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Control of walking robots based on manipulation of the zero moment point

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SUMMARY

In this paper, a new application of the ZMP (Zero Moment Point) control law is presented. The objective of this control method is to obtain a smooth and soft motion based on a real-time control. In the controller, the ZMP is treated as an actuating signal. The coordinates of the robot body are fed back to obtain its position. The proposed control method was applied on two different biped robots, and its validity is verified experimentally.

KEYWORDS: Walking robots; zero moment point; Control Law; Actuating Signal.

1. INTRODUCTION

The motion of the industrial robot manipulators is usually expressed in terms of joint angles trajectories, which are planned to execute a given task such as picking and placing, spray painting, arc welding, etc. Therefore, the control of industrial robots concentrates on the joint angle trajectories to track the reference ones. On the other hand, the control problem for a walking robot is not simple, because many control objectives must be considered. The designed motion must ensure that the robot gets in the goal position without falling down.

For stable walking, the Zero Moment Point (ZMP)[1] is a well known concept for the synthesis of the walking pattern. ZMP is the point where the resultant of the reaction force is applied. As long as the foot is in contact with the ground along its entire lower surface, the ZMP is inside the sole. In many papers proposed so far, the reference trajectories of the joint angles are calculated based on a prescribed trajectory for the ZMP. If the joint angles track the reference trajectories with sufficient accuracy, the ZMP tracks the prescribed line inside a stable region. The stable region is chosen to preserve the contact between the foot and the ground. The tracking control of the joint angles influences the stability of walking robot. If the tracking error occurs, the ZMP deviates from the prescribed line. In the worst case, the ZMP get outside the stable region. Several methods are proposed for compensating the tracking error of ZMP. Vukobratovich et al.[1] introduced an additional input for one of the joint motors to eliminate the error of the ZMP. Takanishi et al.[2] compensated for the effect of external forces through the trunk motion. Li et al.[3] introduced a learning control method for decreasing the ZMP error. In the Honda humanoid robot,[4] the prescribed ZMP is revised during walking when the error of ZMP occurs.

The conventional D.C. motors for industrial robot manipulators are controlled based on independent joint controller, which is integrated into the motor drviers. Therefore, the control method based on the trajectory planning for each independent joint is easy to apply, because the stability problem is solved by expressing the motion planning in terms of each joint angle trajectory.

On the other hand, the ZMP method requires accurate tracking, and hence the motors must be controlled based on high gain feedback, by using gears with a high reduction rate. Because of the need for a rigid joint angle control, it is difficult to realize smooth and soft motion like that of human beings.

Real time control is important for the walking robot to adapt to the environment changes when the robot interacts with the environment. However, if the trajectories for joint angles are planned beforehand, it is difficult to modify the motion while the robot is walking.

This paper is concentrated on the problems related with the ZMP concept, which are:

1. Implementation of the ZMP control method based on a real time controller; and
2. Relaxing the requirement for strict tracking control.

The objective of this research is to control the motion of the robot through ZMP manipulation, where the ZMP is used as an actuating signal for the motion control of the walking robot. In the control system we introduce a minor feedback loop in order to manipulate the ZMP. The trunk position of the robot is fed back to determine the ZMP, so its position follows the desired trajectory. In contrast with previous ZMP methods, the desired trajectory can be given independently of the ZMP restriction.

This paper is organized as follows: In Section 2, a simplified walking model is considered. A control law, based on ZMP feedback control method is proposed in Section 3. In Section 4, application of the ZMP control algorithm and experimental results are presented. Finally, some conclusions are given in Section 5.

2. ANALYSIS BY USING A SIMPLE MODEL

The motion of a biped robot with multiple degrees of freedom is described by a high order nonlinear differential equation, which is often simplified before defining the

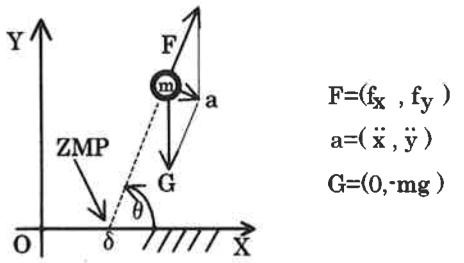

Fig. 1. Motion of a point mass.

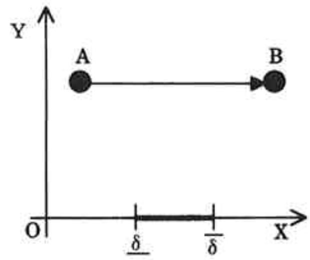

Fig. 2. A possible trajectory of the point mass.

control problem. For example, Miyazaki and Arimoto[5] reduced the order of the dynamic equation by using the singular perturbation method. Mita et al.[6] linearized the equation around the command position of the robot. To explain the control problems of walking robots we consider a simplified model. The model is shown in Figure 1, where m is the mass, G is the gravity force, F is the reaction force from the ground.

Although the reaction force is transmitted through some mechanical structure, it is not specified. The point mass has two degrees of freedom in the sagital plane. We consider the motion control with respect to the ground. The point mass is controllable if a desired reaction force is applied from the ground. However, in the case of walking robot, it is not always possible to control the motion through a desired reaction force, because the reaction force is restricted by the condition of the foot contact. Therefore, the motion control problem is considered under the restriction on the ZMP, because it has a strong connection with the foot contact.

The dynamic equation of the point mass will be:

$$m\ddot{x}=f_x, \tag{1}$$

$$m\ddot{y}=f_y-mg, \tag{2}$$

where m is the mass, g is the gravity acceleration, f_x and f_y are components of the reaction force. These values can be expressed in terms of the ZMP position and the reaction forces magnitude as follows:

$$f_x=\|F\|\cos\theta, \tag{3}$$

$$f_y=\|F\|\sin\theta, \tag{4}$$

$$\cos\theta=\frac{x-\delta}{\sqrt{(x-\delta)^2+y^2}}, \tag{5}$$

$$\sin\theta=\frac{y}{\sqrt{(x-\delta)^2+y^2}}, \tag{6}$$

where δ is the coordinate of the ZMP. The ZMP is found as an intersection of the ground line with the reaction force vector. By the geometrical relations, δ is represented in terms of x and y as

$$\delta=\frac{x(\ddot{y}+g)-y\ddot{x}}{\ddot{y}+g}. \tag{7}$$

Generally, in the control problem of walking robots, the ZMP must be inside a stable region during walking. In the leg mechanism, the foot is in contact with the ground along its entire lower surface as long as the ZMP is inside the lower surface. If the foot contact is lost, the reaction force can not be transmitted to the upper part of the robot. In this sense, the point mass is controllable if the ZMP is inside a bounded region.

Let us consider a possible trajectory for the point mass under the condition that the ZMP is inside a bounded region. In order to simplify the problem, the vertical position y is fixed during the motion. In Figure 2, A and B are, respectively, the starting and the end point of motion, $[\underline{\delta}<\bar{\delta}]$ is the allowable region for the ZMP. The trajectory distance AB is longer than the region of ZMP, because in the case of dynamic walking, the step length is larger than the foot length. Since the ZMP is affected by the position and acceleration of the point mass, the $x(t)$ and $\ddot{x}(t)$ must be determined to get the ZMP trajectory. An efficient solution for this problem is determining $x(t)$ as a solution of the following differential equation:

$$\delta=\frac{xg-y\ddot{x}}{g}, \qquad \underline{\delta}\leq\delta\leq\bar{\delta}, \tag{8}$$

which is obtained by keeping y constant in equation (7). The coordinate of ZMP must be determined within the region: $[\underline{\delta}<\bar{\delta}]$. Consider that the ZMP move from $\underline{\delta}$ to $\bar{\delta}$ at a constant velocity during the motion, $\delta(t)$ is given as follows:

$$\delta(t)=\underline{\delta}+\frac{\bar{\delta}-\underline{\delta}}{T}t,$$

where T is the motion period of time. The solution $x(t)$ is calculated as:

$$x(t)=(x_0-\underline{\delta})\cosh\frac{1}{\alpha}t+\alpha\left(\nu_0-\frac{\bar{\delta}-\underline{\delta}}{T}\right)\sinh\frac{1}{\alpha}t+\frac{\bar{\delta}-\underline{\delta}}{T}t+\underline{\delta},$$

where, $x_0=x(0)$, $\nu_0=\nu(0)$ and $\alpha=\sqrt{\frac{y}{g}}$. An example of $x(t)$ is plotted in Figure 3.

If the point mass follows this trajectory, then the ZMP is always inside the region $[\underline{\delta}<\bar{\delta}]$. In the conventional method based on the ZMP concept, the trajectories of the joint angles must be prepared in advance of the control execution. Then, in the control execution, the motion is controlled to track the trajectory. If the motion tracks the desired one precisely, then the ZMP is always inside the stable region. If we consider a leg mechanism, the trajectory can be expressed in terms of the joint angles, instead of the

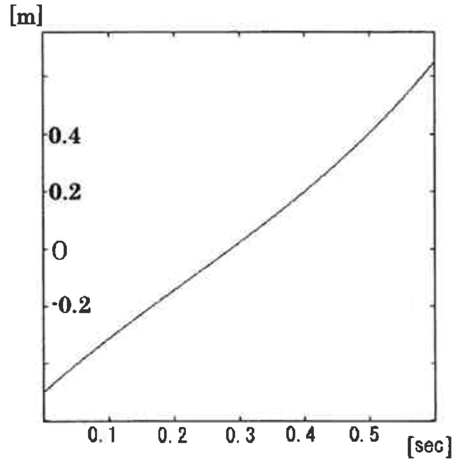

Fig. 3. Trajectory of x.

coordinates of the mass. In most of industrial robot manipulators, the servo motor of each joint is controlled by an individual controller that controls the rotation angle of the motor shaft. Therefore, a desired motion would be simply realized by providing the reference trajectory for each motor controller. Thus each joint tracks the desired trajectory.

However, in practical situations, controlling position and acceleration to track their reference ones is often difficult, because of the disturbances such as frictions, backlashes in the transmissions, and torque ripple of the electric motors.

3. MANIPULATION OF THE ZMP

From a control point of view, the motion of an object is governed by the forces acting on it. In the simple point mass system, these forces are the reaction force from the ground and the gravity force. Since the gravity force is fixed, in order to control the motion, the reaction force must be determined according to the desired motion. On the other hand, in the case of walking robots, in order to apply a desired reaction force to the trunk of the robot, firm contact is necessary between the foot and the ground. These requirements oppose each other because the condition of the foot contact is influenced by the ZMP, which depends on the motion of the robot.

In this section, a control method is proposed in order to solve this problem. The control method is described by applying it to the simple system described in the previous section. The following assumptions are considered:

[A1.] Position of the ZMP can be given arbitrarily within the lower surface of the foot in contact with the ground.

[A2.] Arbitrarily positive reaction force can be applied to the point mass on condition that the ZMP is inside some bounded stable region (The stable region is determined by the sole area of the foot, when the leg mechanism is taken into consideration).

Following these assumptions, our problem is to control the position (x,y), by choosing an appropriate δ (the coordinate of the ZMP) in order to supply an appropriate reaction force f. Our idea for this control problem is a simple feedback control law, where the position (x,y) is fed back to obtain f and δ.

The vertical position y is controlled by the following feedback control:

$$f_y = h_{ref} - y - \dot{y} + mg, \tag{9}$$

where h_{ref} is the reference of the vertical position.

Applying this control, the following equation is satisfied:

$$m\ddot{y} + \dot{y} + y = h_{ref}. \tag{10}$$

So, y converges to h_{ref} exponentially.

Motion of the mass in the x direction is derived from equations (1), (3), and (9) as follows:

$$m\ddot{x} = \frac{\cos\theta}{\sin\theta}(h_{ref} - y - \dot{y} + mg),$$

$$= \frac{x-\delta}{y}(h_{ref} - y - \dot{y} + mg). \tag{11}$$

Defining β as:

$$\beta = \frac{my}{h_{ref} - y - \dot{y} + mg}, \tag{12}$$

equation (11) can be rewritten as:

$$\beta\ddot{x} - x + \delta = 0. \tag{13}$$

Since y converges to h_{ref} exponentially, in the same way β converges to $\frac{h_{ref}}{g}$.

Considering the ZMP position δ as an actuating signal, x is controlled by applying the following feedback control.

$$\delta = x + \dot{x} - v_{ref}, \tag{14}$$

where v_{ref} is the velocity reference.

By using the Lyapunov function, it is easy to understand the convergence of $\dot{x}$ to v_{ref}

$$V = (\dot{x} - v_{ref})^2. \tag{15}$$

Differentiating V along the solution of Equation (13) and Equation (14) yields

$$\dot{V} = -\frac{2}{\beta}(\dot{x} - v_{ref})^2, \tag{16}$$

$$= \frac{-2}{\beta}V.$$

Therefore, if a constant ε exists such that $\frac{2}{\beta} \geq \varepsilon > 0$,

$$\lim_{t\to\infty}\dot{x} = v_{ref}. \tag{17}$$

Consequently, the motion of the point mass is controlled to move in a constant vertical position with a constant velocity.

This can be applied on the control of walking robots moving with constant velocity. According to the condition: $\frac{2}{\beta} \geq \varepsilon > 0$, β must be positive for the stability. Since β converges to $\frac{h_{ref}}{g}$ exponentially, there exists an initial time t_0 such that $\beta > 0$ for $t_0 < t$. To satisfy this condition, the gravity term is needed. This is also true for human walking.

For the regulation of x, the control law equation (14) should be changed to

$$\delta = 2x - x_{ref} + \dot{x}, \qquad (18)$$

where x_{ref} is the reference for x.

In the case of a real robot, the ZMP position must be limited inside the sole length which determine the interval where δ can move.

The block diagram of the control system is shown in Figure 4.

4. APPLICATIONS TO BIPED WALKING ROBOTS

In this section, two applications of the proposed control method to biped robots are presented. The mechanism of each robot is restricted in the sagital plane, and mass of the leg is assumed to be small enough which can be neglected.

4.1 Biped robot with articulated joints

The first biped robot is shown in Figure 5. Each leg of the robot has two degrees of freedom. Four D.C. motors, that actuate the joints through reduction gears, timing belts and parallelogram linkages are attached at the trunk of the robot. Each joint has a circular encoder. The total weight of the robot is 1.54 kg, where each leg is 0.32 kg. Length of each link is 138 mm.

Neglecting the legs mass, the mechanism can be presented as shown in Figure 6. For this model, the dynamical equations are given as follows:

$$\cos\theta_1 \ddot{x} - \sin\theta_1 \ddot{y} - \sin\theta_1 g = \frac{\tau_1}{ml}, \qquad (19)$$

$$\cos\theta_2 \ddot{x} - \sin\theta_2 \ddot{y} - \sin\theta_2 g = \frac{\tau_2}{ml}, \qquad (20)$$

where (x, y) are coordinates of the trunk, m is the mass, l is the link length, g is gravity acceleration, τ_1 and τ_2 are joint torque. Based on these equations, the trunk motion control is derived as follows:

The ZMP position x_{ZMP} is given in the same form as the equation (7):

$$x_{ZMP} = \frac{x(\ddot{y} + g) - y\ddot{x}}{\ddot{y} + g}. \qquad (21)$$

From equations (19), (20) and (21), x_{ZMP} can be expressed in terms of τ_1 and τ_2 as:

$$x_{ZMP} = \frac{(\tau_1 + \tau_2)(\sin\theta_1 \cos\theta_2 - \cos\theta_1 \sin\theta_2)l}{\cos\theta_2 \tau_1 - \cos\theta_1 \tau_2}. \qquad (22)$$

By using (22), it is possible to locate the ZMP at a desired position by constraining the joint torque τ_1 and τ_2. The constraint for $x_{ZMP} = \delta$ can be written as:

$$\alpha(\theta_1, \theta_2, \delta)\tau_1 + \beta(\theta_1, \theta_2, \delta)\tau_2 = 0, \qquad (23)$$

where

$$\alpha = (\sin\theta_1 \cos\theta_2 - \cos\theta_1 \sin\theta_2)l - \delta c_2,$$
$$\beta = (\sin\theta_1 \cos\theta_2 - \cos\theta_1 \sin\theta_2)l - \delta c_1, \qquad (24)$$

By introducing a new input signal u, the equation (23) is satisfied if:

$$\tau_1 = \beta u, \qquad (25)$$

$$\tau_2 = -\alpha u. \qquad (26)$$

Substituting (23), (24), (25), (26) into (19) and (20), with the assumption that $\sin\theta_1 \cos\theta_2 - \cos\theta_1 \sin\theta_2 \neq 0$, yields:

$$ml\ddot{x} = \frac{\sin\theta_1 \alpha + \sin\theta_2 \beta}{\sin\theta_1 \cos\theta_2 - \cos\theta_1 \sin\theta_2} u$$
$$= -\{(\sin\theta_1 + \sin\theta_2)l + \delta\}u$$
$$= -(x + \delta)u, \qquad (27)$$

$$ml\ddot{y} = \frac{\cos\theta_1 \alpha + \cos\theta_2 \beta}{\sin\theta_1 \cos\theta_2 - \cos\theta_1 \sin\theta_2} u$$
$$= -(\cos\theta_1 + \cos\theta_2)lu$$
$$= -yu. \qquad (28)$$

Using these equations, we can define the feedback control law in terms of δ and u. Let (x_{ref}, y_{ref}) be the reference for the trunk position, then the control law is written as:

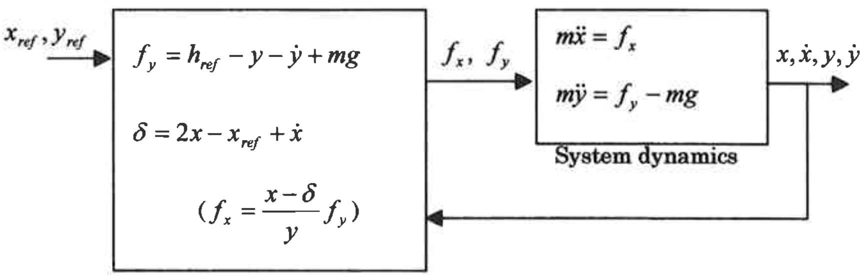

Fig. 4. Block diagram of the control system.

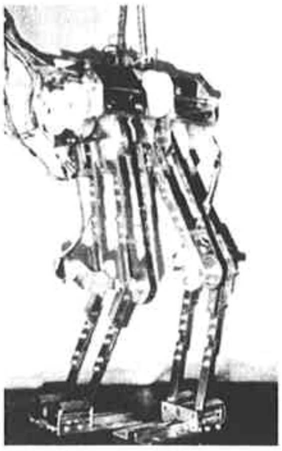

Fig. 5. Biped robot based on leg mechanism 1.

$$\delta = x - k_{px}(x - x_{ref}) + k_{vx}\dot{x}, \quad (29)$$

$$u = \frac{1}{y}\{k_{py}(y - y_{ref}) + k_{vy}\dot{y} - lmg\}, \quad (30)$$

where k_p and k_v are positive feedback gains. Following (29) and (30), the joint torque is obtained from (25), (26) and (24). Figure 7 shows the block diagram of the control system.

Based on this control law, walking experiments are carried out. A video capture of an experimental result, where the vertical position and walking velocity have been constant, is shown in Figure 8.

Figure 9 shows the trunk position measured by the circular encoders and variation of δ during walking. The control law is confirmed to work well in the sense that the trunk position trucks the reference trajectory. However, the stability of walking was not sufficient because the robot fell down after the fifth step. The cause for this instability is the effect of friction and the mass of the lifted leg which is neglected. Due to these effects, the positioning of the ZMP by the constraint (23) is not satisfied with sufficient accuracy. During the experiments, in order to prevent the robot from falling down, a different limit of the ZMP interval must be selected for each step. To overcome this difficulty, other types of sensors such as reaction force sensors and contact detection sensors are necessary.

4.2 Biped robot with prismatic joints

The biped robot, which is developed based on the second leg mechanism, is presented in Figure 10. The leg has two

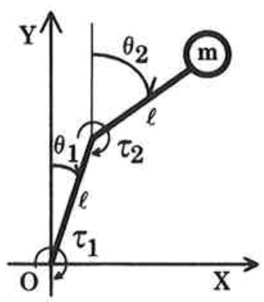

Fig. 6. Model of the leg mechanism 1.

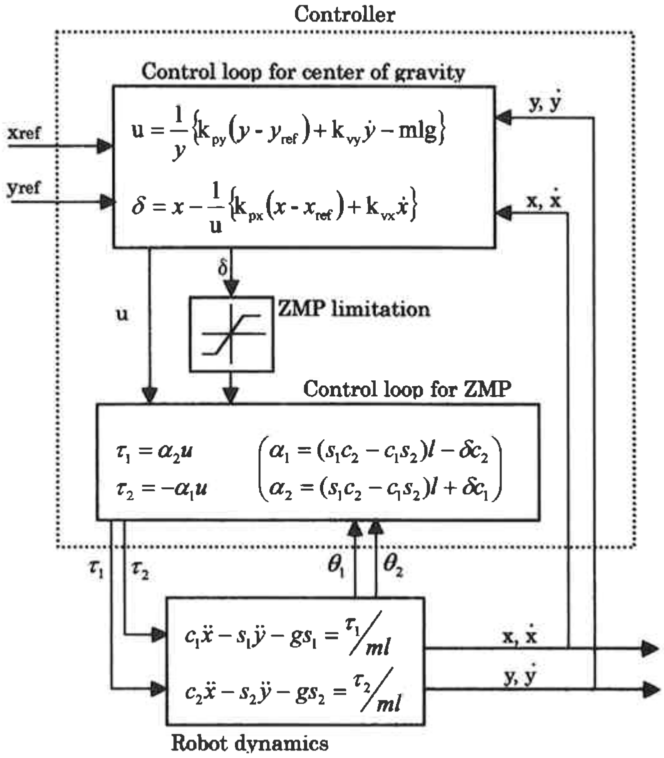

Fig. 7. Block diagram of the control system.

degrees of freedom in sagital plane, one translation and one rotational motion. The translation, which is realized by a prismatic joint, is driven by a D.C. motor attached at the body of the robot, and transmitted through plastic racks and pinions. The other degree of freedom is the rotational motion at the ankle joint. This motion is transmitted through plastic gears and a parallelogram linkage. The total weight of the robot is 1.75 kg, where each leg is 0.16 kg. The length of the leg is 400 mm.

Neglecting the legs mass, the robot can be presented schematically, as shown in Figure 11. The dynamical equations for this system are given as follows:

$$ml\cos\theta\cdot\ddot{x} - ml\sin\theta\cdot\ddot{y} - mgl\sin\theta = \tau, \quad (31)$$

$$m\sin\theta\cdot\ddot{x} + m\cos\theta\cdot\ddot{y} + mg\cos\theta = f, \quad (32)$$

In this case, the constraint for $x_{ZMP} = \delta$ can be written as:

$$\delta lf\cos\theta + (l - \delta\sin\theta)\tau = 0. \quad (33)$$

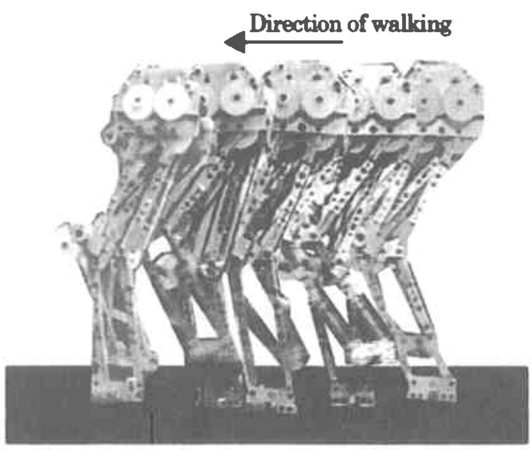

Fig. 8. Experimental walking result.