February 16, 2013
Toyama International Workshop on
Higgs as a Probe of New Physics
(HPNP 2013)

Testing Custodial Symmetry
in Higgs Sector of
Georgi-Machacek Model

Cheng-Wei Chiang (蔣正偉) National Central University
Academia Sinica
National Center for Theoretical Sciences

in collaboration with Kei Yagyu
Plan of Talk

I. Higgs Triplet Model
II. Georgi-Machacek Model
III. Empirical Constraints
IV. GM Higgs Pheno at LHC
V. Summary
Higgs Triplet Model
Introduce a triplet complex scalar field $\Delta: (1,3,2)$ in adjoint rep:

$$\Delta = \frac{\delta^+}{\sqrt{2}} \begin{pmatrix} \delta^+ \\delta^0 \\delta^+ \end{pmatrix}$$

with the $SU(2)_L \times U(1)_Y$ invariant potential

$$\mathcal{L} \supset (D_\mu \Phi)\dagger (D^\mu \Phi) - m^2 (\Phi\dagger \Phi) - \lambda (\Phi\dagger \Phi)^2$$

$$+ \text{Tr}(D_\mu \Delta)\dagger (D^\mu \Delta) - M_\Delta^2 \text{Tr}(\Delta\dagger \Delta) - \frac{\mu}{\sqrt{2}} (\Phi^T i\sigma_2 \Delta\dagger \Phi)$$

$$- \lambda_i (\text{quartic terms})$$

$$- h_{ij} \psi_i^T C i\sigma_2 \Delta \psi_j + \text{h.c.}$$

All $\lambda_i$'s = 0 considered by most people for simplicity.

Konetschny, Kummer 1977; Schechter, Valle 1980; Cheng, Li 1980; Gelmini, Roncadelli 1981
In the case of small $v_{\Delta}$, both $H^{\pm\pm}$ and $H^{\pm}$ decay dominantly into leptonic final states, desirable at hadron colliders.

Perez et al. 2008
Discovery potential for $H^{\pm\pm}$ at 14-TeV LHC through the $\geq 3\ell$ mode is significantly better than the $4\ell$ mode.
• Four simple benchmark points (BP’s):

<table>
<thead>
<tr>
<th></th>
<th>$ee$</th>
<th>$e\mu$</th>
<th>$\mu\mu$</th>
<th>$e\tau$</th>
<th>$\mu\tau$</th>
<th>$\tau\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BP1 (normal hierarchy)</td>
<td>0</td>
<td>0.01</td>
<td>0.3</td>
<td>0.01</td>
<td>0.38</td>
<td>0.3</td>
</tr>
<tr>
<td>BP2 (inverted hierarchy)</td>
<td>0.50</td>
<td>0</td>
<td>0.125</td>
<td>0</td>
<td>0.25</td>
<td>0.125</td>
</tr>
<tr>
<td>BP3 (degenerate neutrinos)</td>
<td>1/3</td>
<td>0</td>
<td>1/3</td>
<td>0</td>
<td>0</td>
<td>1/3</td>
</tr>
<tr>
<td>BP4 (equal branching ratios)</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
</tr>
</tbody>
</table>
• In the case of large $v_\Delta$, both $H^{\pm\pm}$ and $H^{\pm}$ couple dominantly with weak bosons instead.

• Required luminosity for a $5\sigma$ discovery of $H^{\pm\pm}$: with 10/fb, 8-TeV (14-TeV) LHC can reach up to 450 (800) GeV.
• Based on realistic neutrino masses, perturbativity demands $v_\Delta \geq 1 \text{ eV}$.

• Non-zero Higgs triplet VEV leads to

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} < 1$$

Current $\rho^{\text{exp}} \approx 1.0004^{+0.0008}_{-0.0004}$ requires $v_\Delta \leq \text{ a few GeV}$.

PDG 2008; Abada et al 2007
• Include SM isospin doublet field $\phi$: (2,1/2) and isospin triplet fields $\chi$: (3,1) and $\xi$: (3,0)

$$\Phi = \begin{pmatrix} \phi^0* & \phi^+ \\ \phi^- & \phi^0 \end{pmatrix}, \quad \Delta = \begin{pmatrix} \chi^0* & \xi^+ & \chi^{++} \\ \chi^- & \xi^0 & \chi^+ \\ \chi^{--} & \xi^- & \chi^0 \end{pmatrix}$$

transformed under $SU(2)_L \times SU(2)_R$ as $\Phi \rightarrow U_L \Phi U_R^\dagger$ and $\Delta \rightarrow U_L \Delta U_R^\dagger$ with $U_{L,R} = \exp(i \theta_{L,R}^a T^a)$ and $T^a$ being corresponding $SU(2)$ generators.

• Will take $v_\chi = v_\xi \equiv v_\Delta$ (aligned VEV).

$\rightarrow SU(2)_L \times SU(2)_R \rightarrow$ custodial $SU(2)_V$

$\rightarrow \rho = 1$ at tree level
Most general Higgs potential:

\[ V_H = m_1^2 \text{tr}(\Phi^\dagger \Phi) + m_2^2 \text{tr}(\Delta^\dagger \Delta) \]

\[ + \lambda_1 \text{tr}(\Phi^\dagger \Phi)^2 + \lambda_2 [\text{tr}(\Delta^\dagger \Delta)]^2 \]

\[ + \lambda_3 \text{tr}[(\Delta^\dagger \Delta)^2] + \lambda_4 \text{tr}(\Phi^\dagger \Phi) \text{tr}(\Delta^\dagger \Delta) \]

\[ + \lambda_5 \text{tr} \left( \Phi^\dagger \frac{\tau^a}{2} \Phi \frac{\tau^b}{2} \right) \text{tr}(\Delta^\dagger t^a \Delta t^b) \]

\[ + \mu_1 \text{tr} \left( \Phi^\dagger \frac{\tau^a}{2} \Phi \frac{\tau^b}{2} \right) (P^\dagger \Delta P)^{ab} \]

\[ + \mu_2 \text{tr} (\Delta^\dagger t^a \Delta t^b) (P^\dagger \Delta P)^{ab} \]

All terms kept in our analysis

\[ P = \begin{pmatrix} -1/\sqrt{2} & i/\sqrt{2} & 0 \\ 0 & 0 & 1 \\ 1/\sqrt{2} & i/\sqrt{2} & 0 \end{pmatrix} \]
Tadpole Conditions

- Tadpole conditions give

\[
m_1^2 = -v^2 \left(2c_H^2 \lambda_1 + \frac{3}{8}s_H^2 \lambda_4 + \frac{3}{16}s_H^2 \lambda_5 \right) + \frac{3}{8}s_H^2 M_1^2
\]

\[
m_2^2 = -v^2 \left(\frac{3}{4}s_H^2 \lambda_2 + \frac{1}{4}s_H^2 \lambda_3 + c_H^2 \lambda_4 + \frac{1}{2}c_H^2 \lambda_5 \right)
+ \frac{1}{2}c_H^2 M_1^2 + \frac{1}{4}M_2^2
\]

where

\[
M_1^2 = -\frac{v \mu_1}{\sqrt{2} s_H}, \quad M_2^2 = -3\sqrt{2} s_H v \mu_2
\]

\[
v^2 = v_\phi^2 + 8v_\Delta^2 = \frac{1}{\sqrt{2} G_F} , \quad \tan \theta_H = \frac{2\sqrt{2} v_\Delta}{v_\phi}
\]

\[
s_H = \sin \theta_H , \quad c_H = \cos \theta_H
\]
The triplet field $\Delta$: $3 \otimes 3 = 5 \oplus 3 \oplus 1$ under $SU(2)_V$.  

The doublet field $\Phi$: $2 \otimes 2 = 3 \oplus 1$ under $SU(2)_V$.  

$3$ of $\Phi$ can be identified as the NG bosons of the SM as long as there is no mixing between the $3$’s of $\Delta$ and $\Phi$.  

The $5$-plet, $3$-plet and singlet originating from $\Delta$ are  

$$H_5^{\pm\pm} = \chi^{\pm\pm} , \quad H_5^{\pm} = \frac{1}{\sqrt{2}}(\chi^\pm - \xi^\pm) , \quad H_5^0 = \frac{1}{\sqrt{3}}(\chi_r - \sqrt{2}\xi_r) ,$$  

$$\tilde{H}_3^{\pm} = \frac{1}{\sqrt{2}}(\chi^\pm + \xi^\pm) , \quad \tilde{H}_3^0 = \chi i ,$$  

$$\tilde{H}_1^0 = \frac{1}{\sqrt{3}}(\xi_r + \sqrt{2}\chi_r) .$$

$CP$-even

$CP$-odd

$CP$-even

can in general mix with $3$ and $1$ from Higgs doublet fields
Mass Eigenstates

- Mass eigenstates are related to above fields by

\[
\begin{pmatrix}
\phi_i \\
\tilde{H}_3^0
\end{pmatrix} = U_{\text{CP-odd}} \begin{pmatrix}
G^0 \\
H_3^0
\end{pmatrix}, \quad U_{\text{CP-odd}} = \begin{pmatrix}
c_H & -s_H \\
s_H & c_H
\end{pmatrix}
\]

\[
\begin{pmatrix}
\phi^\pm \\
\tilde{H}_3^\pm \\
H_5^\pm
\end{pmatrix} = U_\pm \begin{pmatrix}
G^\pm \\
H_3^\pm \\
H_5^\pm
\end{pmatrix}, \quad U_\pm = \begin{pmatrix}
U_{\text{CP-odd}} \\
0 \\
0 \\
0 & 0 & 1
\end{pmatrix}
\]

\[
\begin{pmatrix}
\phi_r \\
\tilde{H}_1^0 \\
H_5^0
\end{pmatrix} = U_{\text{CP-even}} \begin{pmatrix}
h \\
H_1^0 \\
H_5^0
\end{pmatrix}, \quad U_{\text{CP-even}} = \begin{pmatrix}
c_\alpha & -s_\alpha & 0 \\
s_\alpha & c_\alpha & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

$G^\pm$ and $G^0$ are the NG bosons for the longitudinal components of the $W^\pm$ and $Z$ bosons.
Mass Relation

- In the decoupling limit ($v_\Delta \to 0$ or equivalently $s_H \to 0$), the mass formulae of the Higgs bosons reduce to

\[ m_{H_5}^2 = -\frac{3}{2} \lambda_5 v^2 + M_1^2 + M_2^2 \]

\[ m_{H_3}^2 = -\frac{1}{2} \lambda_5 v^2 + M_1^2 \]

\[ m_{H_1}^2 = M_1^2 - \frac{1}{2} M_2^2 \]

\[ m_h^2 = 8 \lambda_1 v^2. \]

leading to the following relation (reducing one parameter)

\[ m_{H_1}^2 = \frac{3}{2} m_{H_3}^2 - \frac{1}{2} m_{H_5}^2 \]

which is a good approximation even not in the limit.
In scalar mass eigenstates, the Yukawa interactions between leptons of one family and $\Delta$ are

$$\mathcal{L}_\nu = \frac{2\sqrt{2}m_\nu}{s_H v} H_5^+ + e_i^c P_L e_i - \frac{2\sqrt{2}m_\nu}{s_H v} \left( H_5^+ + c_H H_3^+ + s_H G^+ \right) \bar{\nu}_i^c P_L e_i \right]$$

$$+ \frac{2m_\nu}{s_H v} \left[ \frac{1}{\sqrt{3}} \left( H_5^0 + \sqrt{2} s_\alpha h + c_\alpha H_1^0 \right) + i(G^0 s_H + H_3^0 c_H) \right] \bar{\nu}_i^c P_L \nu_i + \text{h.c.}$$

Usual Yukawa interactions between fermions of one family and $\phi$ are

$$\mathcal{L}_Y = - \sum_{f=u,d,e} \frac{m_f}{v} \left[ \frac{c_\alpha}{c_H} \bar{f} f h - \frac{s_\alpha}{c_H} \bar{f} f H_1^0 + i \text{Sign}(f) \tan \theta_H \bar{f} \gamma_5 f H_3^0 \right]$$

$$- \frac{\sqrt{2}V_{ud}}{v} \left[ \tan \theta_H \bar{u} (m_u P_L - m_d P_R) d H_3^+ \right] + \frac{\sqrt{2}m_e}{v} \tan \theta_H \bar{e} P_R e H_3^+$$

+ h.c. \hspace{1cm} \text{Sign}(u)=+1, \text{Sign}(d,e)=-1

**only $H_3$ with quarks via mixing**
**Gauge-Gauge-Scalar Couplings**

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Coefficient</th>
<th>Vertex</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H^\pm W^\mp W^\mp_\nu$</td>
<td>$\frac{g^2}{2\sqrt{2}} s_H v g_{\mu\nu}$</td>
<td>$H^0_1 Z_\mu Z_\nu$</td>
<td>$-\frac{g_2^2}{12} (3s_\alpha c_H - 2\sqrt{6}c_\alpha s_H) v g_{\mu\nu}$</td>
</tr>
<tr>
<td>$H^\pm W^\mp Z_\nu$</td>
<td>$\mp \frac{g_2}{2} s_H v g_{\mu\nu}$</td>
<td>$h W^+<em>\mu W^-</em>\nu$</td>
<td>$\frac{g_2^2}{6} (3c_\alpha c_H + 2\sqrt{6}s_\alpha s_H) v g_{\mu\nu}$</td>
</tr>
<tr>
<td>$H^0 W^+ W^-_\nu$</td>
<td>$-\frac{g^2}{2\sqrt{3}} s_H v g_{\mu\nu}$</td>
<td>$h Z_\mu Z_\nu$</td>
<td>$\frac{g_2^2}{12} (3c_\alpha c_H + 2\sqrt{6}s_\alpha s_H) v g_{\mu\nu}$</td>
</tr>
<tr>
<td>$H^0_0 Z_\mu Z_\nu$</td>
<td>$\frac{g_2^2}{2\sqrt{3}} s_H v g_{\mu\nu}$</td>
<td>$G^\pm W^\mp_\mu A_\nu$</td>
<td>$\pm e m_W g_{\mu\nu}$</td>
</tr>
<tr>
<td>$H^0 W^+ W^-_\nu$</td>
<td>$-\frac{g^2}{6} (3s_\alpha c_H - 2\sqrt{6}c_\alpha s_H) v g_{\mu\nu}$</td>
<td>$G^\pm W^\mp_\mu Z_\nu$</td>
<td>$\mp e s_W m_Z g_{\mu\nu}$</td>
</tr>
</tbody>
</table>

**TABLE III:** Gauge-Gauge-Scalar vertices and the associated coefficients.

- No $H_3$ bosons are involved.
- In models with $\rho = 1$ at tree level and having $H^\pm$ bosons (e.g., 2HDM), the $H^\pm W^\mp Z$ vertex is usually absent at tree level and can only be induced at loop levels. Grifols, Mendez 1980
- This vertex can be used to discriminate models with singly-charged Higgs bosons. Asakawa, Kanemura 2005
  Kanemura, Yagyu, Tanase 2011
CONSTRAINTS ON GEORGI-MACHACEK MODEL
Unitarity/Stability Bounds

- **Perturbative unitarity** bound for the GM model has been studied before to be 
  \[
  12\lambda_1 + 22\lambda_2 + 14\lambda_3 \pm \sqrt{(12\lambda_1 - 22\lambda_2 - 14\lambda_3)^2 + 144\lambda_4^2} < 16\pi
  \]
  Aoki, Kanemura 2008

- **Vacuum stability** condition gives
  \[
  \lambda_1 > 0, \quad \lambda_2 + \lambda_3 > 0, \quad \lambda_2 + \frac{1}{2}\lambda_3 > 0, \\
  -|\lambda_4| + 2\sqrt{\lambda_1(\lambda_2 + \lambda_3)} > 0, \quad \lambda_4 - \frac{1}{4}|\lambda_5| + \sqrt{2\lambda_1(2\lambda_2 + \lambda_3)} > 0.
  \]

- For $\alpha = 0$ (no mixing between $\Phi$ and $\Delta$),
  \[
  \lambda_1 = \frac{m_h^2}{8v^2c_H^2}, \quad \lambda_2 = \frac{m_{H3}^2 - m_{H5}^2 + M^2 + \bar{M}^2}{2v^2}, \quad \lambda_3 = \frac{m_{H5}^2 - M^2}{v^2}, \\
  \lambda_4 = \frac{m_{H3}^2 + m_{H5}^2 - M^2}{4v^2}, \quad \lambda_5 = \frac{m_{H3}^2 - m_{H5}^2 + M^2}{v^2}.
  \]
Unitarity/Stability Bounds

- Constraints to be imposed on later analyses.
• Excluded parameter space in the $m_{H^3}$-$v_\Delta$ plane using the $R_b^{\text{exp}} = 0.21629 \pm 0.00066$.

PDG 2010

• Upper bound on $v_\Delta$ increases monotonically with $m_{H^3}$.

• The 2$\sigma$ bound is about 25 GeV more relaxed than 1$\sigma$ bound.

• Constraint from the $b \to s\gamma$ data for the GM model is similar to that in the Type-I 2HDM and is milder than the $R_b$ constraint.

Barger, Hewett, Phillips 1990
Aoki, Kanemura, Tsumura, Yagyu 2009
HIGGS DECAYS
General Remarks

• Decay rates of new Higgs bosons generally depend on $m_{H5}$, $m_{H3}$ and $m_{H1}$ (related by approximate mass relation), $v_\Delta$, and mixing angle $\alpha$.

• Fix $m_h = 125 \text{ GeV}$ and $\alpha = 0$ to be specific.

• Decay rates now depend upon $v_\Delta$, $m_{H3}$ and mass splitting between 5-plet and 3-plet:

$$\Delta m \equiv m_{H3} - m_{H5}$$

• Possible mass spectra:
  • $\Delta m = 0 \iff m_{H5} = m_{H3} = m_{H1}$
  • $\Delta m > 0 \iff m_{H1} > m_{H3} > m_{H5}$
  • $\Delta m < 0 \iff m_{H5} > m_{H3} > m_{H1}$
CONTour PLOTS FOR H₅ DECAYS

- **doubly charged**
- **singly charged**
- **neutral**

\[ m_{H_5} > m_{H_3} \]

solid: 50%; dashed: 90%
Contour plots for $H_3$ decays

$m_{H_3} < m_{H_3}$

$m_{H_5} > m_{H_3}$

solid: 50%; dashed: 90%
**In Short**

\[ \Delta m < 0 \]

\[ H_5^{++} \to H_3^+ W^+ , \ H_3^+ \to H_3^+ Z/H_3^0 W^+ , \ H_5^0 \to H_3^\pm W^\mp/H_3^0 Z \]

\[ H_3^+ \to H_1^0 W^+ , \ H_3^0 \to H_1^0 Z \]

\[ \Delta m > 0 : \]

\[ H_5^{++} \to \ell^+ \ell^+ \]

\[ H_5^+ \to \ell^+ \nu \]

\[ H_5^0 \to \nu \nu \]

\[ H_3^+ \to H_5^{++} W^- / H_5^+ Z / H_5^0 W^+ \]

\[ H_3^0 \to H_5^\pm W^\mp / H_5^0 Z \]

\[ \Delta m > 0 : \]

\[ H_5^{++} \to W^+ W^+ \]

\[ H_5^+ \to W^+ Z \]

\[ H_5^0 \to W^+ W^- / ZZ \]

\[ H_3^+ \to H_5^{++} W^- / H_5^+ Z / H_5^0 W^+ \]

\[ H_3^0 \to H_5^\pm W^\mp / H_5^0 Z \]

\[ \Delta m > 0 : \]

\[ H_5^{++} \to \ell^+ \ell^+ \]

\[ H_5^+ \to \ell^+ \nu \]

\[ H_5^0 \to \nu \nu \]

\[ H_3^+ \to \ell^+ \nu \]

\[ H_3^0 \to \nu \nu . \]

\[ H_5^{++} \to W^+ W^+ , \ H_5^+ \to W^+ Z , \ H_5^0 \to W^+ W^- / ZZ , \]

\[ H_3^+ \to \tau^+ \nu / c\bar{s} / t \bar{b} , \ H_3^0 \to b \bar{b} . \]
HIGGS PHENOMENOLOGY AT LHC
Production Channels

• **Drell-Yan (DY) process**
  - H₅, H₃ produced in pairs via γ and Z, e.g., pp → H₅H₅ and H₃H₃. Cross section determined by the gauge coupling, m₅ and m₃, independent of the value of vₐ.

• **Mixed Drell-Yan (mDY) process**
  - H₅, H₃ produced at the same time, e.g., pp → H₅H₃. Cross section proportional to $\cos^2\theta_H$, relatively suppressed than DY process, especially for large vₐ.

• **Weak vector boson fusion (VBF) process**
  - Single production of H₅ via qQ → H₅ process. Cross section proportional to $v^2$, thus important for large vₐ.

• **Weak vector boson associated (associated VBF) process**
  - H₅ also produced in association with a weak gauge boson, e.g., qQ’ → H₅V. Cross sections proportional to vₐ too, thus also important when VBF process is important.

• **Yukawa process**
  - H₃ produced via Yukawa interactions as gg → H₃⁰ via gluon fusion. t-channel H₃± and H₃⁰ modes: gb → tH₃⁻ and gb → bH₃⁰. Cross sections proportional to $\tan^2\theta_H$.

• **Top quark decay**
  - For m₃ < mₜ, H₃± produced from the top quark decay. Decay rate of t → bH₃± depending on $\tan^2\theta_H$. 
VBF and associated VBF can be useful to discriminate the GM model from others with doubly-charged Higgs bosons and to test the mass degeneracy of $H_5$.

- In HTM, VBF and associated VBF productions of $H_5^{\pm\pm}$ are much suppressed due to tiny triplet VEV.
- In GM model, $v_\Delta \sim O(10 \text{ GeV})$ possibly, so that these production processes become useful.

mDY process is also a unique feature of the GM model because $H_5$ and $H_3$ having different decay properties (Region II in particular) are produced at the same time.

- This process is useful not only to test mass degeneracy of $H_3$ but also to distinguish the model from others that also have $H^{\pm\pm}$ and/or $H^\pm$ bosons.
PRODUCTION OF $H_{5,3}$ Bosons

Dominant production mechanism is the VBF process for a sufficiently large $m_{H5}$.

red: VBF
black/green: mDY
blue: associated VBF
One Benchmark Point

- Consider as an example: $m_{H^3} = 150$ GeV, $\Delta m = 10$ GeV (i.e., $m_{H^5} = 140$ GeV) and $v_\Delta = 20$ GeV in Region II.
- 5-plet Higgs bosons decay into gauge boson pairs $\sim 100\%$; BR’s of $H^5_0 \rightarrow W^+W^-$ and $ZZ$ are $2/3$ and $1/3$ by Clebsch.
- $H_{3^\pm}$ decays to $\tau^\pm\nu$ at $66\%$ and cs at $29\%$, and $H_{3^0}$ decays to $bb$ at $89\%$.
- BR of $t \rightarrow H_{3^+}b$ is $\sim 0.4\%$, consistent with upper limit of 2-3% for $m_{H^3}$ between 80 and 160 GeV, assuming that the charged Higgs boson decays to $\tau\nu$ at 100%.
- Will use VBF and associated VBF to fix $H_5$; and use mDY to fix $H_3$.  

CMS 2012
**Signal Events**

- For 5-plet bosons, VBF’s are
  \[ qQ \rightarrow H_5^{\pm\pm} q' Q' \rightarrow W^\pm W^\pm jj, \]
  \[ qQ \rightarrow H_5^{\pm} q' Q' \rightarrow W^\pm Zjj, \]
  \[ qQ \rightarrow H_5^0 q' Q' \rightarrow W^\pm W^\mp jj / ZZjj. \]

- Associated VBF’s are
  \[ q\bar{q}' \rightarrow H_5^{\pm\pm} W^\mp \rightarrow W^\pm W^\pm jj \]
  \[ q\bar{q} \rightarrow H_5^{\pm} W^\mp \rightarrow W^\pm Zjj, \quad q\bar{q}' \rightarrow H_5^0 Z \rightarrow W^\pm Zjj, \]
  \[ q\bar{q} \rightarrow H_5^0 Z \rightarrow W^+ W^- jj / ZZjj, \]
  \[ q\bar{q}' \rightarrow H_5^0 W^\pm \rightarrow W^+ W^- jj / ZZjj, \]

where associated weak gauge bosons are assumed to decay *hadronically* to have same final states as VBF.
Signal/Background Events

- Consider further that the weak gauge bosons produced from the decay of $H_5$ decay \textit{leptonically}:

  \begin{align*}
  H_{5}^{\pm\pm} & : \ell^{\pm}\ell^{\pm}\text{jj}\slashed{E}_T \\
  H_{5}^{\pm} & : \ell^{\pm}\ell^{\pm}\ell^{\mp}\text{jj}\slashed{E}_T \\
  H_{5}^{0} & : \ell^{\pm}\ell^{\mp}\text{jj}\slashed{E}_T
  \end{align*}

- $\ell^{\pm}$ denotes collectively light leptons $e^{\pm}$ and $\mu^{\pm}$.

- Corresponding background events for these signal events are from the $W^{\pm}W^{\pm}\text{jj}$ for the $H_{5}^{\pm\pm}$, $W^{\pm}Z\text{jj}$ for the $H_{5}^{\pm}$, and $t\bar{t}$, $W^{\pm}W^{\mp}\text{jj}$ and $ZZ\text{jj}$ for $H_{5}^{0}$. 
Distributions

\[ H_5^{±±} : \ell^± \ell^± jj \not{E}_T \]

\[ H_5^{±} : \ell^± \ell^± \ell^± jj \not{E}_T \]

\[ H_5^{0} : \ell^± \ell^± jj \not{E}_T \]
Kinematic Cuts

• Basic cuts:

\[ p_T^j > 20 \text{ GeV}, \quad p_T^\ell > 10 \text{ GeV}, \]
\[ |\eta^j| < 5, \quad |\eta^\ell| < 2.5, \quad \Delta R^{jj} > 0.4 \]

• Additional cuts:

\[ \Delta \eta^{jj} > 3.5 \ (> 4.0 \ \text{for } \ell^\pm \ell^\mp jj E_T), \]
\[ 50 < M_T < 150 \text{ GeV} \]
# Significance Improvements

<table>
<thead>
<tr>
<th>Cuts</th>
<th>( \ell^+\ell^- jj E_T )</th>
<th>( \ell^+\ell^- \ell^+ jj E_T )</th>
<th>( \ell^+\ell^- \ell^\mp jj E_T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.71 (8.72)</td>
<td>3.48 (8.13)</td>
<td>13.8 (21.2)</td>
</tr>
<tr>
<td>(\Delta \eta^{jj})</td>
<td>1.82 (5.68)</td>
<td>0.20 (0.65)</td>
<td>12.8 (22.6)</td>
</tr>
<tr>
<td></td>
<td>0.33 (0.98)</td>
<td>4.42 (15.6)</td>
<td>1.51 (2.41)</td>
</tr>
<tr>
<td></td>
<td>0.51 (1.42)</td>
<td>30.7 (1.99\times10^2)</td>
<td>0.91 (1.00)</td>
</tr>
<tr>
<td>(M_T)</td>
<td>1.80 (5.58)</td>
<td>0.05 (0.12)</td>
<td>13.2 (23.4)</td>
</tr>
<tr>
<td></td>
<td>0.33 (0.98)</td>
<td>0.07 (0.46)</td>
<td>5.22 (8.17)</td>
</tr>
<tr>
<td></td>
<td>0.48 (1.36)</td>
<td>11.4 (67.4)</td>
<td>1.39 (1.64)</td>
</tr>
<tr>
<td>(b)-jet veto</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>0.48 (1.36)</td>
<td>1.82 (10.8)</td>
<td>3.16 (3.90)</td>
</tr>
</tbody>
</table>

**Table I:** Signal and background cross sections in units of fb after each kinematic cut, along with the significance \(S\) defined by Eq. (48) based on an integrated luminosity of 100 fb\(^{-1}\). The numbers without (with) parentheses correspond to the case with a CM energy of 8 TeV (14 TeV). The signal cross section includes contributions from both the VBF production and the vector boson associated production processes. For the \(\ell^+\ell^- \ell^\mp jj E_T\) events, we further impose the requirement of the \(b\)-jet veto for each jet to reduce the background, where the \(b\)-tagging efficiency is taken to be 0.6. **ATLAS 2011**
Production of $H_3$ Bosons

- Consider mDY processes:
  
  \[
  pp \rightarrow H_5^{\pm \pm} H_3^\mp \rightarrow W^{\pm} W^{\pm} cs
  \]
  
  \[
  pp \rightarrow H_5^{\pm} H_3^\mp \rightarrow W^{\pm} Z cs
  \]
  
  \[
  pp \rightarrow H_5^{\pm} H_3^{0} \rightarrow W^{\pm} Z b \bar{b}
  \]
  
  \[
  pp \rightarrow H_5^{0} H_3^{\pm} \rightarrow W^{+} W^{-} cs/Z Z cs
  \]
  
  \[
  pp \rightarrow H_5^{0} H_3^{0} \rightarrow W^{+} W^{-} b \bar{b}/Z Z b \bar{b}
  \]

where **leptonic** decays of the weak gauge bosons from the $H_5$ decays are also assumed in this analysis.

⇒ same final states as VBF and associated VBF
Distribution

- Difference between mDY and VBF is in $\Delta \eta_{jj}$ distribution.
  - In mDY process, dijets in the final state come from 3-plet Higgs boson decays, not the external quark jets.
  - Events from mDY process concentrates in the $\Delta \eta_{jj} < 2.5$ region for all three cases.

- $M_T$ distributions from mDY process and VBF + associated VBF are almost the same.
  - Leptons plus missing transverse energy system come from the decays of $H_5$ in both processes

$\Rightarrow$ use same $M_T$ cut, but not $\Delta \eta_{jj}$ cut
### Significance Improvements

<table>
<thead>
<tr>
<th>Cuts</th>
<th>$H_5^{\pm\pm}jj$</th>
<th>$H_5^{\pm\pm}H_3^{\mp}$</th>
<th>$W^{\pm}W^{\pm}jj$</th>
<th>$S$</th>
<th>$H_5^{\pm}jj$</th>
<th>$H_5^{\pm}H_3^{\mp,0}$</th>
<th>$W^{\pm}Zjj$</th>
<th>$S$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Basic</strong></td>
<td>3.71 (8.72)</td>
<td>0.72 (1.63)</td>
<td>3.48 (8.13)</td>
<td>15.8 (24.1)</td>
<td>0.61 (1.60)</td>
<td>0.53 (1.21)</td>
<td>45.9 ($1.39\times10^2$)</td>
<td>1.66 (2.36)</td>
</tr>
<tr>
<td><strong>$M_T$</strong></td>
<td>3.65 (8.57)</td>
<td>0.71 (1.60)</td>
<td>1.02 (2.20)</td>
<td>18.8 (28.9)</td>
<td>0.61 (1.60)</td>
<td>0.53 (1.21)</td>
<td>1.16 (3.42)</td>
<td>7.52 (11.3)</td>
</tr>
</tbody>
</table>

TABLE II: Signal and background cross sections in units of fb after each kinematic cut, along with the significance based on an integrated luminosity of 100 fb$^{-1}$. The numbers without (with) parentheses correspond to the case with a CM energy of 8 TeV (14 TeV).

- BR’s of $H_3^{\pm}\to cs$ and $H_3^0\to bb$ are $\sim30\%$ and $90\%$, respectively.
- Trilepton events mainly come from $H_5^{\pm\pm}H_3^0$ production.
• Mass degeneracy between $H_{3\pm}$ and $H_{3^0}$ can be readily established from the mDY process.

FIG. 14: Invariant mass distribution for the dijets system. The bin size in this distribution is 5 GeV. The distributions for the signal events are divided into those from the VBF process and vector boson associated process (green dashed curve), the mDY process (blue dashed curve) and the sum of them (red solid curve). The integrated luminosity and the CM energy are assumed to be 100 fb$^{-1}$ and 8 TeV, respectively.
\[ R_{\gamma\gamma, Z\gamma} = \frac{\sigma(gg \to h)_{GM} \times BR(h \to \gamma\gamma, Z\gamma)_{GM}}{\sigma(gg \to h)_{SM} \times BR(h \to \gamma\gamma, Z\gamma)_{SM}} \]

- For larger \( v_\Delta \), \( R_{\gamma\gamma, Z\gamma} \) tend to have smaller values because the \( hW^+W^- \) coupling gets smaller.
Summary

• At LHC, the GM model with custodial symmetry can be tested by observing:
  • SM-like $h$, a pair of $H^{\pm\pm}$, two pairs of $H^{\pm}$, one CP-odd $A$ and three CP-even $H$’s
  • Mass degeneracy among same SU(2)$_V$ multiplet, subject to small EM corrections at O(100 MeV)
  • 5-plet and singlet $H$’s couple to weak boson pairs, but not fermion pairs via usual Yukawa couplings at tree level; 3-plet $H$’s couple to fermion pairs, but not weak boson pairs
  • Isospin triplet VEV $\sim$ O(10 GeV) possible
  • VBF, associated VBF and mDY processes are useful for discovering and determining Higgs masses at LHC.
  • Predictions for $h\rightarrow\gamma\gamma$ and $Z\gamma$ rates are estimated.