

Optimal trajectory generation for a prismatic joint biped robot using genetic algorithms

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Abstract

In this paper, a prismatic joint biped robot trajectory planning method is proposed. The minimum consumed energy is used as a criterion for trajectory generation, by using a real number genetic algorithm as an optimization tool. The minimum torque change cost function and constant vertical position trajectories are used in order to compare the results and verify the effectiveness of this method. The minimum consumed energy walking is stable and the impact of the foot with the ground is very small. Experimental investigations of a prismatic joint biped robot confirmed the predictions concerning the consumed energy and stability. © 2002 Elsevier Science B.V. All rights reserved.

Keywords: Biped robots; Genetic algorithm; Minimum energy; ZMP

1. Introduction

In our laboratory, we have developed different kinds of biped robots. One of them is the prismatic joint biped robot [1]. In the previous papers related with the prismatic joint biped robot, the objective has been to verify a new control method based on the manipulation of Zero Moment Point (ZMP) [2]. In most of the experiments, the vertical position of the point mass is the considered constant. The results of the control method have been very good and the point mass trajectory followed the prescribed trajectory accurately. But sometimes, the impact of the foot with the ground influences the motion stability. The prismatic joint biped robot is actuated by external power supply. Because it is a locomotion system, it is more convenient that a battery

actuates the motors. In order to have a long operation time of the battery, the consumed energy must be reduced. In this paper, we propose a new trajectory generation method based on minimum Consumed Energy (CE). Our objective is to reduce the CE during walking and guarantee a stable motion of biped robot.

The biped robot optimal gait generation, which belongs to an optimization problem, is considered in [3–5]. In these works, the conventional optimization methods are used to generate the optimal gait of a five-link biped robot. In these methods, the objective is to optimize the joint angle trajectories. On the other hand in the prismatic joint biped robot, the problem is to determine the optimal point mass trajectory during walking.

In this paper, we use a real number Genetic Algorithm (GA) as an optimization tool because GA has been known to be robust for search and optimization problems [6]. GA has been used to solve difficult

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problems with objective functions that do not possess properties such as continuity, differentiability, etc. It manipulates a family of possible solutions that allows the exploration of several promising areas of the solution space at the same time. Also GA makes handling the constraints easy by using a penalty function vector, which converts a constrained problem to an unconstrained one. In our work, the most important constraint is the stability, which is verified by the ZMP concept. The optimal motion generated by GA is stable and the ZMP is all the time inside the sole region.

In experiments, we see that the friction force in the translation motion cannot be neglected. We considered it as the Coulomb friction. But the difficulty to calculate it exactly gave some differences between the simulation and experimental results. By improving the robot design, we are trying to reduce the friction as much as possible. Therefore the simulation and experimental results would be closer to each other. We compared the energy needed for minimum CE, minimum Torque Change (TC) and constant vertical position trajectories. The simulation and experiments show that the minimum CE walking is stable and the impact of the foot with the ground is small.

This paper is organized as follows. In Section 2 are presented the structure and control method of the prismatic joint biped robot. Section 3 gives a brief explanation of GA. The problem formulation is treated in Section 4. The proposed method is presented in Section 5. Simulation and experimental results are given in Section 6. Finally some conclusions are presented in Section 7.

2. Biped model

The biped robot is presented in Fig. 1. Each leg has two degrees of freedom in the sagittal plane, one translation and the other a rotational motion. A DC motor, attached at the body of the robot, activates the translation motion which is realized by a prismatic joint and transmitted through plastic racks and pinions. The second degree of freedom is the rotational motion at the ankle joint. This motion is transmitted through plastic gears and parallelogram linkage, which forms the leg itself. The total weight of the robot is 1.75 kg where each leg is 0.16 kg. The leg length is 400 mm

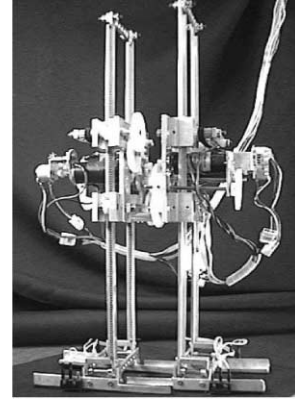


Fig. 1. Prismatic joint biped robot.

and the sole length is 80 mm. Neglecting the legs mass, the robot can be presented schematically as shown in Fig. 2. An advantage of the prismatic joint biped robot is that the reaction force passes approximately through the center of mass of the robot. Therefore the modeling error between the real robot and the dynamic model (Fig. 2) is sufficiently small. The dynamical equations for this system are given as follows:

$$ml \cos \theta \cdot \ddot{x} - ml \sin \theta \cdot \ddot{y} - mgl \sin \theta = \tau, \quad (1)$$

$$m \sin \theta \cdot \ddot{x} + m \cos \theta \cdot \ddot{y} + mg \cos \theta + f_f = f, \quad (2)$$

where $f_f = F_c \operatorname{sgn}(\dot{y} \cos \theta)$ is the Coulomb friction with magnitude F_c . The robot motion control is realized through ZMP manipulation, where the ZMP is used as an actuating signal [1]. The constraint $x_{\text{ZMP}} = \delta$ is written:

$$\delta l f \cos \theta + (l - \delta \sin \theta) \tau = 0. \quad (3)$$

By introducing a new input signal u , Eq. (3) is satisfied if

$$f = (l - \delta \sin \theta) u, \quad (4)$$

$$\tau = -\delta l \cos \theta \cdot u. \quad (5)$$

Substituting (4) and (5) into (1) and (2) yields

$$m \ddot{x} = (x - \delta) u, \quad (6)$$

$$m \ddot{y} = y u. \quad (7)$$

Using these equations, we can define the feedback control law in terms of δ and u . Let $(x_{\text{ref}}, y_{\text{ref}})$ be the reference points for the trunk position, then the control

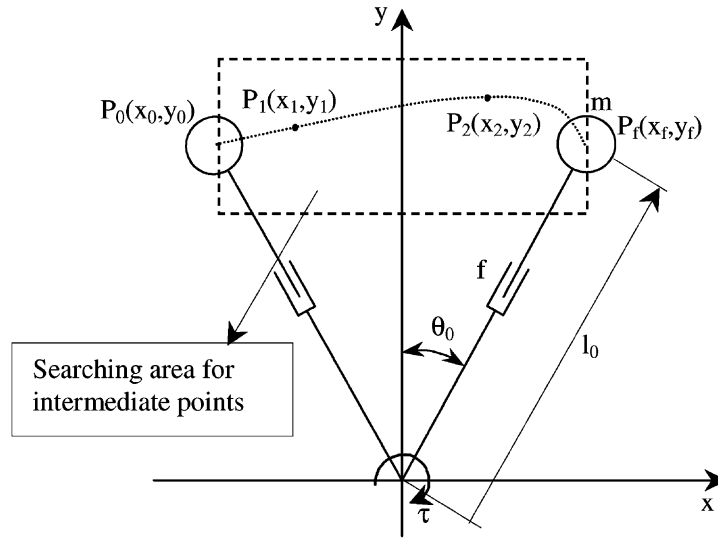


Fig. 2. Leg model.

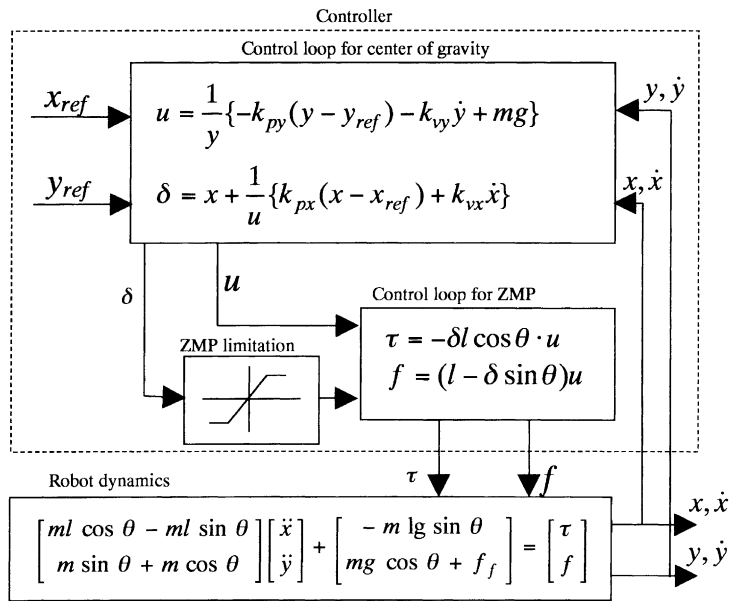


Fig. 3. Block diagram of control system.

law, which is presented in Fig. 3 is written as:

$$\delta = x + \frac{1}{u} \{k_{px}(x - x_{ref}) + k_{vx}\dot{x}\}, \quad (8)$$

$$u = \frac{1}{y} \{-k_{py}(y - y_{ref}) - k_{vy}\dot{y} + mg\}. \quad (9)$$

3. GA

GA is a search algorithm based on the mechanics of natural selection and population genetics. The search mechanism is based on the interaction between indi-

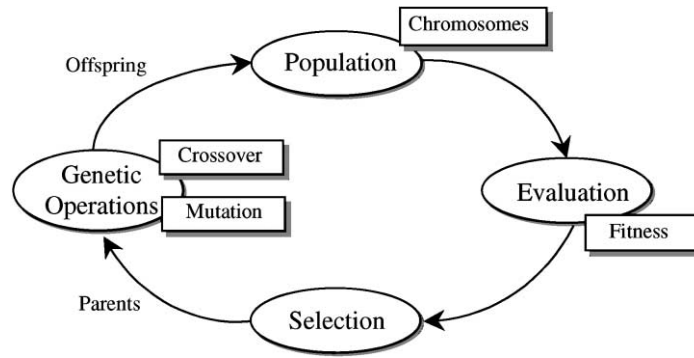


Fig. 4. GA cycle.

viduals and the natural environment. GA comprises a set of individuals (the population) and a set of biologically inspired operators (the genetic operators). The individuals have genes which are the potential solutions for the problem. The genetic operators are crossover and mutation. GA generates a sequence of populations by using genetic operators among individuals. Only the most suited individuals in a population can survive and generate offspring, thus transmitting their biological heredity to the new generation.

GA operates through a simple cycle of four stages, as shown in Fig. 4. Each cycle produces a new generation of possible solutions for a given problem. At the first stage, an initial population of potential solutions is created as a starting point for the search. In the next stage, the performance (fitness) of each individual is evaluated with respect to the constraints imposed by the problem. Based on each individual's fitness, a selection mechanism chooses "parents" for the crossover and mutation operators. The crossover operator takes two chromosomes and swaps part of their genetic information to produce new chromosomes. The mutation operator introduces new genetic structures in the population by randomly modifying some of the genes, helping the search algorithm to escape from local minima's traps. The offsprings produced by the genetic manipulation process are the next population to be evaluated. GA can replace either a whole population or its less fitted members only. The creation–evaluation–selection–manipulation cycle repeats until a satisfactory solution to the problem is found or some other termination criteria are met.

4. Minimum energy trajectory problem

We define the minimum energy trajectory as a path to connect the starting and terminal points, $P(x_0, y_0)$ and $P(x_f, y_f)$, respectively. The objective is to find the point mass trajectory which minimizes the CE. It can be assumed that the energy to control the robot position is proportional to the integration of the square of the torque with respect to time. Because the manipulator joints are driven by torque, then the unit of torque (Nm) is equal to the unit of energy (Joule). So, the cost function (J) can be defined as the following expression:

$$J = \frac{1}{2} \left(\int_0^{t_f} \tau^T \tau dt + \int_0^{t_f} C dt \right), \quad (10)$$

where t_f is the step time, τ is the torque vector and C is the constraint function given as follows:

$$C = \begin{cases} 0 & \text{if the constraints are satisfied,} \\ c_i & \text{if the constraints are not satisfied,} \end{cases}$$

where c denotes the penalty function vector.

We consider the following constraints for our system:

1. The ZMP is all the time in the sole region.
2. The point mass position should not exceed the leg length.

To clarify the effect of the proposed method, the minimum CE trajectory is compared with the trajectory that minimizes the rate of change of torque [7].

The cost function is as follows:

$$J_{\text{torque change}} = \frac{1}{2} \left(\int_0^{t_f} \left(\frac{d\tau}{dt} \right)^T \left(\frac{d\tau}{dt} \right) dt + \int_0^{t_f} C dt \right). \quad (11)$$

5. GA variables and proposed method

To have a repetitive and continuous motion, the initial and final vertical positions of the point mass are considered to be the same at the beginning and end of the step. For a given step length, the initial and final horizontal positions are known. Also the horizontal velocity is considered to be the same at the beginning and end of the step. In order to find the optimal values of the initial vertical position and horizontal velocity, we consider them as two variables of GA. The GA generates the coordinates of two intermediate points, $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, in first and second half of the step (Fig. 2), and their respective passing times t_1 and t_2 . The GA searching intervals of x_1 and x_2 are $[-\text{step_length}/2; 0]$ and $[0; \text{step_length}/2]$, respectively. The y_1 and y_2 are considered to be proportional with the initial vertical position. In this way, we limit the GA searching area which influences the time and convergence of the GA. This has no effect on the optimal trajectory result for the following reasons:

1. Based on the simulation results, the difference between the maximum and the initial vertical position is not larger than 0.1 m.
2. The step time during the experiments is short. Therefore the body mass cannot change the vertical position in a wide range.

The coordinates y_1 and y_2 can be written as follows:

$$y_1 = k_1 y_0, \quad (12)$$

$$y_2 = k_2 y_0. \quad (13)$$

The optimal values of y_1 and y_2 are determined by optimizing the coefficients k_1 and k_2 . The point mass coordinates, x and y , are presented as polynomials of time. The degree of the polynomials is determined based on the number of constraints and the coefficients are calculated to satisfy them.

The block diagram of the proposed method is presented in Fig. 5. Based on the initial conditions and range of the searching variables, an initial population is generated. Each individual in the population

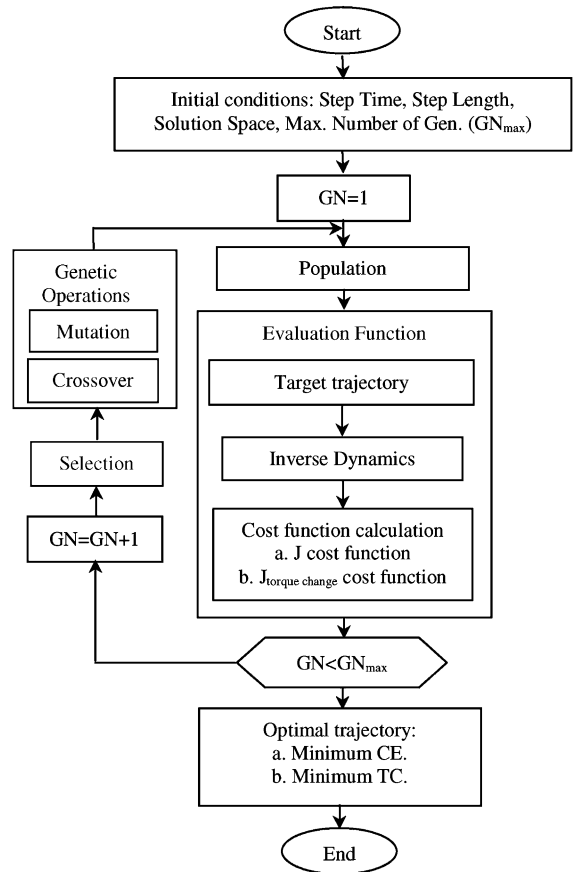


Fig. 5. The block diagram of the proposed method.

presents a possible point mass trajectory. The torque τ_1 and force f_2 are calculated for every individual based on the inverse dynamics of biped robot, Eqs. (1) and (2). To calculate the minimum CE and minimum TC cost functions, the prismatic joint force f_2 is converted in its corresponding torque as follows:

$$\tau_2 = f_2 r, \quad (14)$$

where r is the radius of the applied force.

The cost function is calculated for minimum CE or minimum TC, respectively. The value of the cost function is attached to every individual in the population. The GA moves from generation to generation, selecting parents and reproducing offsprings until the termination criterion (maximum number of generations GN_{max}) is met.

Table 1
GA functions and parameters

Function name	Parameters
Arithmetic crossover	2
Heuristic crossover	[2, 3]
Simple crossover	2
Uniform mutation	4
Non-uniform mutation	[4, $G_{n_{max}}$ 3]
Multi-non-uniform mutation	[6, $G_{n_{max}}$ 3]
Boundary mutation	4
Normalized geometric selection	0.08

The optimal motion is generated for minimum CE or minimum TC based on the GA results. These trajectories are used to carry out experiments. During the experiments the objective of the controller is that the point mass trajectory follows the minimum CE or minimum TC trajectory, by using joint servo controllers implemented in each joint.

6. Simulation and experimental results

For the optimization of the cost function, a real-value GA was employed in conjunction with the selection, mutation and crossover operators. Many experiments, comparing real-value and binary GA, show that the real-value GA generates better results in terms of the solution quality and CPU time [8]. The GA functions and parameters are shown in Table 1. To ensure a good result of the optimization problem, the best GA parameters are determined by extensive simulations. The maximum number of generations is used as the termination function. The GA converges within 40 generations, as shown in Fig. 6. The average value of J for every generation is shown in Fig. 7. The 40th generation has the lowest value, the same as Fig. 6.

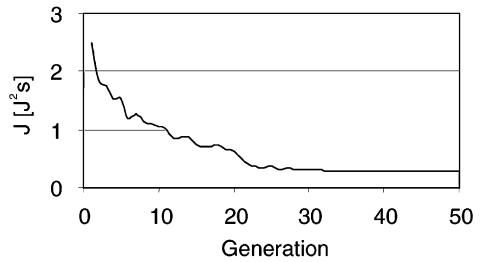


Fig. 6. GA convergence for CE cost function.

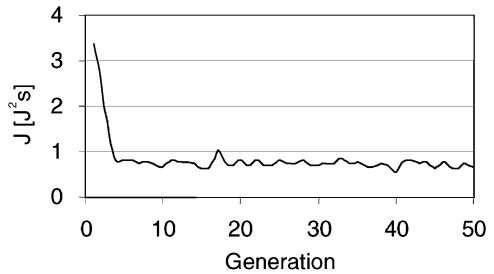


Fig. 7. Average of the cost function J .

In the following the minimum CE, minimum TC and constant vertical position trajectories are analyzed in terms of the energy consumption and stability for the step length and step time, 0.07 m and 0.8 s, respectively. The magnitude of the Coulomb friction is considered 12 N. The values of torque for minimum CE and minimum TC cost functions are presented in Figs. 8(a) and 8(b), respectively. The torque changes more smoothly when the minimum TC is used as cost function, and the value of the torque is higher compared with minimum CE. The point mass trajectories are presented in Fig. 9. Based on the simulation results, we see that the vertical position of body mass changes when minimum CE is used as cost function. Therefore

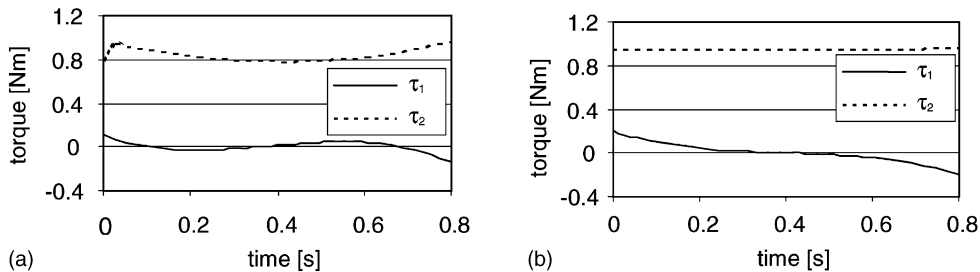


Fig. 8. Torque value: (a) CE; (b) TC.

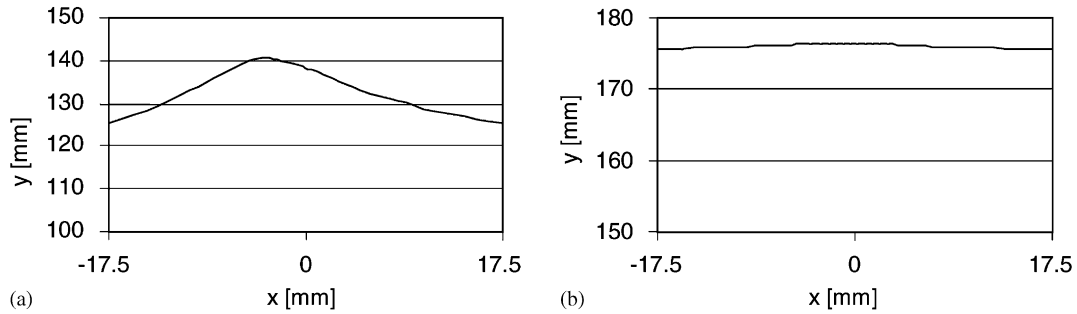


Fig. 9. Point mass trajectory: (a) CE; (b) TC.

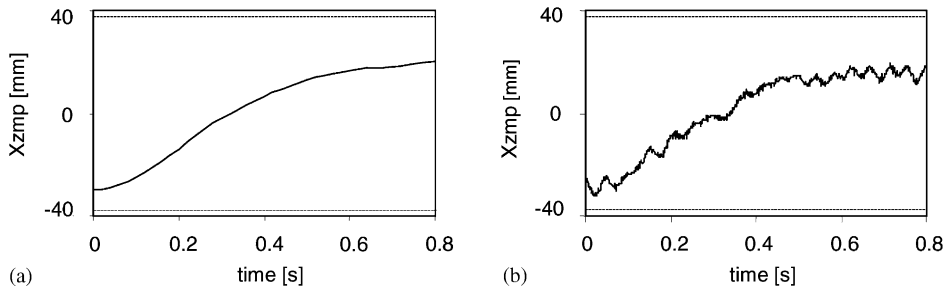


Fig. 10. ZMP position: (a) simulation; (b) experiment.

the prismatic joint will be active during walking. The ZMP trajectories of minimum CE walking motion are presented in Figs. 10(a) and 10(b) for the simulation and experiment, respectively. From these figures, it is clear that the ZMP is all the time inside the sole region presented by dotted lines, which guaranty a stable walking motion.

The values of J for minimum CE, minimum TC and constant vertical position trajectories are presented in Fig. 11. The minimum CE trajectory reduces the energy by nearly 30% compared with minimum TC trajectory and 24% compared with $y = 100$ mm constant, which has the minimum value of energy consumption compared with other constant vertical position trajectories.

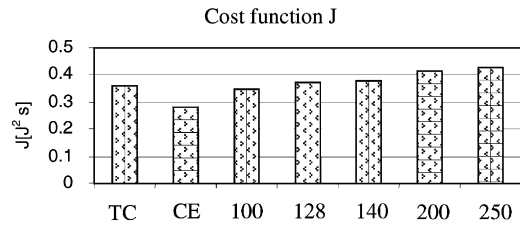


Fig. 11. Energy comparison based on simulation results.

In order to verify the simulation results, the experiments are performed with the prismatic joint biped robot. The CE is considered to be proportional with the armature current i_a . This is justified from the motor supply hardware, schematically shown in Fig. 12.

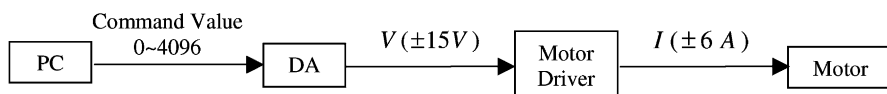


Fig. 12. Block diagram of motor supply hardware system.

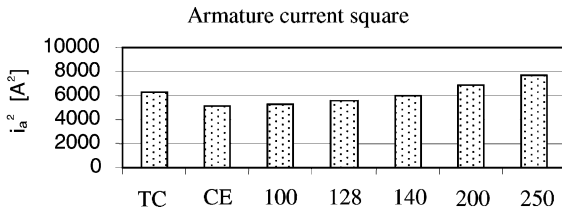
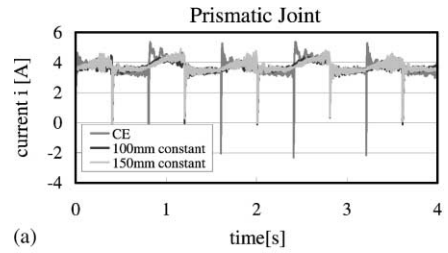
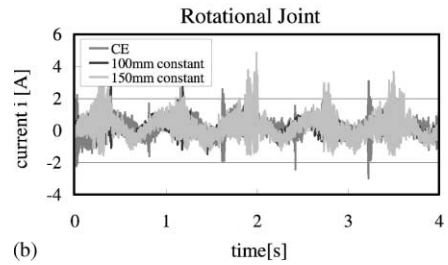


Fig. 13. Energy comparison based on experimental results.

The values of armature current square i_a^2 , measured from the experimental results, are shown in Fig. 13. We see that the minimum CE and $y = 100$ mm constant trajectories consume nearly the same energy. This is also clear from Fig. 14, which shows the armature current i_a , for the rotation and prismatic joints. The simulation and experimental results are a little bit different because of the prismatic joint friction. In Fig. 13, the best trajectory of constant vertical position is $y = 100$ mm, which is the lowest possible vertical position. This is because the translation motion of point mass for constant vertical position trajectories is proportional with the vertical height. So when the vertical height is low the translation motion is minimal. Although we included the Coulomb friction in our simulations, it is very difficult to consider it exactly. The friction in the prismatic joint is mainly caused by the joint construction. For this reason, we are improving this joint in order to reduce the friction.



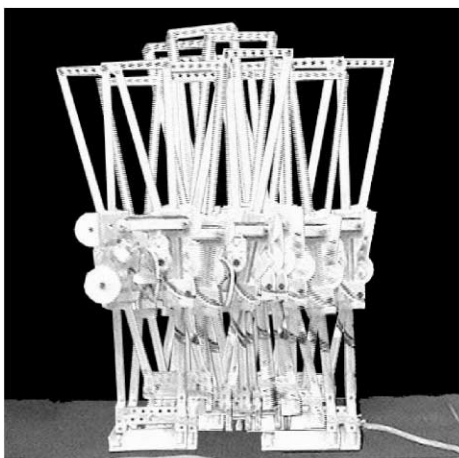
(a)



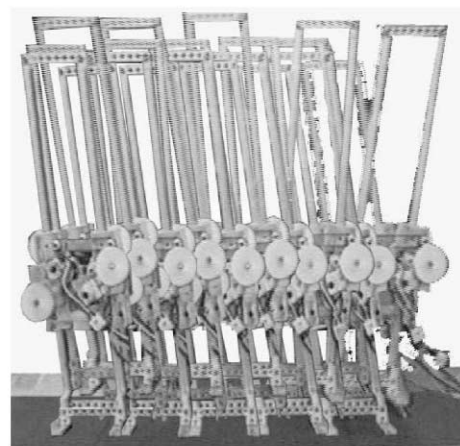
(b)

Fig. 14. Armature current: (a) prismatic joint; (b) rotational joint.

The video captures of two experimental results, minimum CE and 100 mm constant vertical position, are presented in Fig. 15. The walking generated by GA based on minimum CE cost function is stable and the impact of the foot with the ground is very small. On the other hand when the mass point vertical position is constant, the impact is large which sometimes influences the robot stability.



a. Minimum CE.



b. 100 mm constant.

Fig. 15. Video captures of two experimental results: (a) minimum CE; (b) 100 mm constant vertical position.

7. Conclusions

In this paper, we presented a new trajectory generation method for a prismatic joint biped robot generated by GA. The optimal trajectory is generated based on minimum CE. The minimum CE results are compared with the trajectory that minimizes the rate of change of torque, and the trajectories where the vertical position of point mass is considered constant. The optimal trajectory generation is considered without neglecting the stability, which is verified by the ZMP concept. Comparing the simulation and experimental results, we see that there are some differences caused by the friction in the translation motion. Although the Coulomb friction is considered in the simulations, it is very difficult to calculate it exactly. We plan to make more investigations, as well as reconstruct the prismatic joint in order to reduce the friction and bring the simulation and experimental results closer to each other. The simulation and experimental results show that the optimal trajectory generation method has the following advantages:

1. The minimum CE motion is very smooth and the impact of the foot with ground is very small.
2. The optimal motion is stable or the ZMP is all the time inside the sole region.
3. The minimum CE trajectory reduces the energy nearly 24% compared with 100 mm constant vertical position and 30% compared with minimum TC trajectory.

As a future work, we are considering the real-time implementation of the proposed method by using:

1. application of pre-computed optimal trajectories;
2. teaching a neural network based on the GA results;
3. parallel GA.

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