tracking of joint trajectories, which is difficult to be achieved due to disturbances or terrain conditions. Moreover, experimental results show that a smooth and soft motion is generated because rigid servo-control is not necessary.

This paper is organized as follows: In Section 2, the control problem is formulated. Manipulation of the ZMP using ankle joint torque is discussed in Section 3. The control law, based on the ZMP feedback control method, is proposed and verified experimentally in Section 4. In Section 5, the overall control system for walking robots is discussed. Finally, conclusions are provided in Section 6.

2. ZMP and angular momentum of walking robots

The relationship between the ground reaction force and moment is important in order to control the angular momentum of walking robots. We derived two different expressions of the ZMP to find this relationship; one from the body motion and the other one from the foot pressure. The correspondence between these two expressions is important for the implementation of the desired control method.

To formulate the control method, we consider a general rigid body chain, as shown in Fig. 1. This model is the same as the biped robot during the single support phase. To simplify the problem, the motion is constrained to the sagital plane. The

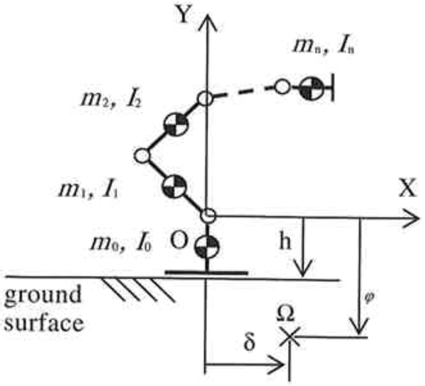

Fig. 1. Rigid body chain.

origin of the coordinate system is located at the ankle joint (the joint of the base link). When the foot (the base link) is in contact with the ground along its entire length, the moment M_Ω with respect to an arbitrary point Ω is the sum of moments created by the inertial and gravitational forces M_I and M_G, as follows:

$$M_\Omega = M_\mathrm{I} + M_\mathrm{G}. \tag{1}$$

M_I and M_G are given as:

$$M_\mathrm{I} = \sum_{i=1}^{n} I_i\dot{\omega}_i + \sum_{i=1}^{n} m_i\{\ddot{x}_i(\varphi - y_i) - \ddot{y}_i(\delta - x_i)\}, \tag{1a}$$

$$M_\mathrm{G} = \sum_{i=0}^{n} m_i g(\delta - x_i), \tag{1b}$$

where (δ, φ) are the coordinates of the point Ω, g is gravitational acceleration, and (x_i, y_i), m_i, I_i and ω_i are the coordinates, mass, moment of inertia, and angular velocity of each link, respectively. This relation is useful to analyze the interaction between the foot and the ground. The ankle joint torque, ZMP, pressure, and friction along the sole of the foot can be derived from this relation. Because the moment with respect to the ZMP is equal to zero ($M_\Omega = 0$), by substituting $\varphi = h$ into Eq. (1), the ZMP coordinate is obtained by solving for δ.

The moment with respect to the ankle joint can be obtained by substituting $(\delta, \varphi) = (0, 0)$ into Eq. (1), as follows:

$$M_A = \sum_{i=1}^{n} I_i\dot{\omega}_i + \sum_{i=1}^{n} m_i(\ddot{y}_i x_i - \ddot{x}_i y_i) - \sum_{i=0}^{n} m_i g x_i. \tag{2}$$

Based on Eqs. (1) and (2), the moment M_Ω can be rewritten as:

$$M_\Omega = M_A - \sum_{i=1}^{n} m_i(\ddot{y}_i - g)\delta + \sum_{i=0}^{n} m_i\ddot{x}_i\varphi. \tag{3}$$

If we substitute the coordinate of the ZMP, δ_ZMP, and the vertical height of the ankle joint h, respectively into δ and φ of Eq. (3), the moment M_Ω becomes zero. This can be written as:

$$0 = M_A - \sum_{i=1}^{n} m_i(\ddot{y}_i - g)\delta_\mathrm{ZMP} + \sum_{i=0}^{n} m_i\ddot{x}_i h. \tag{4}$$

The Eqs. (2)–(4) are useful in controlling the body angular momentum because they give the relationship among the reaction force, ankle joint torque and ZMP position.

An equivalent expression to Eq. (4) can be obtained by considering the equilibrium of moments with respect to the ankle joint. Referring to Fig. 2, the ankle joint torque τ balances the moments of the resultant foot pressure and friction at the

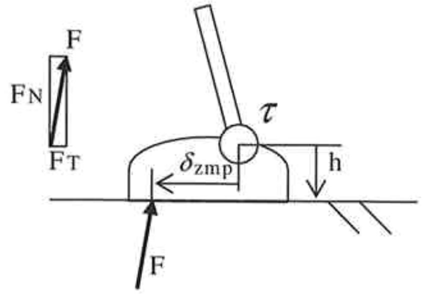

Fig. 2. Equilibrium of moments with respect to the ankle joint.

ZMP. Considering F_N and F_T, respectively the foot pressure and friction at the ZMP, we can write:

$$F_N \delta_{\mathrm{ZMP}} - F_T h + \tau = 0. \tag{5}$$

According to this equation, if the foot pressure and friction are known, it is possible to manipulate the ZMP by changing the ankle joint torque. To have the ZMP in a desired position δ^d_{ZMP}, the ankle joint torque can be calculated as:

$$\tau = F_T h - F_N \delta^d_{\mathrm{ZMP}}. \tag{6}$$

3. Manipulation of ZMP through ankle joint torque

In order to verify the manipulation of the ZMP by changing the ankle joint torque, an experimental setup is developed in our laboratory, as shown schematically in Fig. 3. A weight is attached at the end of a simple pendulum to substitute the

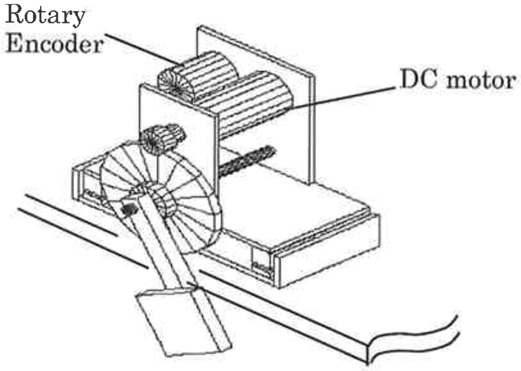

Fig. 3. Schematic of the experimental ankle joint.

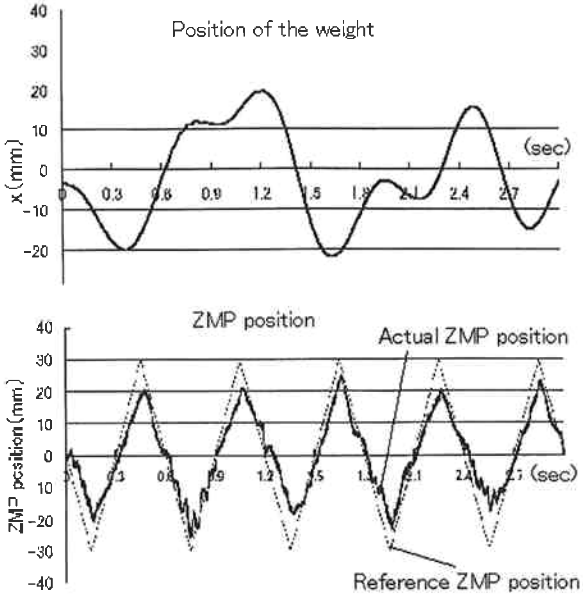

Fig. 4. Reference and actual ZMP position.

robot body. To simplify our experiments, the vertical height of the ankle position is considered to be 0. Thus, the torque at the ankle joint is given as:

$$\tau = -F_N \delta^d_{\mathrm{ZMP}} \tag{7}$$

As we explained in the previous section, the measurement signals of pressure and friction forces are necessary to manipulate the ZMP. There are several methods that can be used to measure these forces. Force sensors can be attached to either the ankle joint or the sole region. The type of pressure sensors used in our experiments is A101-25 Nitta FrexiForce$^{\mathrm{TM}}$. They are attached to the four corners of the sole plate. The foot pressure is obtained by summing the force signals. By using the force sensor data, it is easy to calculate the actual ZMP position.

A set of experimental results is shown in Fig. 4. A saw tooth signal is given as the ZMP reference trajectory. In the stationary state, the ZMP position coincides with the x-coordinate of the center of mass. The results show that the ZMP can be manipulated independently from the robot body position and with a sufficient speed compared with the pendulum motion.

4. Control of angular momentum through ZMP manipulation

Based on the ZMP manipulation method presented in the previous section, we derive a control law for the robot body. In our method, the controlled variable is the body angular momentum with respect to a reference point. The actuating signal of feedback controller is the ZMP. The reference point can be chosen arbitrarily in the

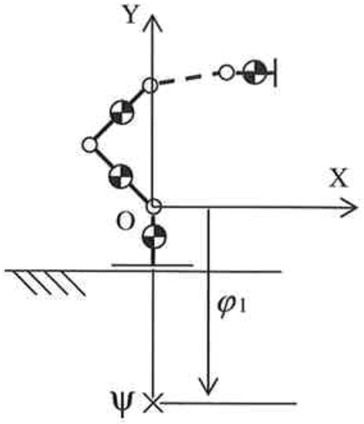

Fig. 5. Reference point for calculating the angular momentum.

region below the ground surface. In this paper, we consider a point just below the ankle joint. The reference point Ψ and its distance from the ankle joint φ_1 are shown in Fig. 5.

The moment with respect to the point Ψ is expressed by substituting $\varphi = \varphi_1$ and $\delta = 0$ into Eq. (1). Therefore, it can be written:

$$M_\Psi = \sum_{i=1}^{n} I_i\dot{\omega}_i + \sum_{i=1}^{n} m_i\{\ddot{x}_i(\varphi_1 - y_i) + \ddot{y}_i x_i\} - \sum_{i=0}^{n} m_i x_i g. \tag{8}$$

In order to derive the control law, a new point Ψ_1 is defined on the horizontal line drawn through point Ψ, as shown in Fig. 6. The body moment with respect to Ψ_1 is given as:

$$M_{\Psi 1} = M_\Psi^* - \sum_{i=1}^{n} m_i(\ddot{y}_i - g)\delta - \sum_{i=0}^{n} m_i x_i g, \tag{9}$$

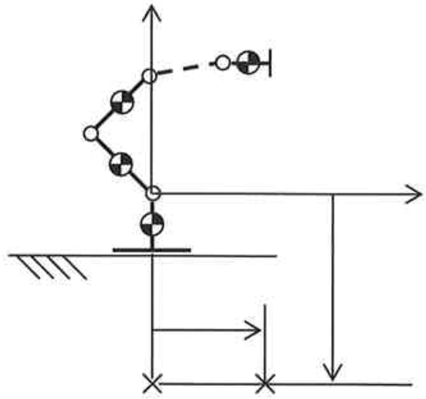

Fig. 6. Ψ and Ψ_1 points.

where M^*_Ψ is the moment due to inertial forces with respect to the point Ψ, given as:

$$M^*_\Psi = \sum_{i=1}^{n} I_i \dot{\omega}_i + \sum_{i=1}^{n} m_i\{\ddot{x}_i(\varphi_1 - y_i) + \ddot{y}_i x_i\}. \tag{10}$$

On the horizontal line, we can find a point $P(\varphi_1, \delta'_{\mathrm{ZMP}})$ with respect to which the moment is equal to zero. This is written as follows:

$$0 = M^*_\Psi - \sum_{i=1}^{n} m_i(\ddot{y}_i - g)\delta'_{\mathrm{ZMP}} + \sum_{i=0}^{n} m_i x_i g. \tag{11}$$

Point P and ZMP are not the same because P is not a point on the ground surface. However, as described in the previous section, P can be manipulated in the same way by substituting φ_1 with h in Eq. (6). Considering K_Ψ the angular momentum with respect to the point Ψ, and by substituting $\dot{K}_\Psi = M^*_\Psi$ into Eq. (10), we obtain:

$$\dot{K}_\Psi = \sum_{i=1}^{n} m_i(\ddot{y}_i - g)\delta'_{\mathrm{ZMP}} - \sum_{i=0}^{n} m_i x_i g. \tag{12}$$

Based on this relation, it is possible to control the angular momentum K_Ψ by using δ'_{ZMP} as the actuating signal. The feedback control law is written as:

$$\delta'_{\mathrm{ZMP}} = \frac{1}{\sum_{i=1}^{n} m_i(\ddot{y}_i - g)}\left(\sum_{i=0}^{n} m_i x_i g + K_\Psi - K_{\mathrm{ref}}\right), \tag{13}$$

where K_{ref} is the reference value of the angular moment. Eq. (13) can be rewritten in terms of the foot pressure signal F_N as follows:

$$\delta'_{\mathrm{ZMP}} = \frac{1}{F_N}\left(\sum_{i=0}^{n} m_i x_i g + K_{\mathrm{ref}} - K_\Psi\right). \tag{14}$$

By substituting Eq. (13) into Eq. (12), the motion of the robot under this control is written:

$$\dot{K}_\Psi = K_{\mathrm{ref}} - K_\Psi. \tag{15}$$

Therefore, the angular momentum K_Ψ converges to the reference value K_{ref}. The selection of K_{ref} and φ_1 depend on the desired gait. For example, when we want the robot to be in a stationary standing position, K_{ref} should be set to zero. When a large value of φ_1 is chosen, the control mainly effects the center of mass's horizontal velocity because it has stronger influence on the angular momentum.

The block diagram of the control system is shown in Fig. 7. The proposed control method is similar with the balance mechanism of humans, where the body balance is maintained by adjusting the point at which the foot pressure is applied.

To verify the angular momentum control law, an experimental robot leg is built, as shown in Fig. 8. DC servomotors actuate the ankle and knee joints. The rotary encoders measure the angle and angular velocity of each joint, making possible to calculate the angular momentum. The ZMP manipulation method, described in the

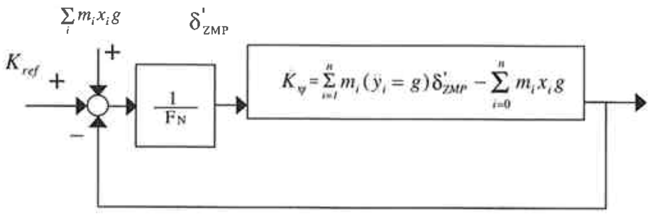

Fig. 7. Block diagram of the control method.

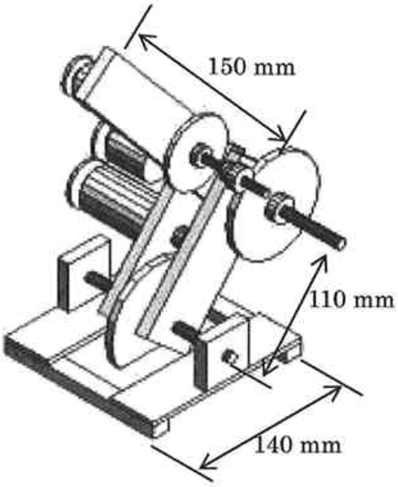

Fig. 8. Experimental robot leg.

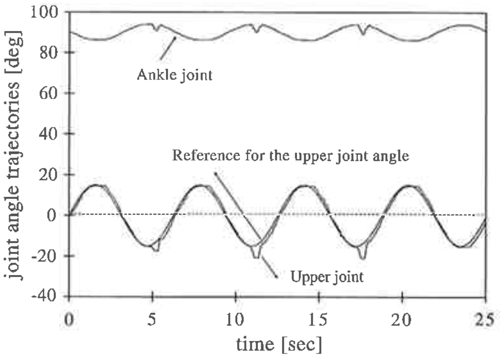

Fig. 9. Joint angle trajectories.

previous section, is applied to the ankle joint. In this experiment, the reference point Ψ is selected to be on the ground surface just below the ankle joint. The feedback control law, Eq. (14), is applied to control the angular momentum. In the first ex-

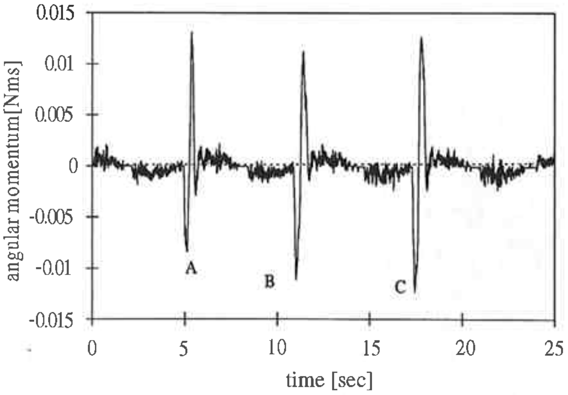

Fig. 10. Angular momentum response for zero input reference.

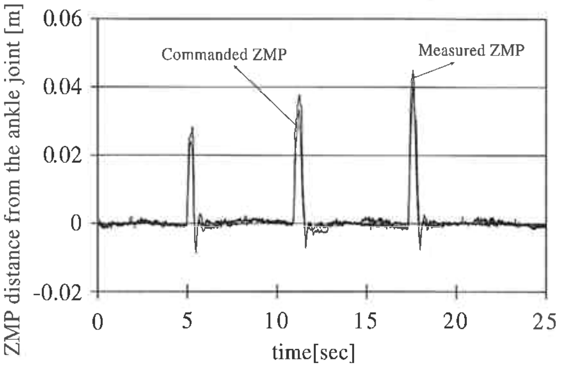

Fig. 11. Commanded and real ZMP positions.

periment, the reference value for the angular momentum is set to zero, while the upper joint is controlled to follow a reference sinusoidal wave (Fig. 9). As shown in Fig. 10, the variation of angular momentum from the reference value is very small. The ZMP is all the time inside the sole region as shown in Fig. 11, where the commanded and real ZMP positions are presented.

In order to verify the stability of the control under disturbances, a tap was applied to the upper link at points A, B, and C. The results show that the control method based on angular momentum works well. The control method provides sufficient stability regardless of the upper link motion, as shown in Fig. 12 where the video capture of the experiment is presented.

In the second experiment, the reference value for the angular momentum was set to 0.005 [N m s]. The variation of the controlled angular momentum is shown in Fig. 13. A small degree of tracking error remains after the angular momentum converges to its reference. The ankle joint friction causes the tracking error. However, the controller is confirmed to work well.