

Higgs Triplet Models

Kei Yagyu (National Central U)



M. Aoki, S. Kanemura, KY, PRD85 (2012),
S. Kanemura, KY, PRD85 (2012),
M. Aoki, S. Kanemura, M. Kikuchi, KY, PLB714 (2012),
M. Aoki, S. Kanemura, M. Kikuchi, KY, PRD87 (2012),
S. Kanemura, M. Kikuchi, KY, 1301.7303

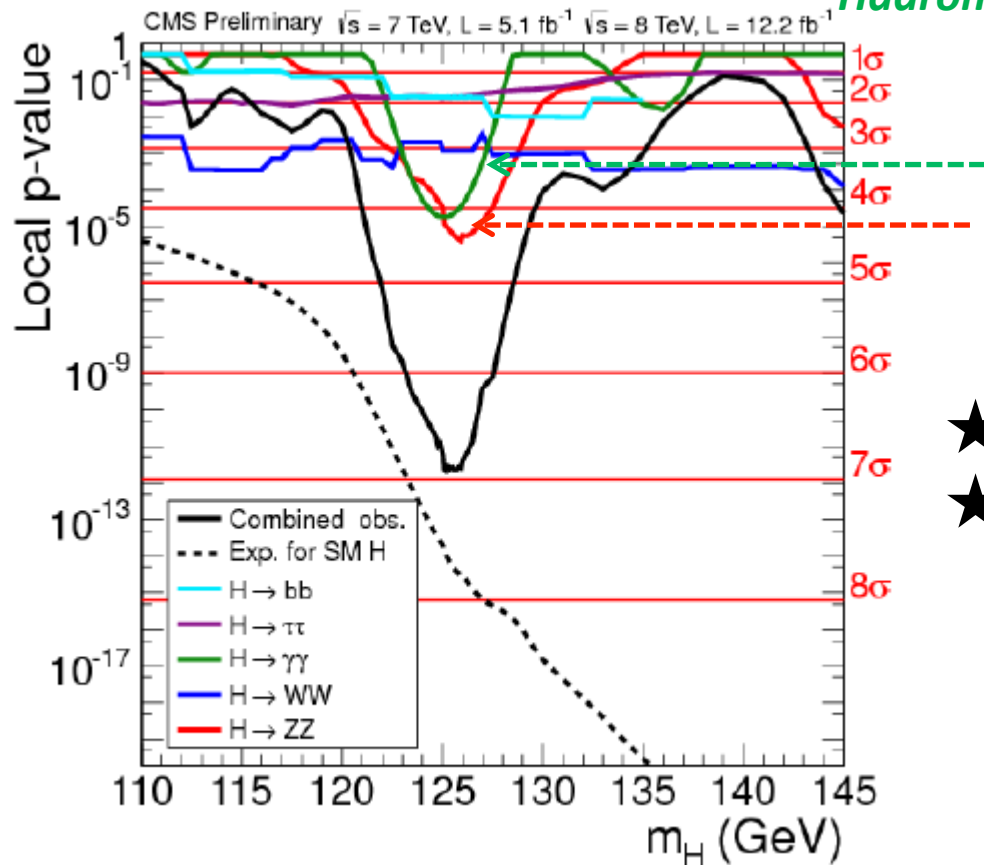
Contents

- Introduction
- Exotic Higgs sectors
- The Higgs triplet model
 - Direct test
 - Indirect test (Kikuchi's poster)
- Summary

Introduction

- ▶ A new particle has been found at around 126 GeV at the LHC.

Hadron Collider Physics Symposium, Nov. 2012



$H \rightarrow \gamma\gamma$

$H \rightarrow Z Z \rightarrow 4l$

★ $H \rightarrow \gamma\gamma : \sim 4\sigma$,

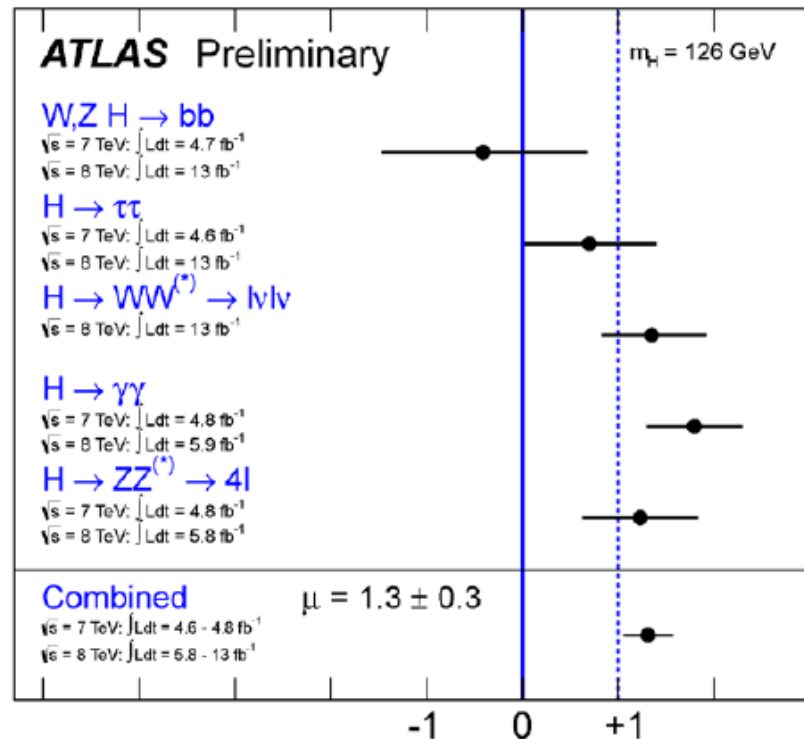
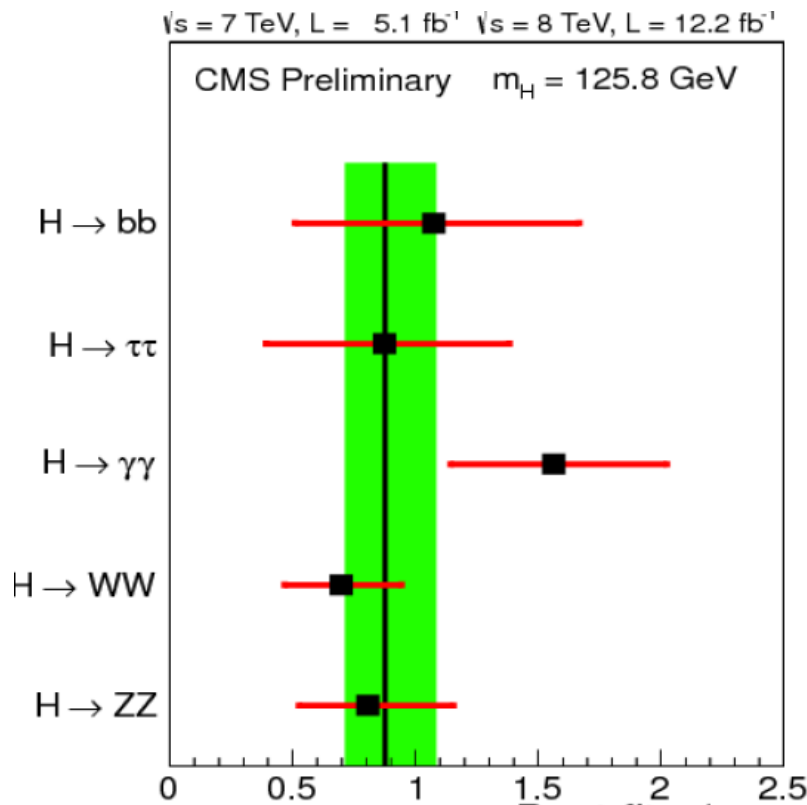
★ $H \rightarrow Z Z^* \rightarrow 4\text{-leptons} : \sim 4\sigma$

The combined significance is about 7σ at 126 GeV.

Introduction

- Signal strength ($\sigma_{\text{obs}}/\sigma_{\text{SM}}$) in each channel

Hadron Collider Physics Symposium, Nov. 2012



***It is most likely the SM Higgs boson,
but it does not necessarily mean that SM is correct!***

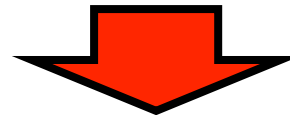
Introduction

Physics Beyond the SM

Neutrino masses

Dark matter

Baryon asymmetry



Introduce

Extended Higgs Sectors

Minimal Higgs Sector
1 doublet: Φ



- Singlets,
- Doublets,
- Triplets, ...



Test

Experiments

EW precision obs.

*126 GeV
Boson*

*Extra Higgs,
 $H^\pm$, A , ... ?*

*Higgs Coupling
Measurements?*

Introduction

Physics Beyond the SM

Neutrino masses

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Baryon asymmetry



Determine

Higgs as a Probe of New Physics !!



Compare

Experiments

EW precision obs.

*126 GeV
Boson*

*Extra Higgs,
 $H^\pm$, A, ... ?*

*Higgs Coupling
Measurements?*

Higgs sector with exotic fields

★ In this talk, we focus on the Higgs sector with *exotic scalar fields*.

Higgs sectors with exotic fields

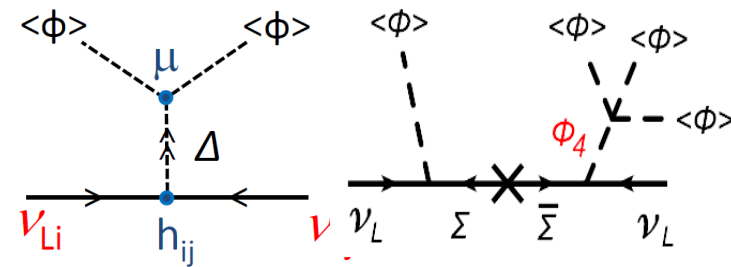
Normal extension

Minimal

Doublets and
Singlets

Exotic Higgs field

→ isospin larger than 1/2
and having a non-zero VEV.

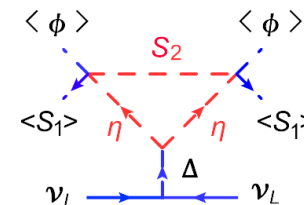


Cheng, Li (1980); Schechter, Valle (1980); Magg, Wetterich, (1980)

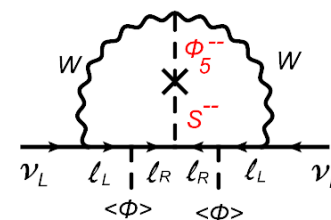
Babu, Nandi and Tavartkiladze (2009)

★ Models with exotic scalar fields

- Type-II seesaw model [a comp. triplet]
- Georgi-Machacek (GM) model [a comp. and a real triplets]
- Model by Babu, Nandi and Tavartkiladze [a quadruplet]
- Radiative seesaw models
 - Model by Kanemura and Sugiyama [a comp. triplet]
 - Model by Chen, Geng and Tsai [a quintuplet]



Kanemura, Sugiyama (2012)



Chen, Geng, Tsai (2012)

Higgs sector with exotic fields

★ There are striking features in Higgs sectors w/ exotic fields.

- Electroweak ρ parameter
- The $H^\pm W^\mp Z$ vertex
- The hZZ/hWW vertex
- Multi-charged Higgs bosons

Higgs sector with exotic fields

★ There are striking features in Higgs sectors w/ exotic fields.

- Electroweak rho parameter
- The $H^\pm W^\mp Z$ vertex
- The hZZ/hWW vertex

$$\rho_{\text{exp}} = 1.0004^{+0.0003}_{-0.0004}$$

$$\rho_{\text{tree}} = \frac{m_W^2}{m_Z^2 \cos^2 \theta_W} = \frac{\sum_i v_i^2 [T_i(T_i+1) - Y_i^2]}{\sum_i 2Y_i^2 v_i^2}$$

Y_i : hypercharge
 T_i : isospin
 v_i : VEV

Model with $\rho_{\text{tree}} = 1$

- A Higgs field satisfying with the relation:

$$T = \frac{1}{2}(-1 + \sqrt{1 + 12Y^2})$$

$$(T, Y) = (1, 0), (1/2, 1/2), (3, 2), (25/2, 15/2)$$

Model without $\rho_{\text{tree}} = 1$

- Usually, exotic Higgs sectors predict $\rho \neq 1$ at the tree level except for some special cases.

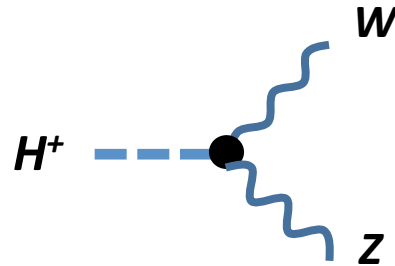
- Model w/ the $Y=0$ ($Y=1$) triplet

The other observables are necessary to discriminate exotic Higgs sectors.

Higgs sector with exotic fields

★ There are striking features in Higgs sectors w/ exotic fields.

- Electroweak rho parameter
- **The $H^\pm W^\mp Z$ vertex**
- The hZZ/hWW vertex



*Grifols, Mendez (1980);
Peyranere, Haber, Irulegui (1991);
Mendez, Pomarol (1991);
Kanemura (2000)*

Effective HWZ vertex: $\mathcal{L}_{\text{eff}} = g_Z m_W \mathbf{F} H^+ W_\mu^- Z^\mu + \text{h.c.}$

$$|\mathbf{F}|^2 = \frac{2g^2}{m_W^2} \sum_i Y_i^2 [T_i(T_i + 1) - Y_i^2] v_i^2 - \frac{1}{\rho_{\text{tree}}^2}$$

Y_i : hypercharge
 T_i : isospin
 v_i : VEV

Model with multi-doublet structure

→ $|\mathbf{F}|^2$ is **zero** at the tree level.

The $H^\pm W^\mp Z$ is induced at 1-loop level.

Model with exotic fields

→ $|\mathbf{F}|^2$ is **non-zero** at the tree level.

In general, $|\mathbf{F}|^2$ and ρ are independent quantities. Measuring $H^\pm W^\mp Z$ vertex can be a useful tool to the ρ to test triplet Higgs models.

Higgs sector with exotic fields

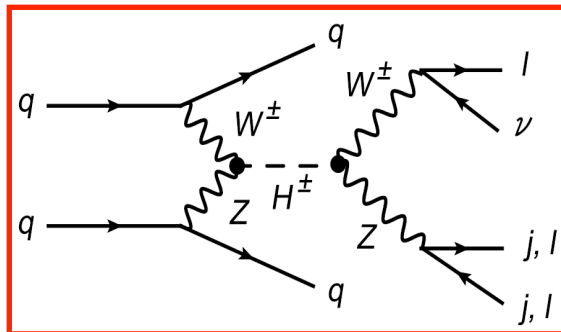
★ There are striking features in Higgs sectors w/ exotic fields.

- Electroweak rho parameter
- **The $H^\pm W^\mp Z$ vertex**
- The hZZ/hWW vertex

★ Measuring the HWZ vertex at the collider experiments

LHC

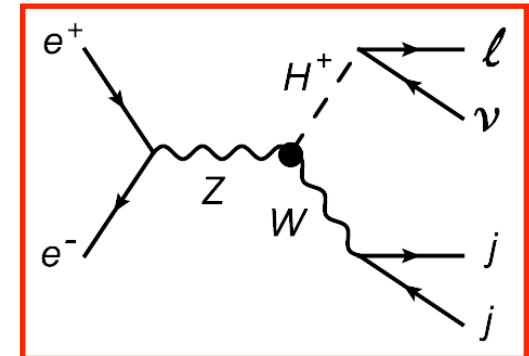
Asakawa, Kanemura, Kanzaki (2007)



$|F|^2 > 0.036$ is required to reach $S/\text{Root}(B)=2$
with $\text{Root}(s) = 14 \text{ TeV}$, Int. luminosity = 600 fb^{-1} ,
 $m_{H^\pm} = 200 \text{ GeV}$.

ILC

Kanemura, KY, Yanase (2011)

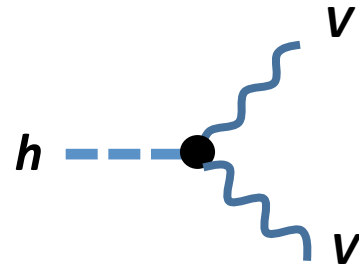


$|F|^2 > 0.001$ is required to reach $S/\text{Root}(B)=2$
with $\text{Root}(s) = 300 \text{ GeV}$, Int. luminosity = 1000 fb^{-1} ,
 $m_{H^\pm} = 150 \text{ GeV}$.

Higgs sector with exotic fields

★ There are striking features in Higgs sectors w/ exotic fields.

- Electroweak rho parameter
- The $H^\pm W^\mp Z$ vertex
- **The hZZ/hWW vertex**



$$g_{hVV} = g_{hVV}^{\text{SM}} \times c_{hVV}$$

The deviation in the hVV coupling:

$$c_{hWW} = \frac{\sum_i 2v_i [T_i(T_i+1) - Y_i^2] R_{ih}}{v_{\text{SM}}}$$

$$c_{hZZ} = \sum_i \frac{2Y_i^2 v_i R_{ih}}{\sqrt{\sum_j Y_j^2 v_j^2}}$$

Y_i : hypercharge
 T_i : isospin
 v_i : VEV

$$v_{\text{SM}}^2 = \sum_i 2v_i^2 [T_i(T_i+1) - Y_i^2] = (246 \text{ GeV})^2 \quad R_{ih} : \text{Mixing angle among CP-even Higgs bosons}$$

Logan (2006)

Model with multi-doublet structure

→ c_{hVV} is **smaller** than 1

Model with exotic fields

→ c_{hVV} **can be larger** than 1

This is because the Clebsh-Gordan coefficient $2[T(T+1) - Y^2]v$ and the factor $4Y^2v$ from exotic fields are larger than those from the doublet.

Higgs sector with exotic fields

α , β and β' are mixing angles for **CP-even**, **singly-charged** and **CP –odd** Higgs bosons.

*Kanemura, Kikuchi, KY
(2013)*

Model	$\tan \beta$	$\tan \beta'$	c_{hWW}	c_{hZZ}
$\phi_1 + \phi_2$ (THDM)	v_{ϕ_2}/v_{ϕ_1}	v_{ϕ_2}/v_{ϕ_1}	$\sin(\beta - \alpha)$	$\sin(\beta - \alpha)$
$\phi + \chi$ (cHTM)	$\sqrt{2}v_\chi/v_\phi$	$2v_\chi/v_\phi$	$\cos \beta \cos \alpha + \sqrt{2} \sin \beta \sin \alpha$	$\cos \beta' \cos \alpha + 2 \sin \beta' \sin \alpha$
$\phi + \xi$ (rHTM)	$2v_\xi/v_\phi$	-	$\cos \beta \cos \alpha + 2 \sin \beta \sin \alpha$	$\cos \alpha$
$\phi + \chi + \xi$ (GM model)	$2\sqrt{2}v_\Delta/v_\phi$	$2\sqrt{2}v_\Delta/v_\phi$	$\cos \beta \cos \alpha + \frac{2\sqrt{6}}{3} \sin \beta \sin \alpha$	$\cos \beta \cos \alpha + \frac{2\sqrt{6}}{3} \sin \beta \sin \alpha$
$\phi + \varphi_7$	$4v_{\varphi_7}/v_\phi$	$4v_{\varphi_7}/v_\phi$	$\cos \beta \cos \alpha + 4 \sin \beta \sin \alpha$	$\cos \beta \cos \alpha + 4 \sin \beta \sin \alpha$

(T, Y)

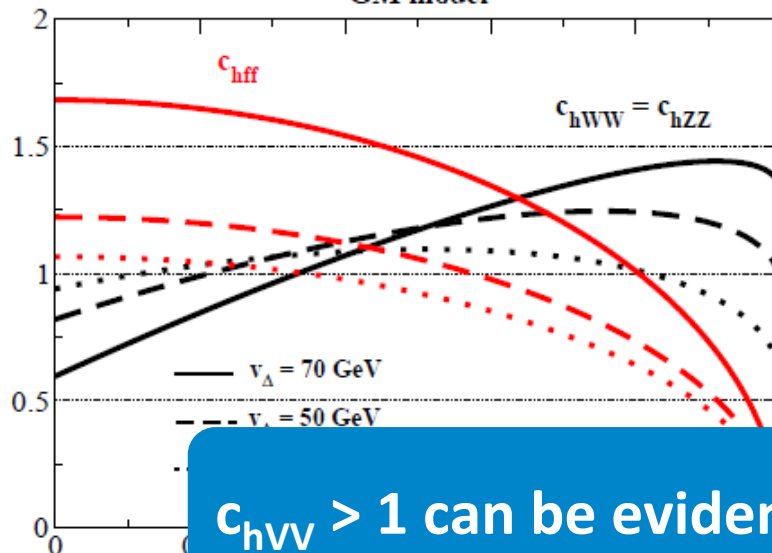
ϕ : (1/2, 1/2)

χ : (1, 1)

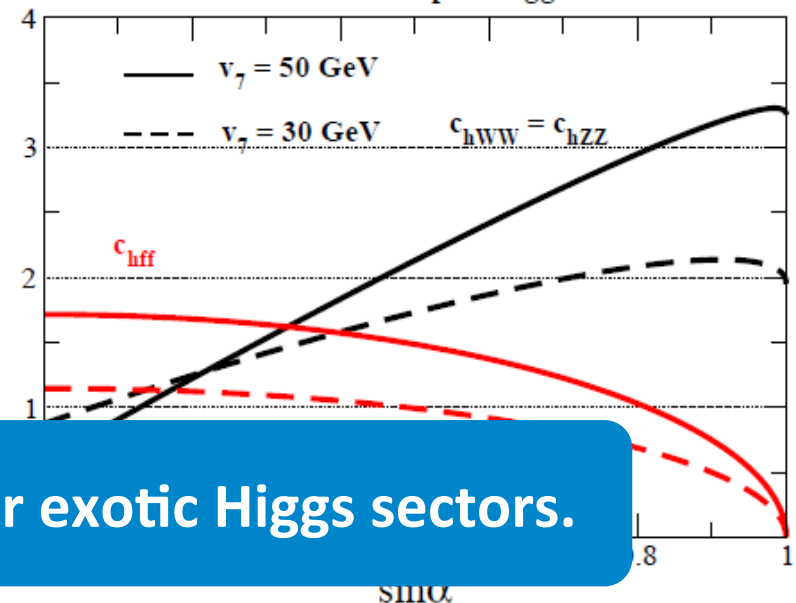
ξ : (1, 0)

ϕ_7 : (3, 2)

GM model



Model with the septet Higgs field



$c_{hVV} > 1$ can be evidence for exotic Higgs sectors.

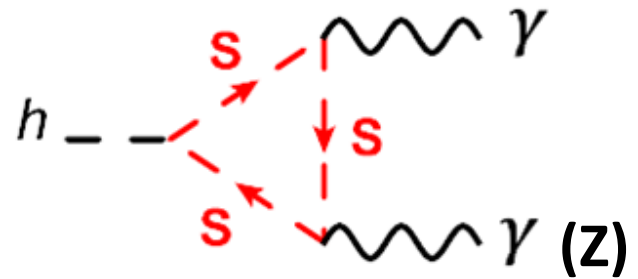
Higgs sector with exotic fields

★ There are striking features in Higgs sectors w/ exotic fields.

- Electroweak rho parameter
- The $H^\pm W^\mp Z$ vertex
- The hZZ/hWW vertex

- **Multi-charged Higgs bosons**

- From the relation $Q = I_3 + Y$, the EM charge of component scalar fields can be expressed as: $\Phi = [\phi^{Q=T+Y}, \dots, \phi^{++}, \phi^+, \phi^0, \phi^-, \phi^{--}, \dots, \phi^{Q=-T+Y}]^T$.
- Discovery of multi-charged Higgs bosons is direct evidence for the exotic Higgs sectors.
- Multi-charged Higgs bosons can affect the $h \rightarrow \gamma\gamma$ decay via the loop effect.
In addition to $h \rightarrow \gamma\gamma$, measuring the $h \rightarrow Z\gamma$ mode is also important to know the isospin of the multi-charged Higgs bosons.



Carena, Wagner, Low (2012), Chiang, KY (2012)

The Higgs Triplet Model (HTM)

*Cheng, Li (1980);
Schechter, Valle, (1980);
Magg, Wetterich, (1980);
Mohapatra, Senjanovic, (1981).*

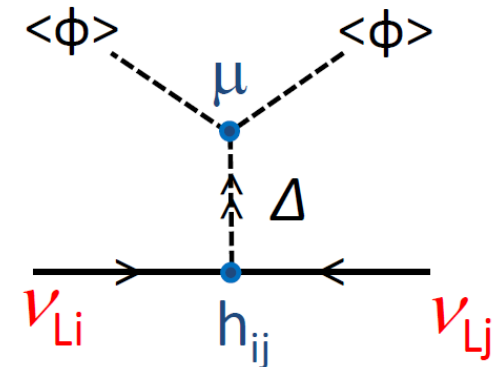
The Higgs triplet field Δ is added to the SM.

	SU(2) _I	U(1) _Y	U(1) _L
Φ	2	1/2	0
Δ	3	1	-2

• Important new interaction terms:

Lepton number breaking parameter

$$\mathcal{L}_{\text{HTM}} = h_{ij} \overline{L}_L^{ci} \cdot \Delta L_L^j + \mu \Phi \cdot \Delta^\dagger \Phi + \dots$$



• Neutrino mass matrix

$$(m_\nu)_{ij} = h_{ij} \frac{\mu \langle \phi^0 \rangle^2}{M_\Delta^2} = h_{ij} v_\Delta$$

M_Δ : Mass of triplet scalar boson.
 v_Δ : Triplet Higgs VEV

• Electroweak rho parameter

$$\rho_{\text{tree}} = \frac{1 + \frac{2v_\Delta^2}{v_\Phi^2}}{1 + \frac{4v_\Delta^2}{v_\Phi^2}} \simeq 1 - \frac{2v_\Delta^2}{v_\Phi^2}$$

The HTM can be
tested at colliders !!

Higgs potential

The Higgs potential

$$V = m^2 \Phi^\dagger \Phi + M^2 \text{Tr}(\Delta^\dagger \Delta) + (\mu \Phi^T i \tau_2 \Delta^\dagger \Phi + \text{h.c.}) \\ + \lambda_1 (\Phi^\dagger \Phi)^2 + \lambda_2 [\text{Tr}(\Delta^\dagger \Delta)]^2 + \lambda_3 \text{Tr}[(\Delta^\dagger \Delta)^2] \\ + \lambda_4 (\Phi^\dagger \Phi) \text{Tr}(\Delta^\dagger \Delta) + \lambda_5 \Phi^\dagger \Delta \Delta^\dagger \Phi$$

$$\Phi = \begin{bmatrix} \frac{1}{\sqrt{2}}(\varphi^+ + v_\Phi + i\chi) \end{bmatrix}$$

$$\Delta = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix}$$

$$\Delta^0 = \frac{1}{\sqrt{2}}(\delta + v_\Delta + i\eta)$$

7 physical scalar states.

$$\Delta^{\pm\pm} = \text{Doubly-charged Higgs}$$

$$\begin{pmatrix} \varphi^0 \\ \delta^0 \end{pmatrix} = R(\alpha) \begin{pmatrix} h^0 \\ H^0 \end{pmatrix}, \quad \begin{pmatrix} \varphi^\pm \\ \Delta^\pm \end{pmatrix} = R(\beta) \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix}, \quad \begin{pmatrix} \chi^0 \\ \eta^0 \end{pmatrix} = R(\beta') \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}$$

NG bosons

$$\tan 2\alpha \simeq \frac{v_\Delta}{v_\Phi} \frac{2v_\Phi^2(\lambda_4 + \lambda_5) - 4M_\Delta^2}{2v_\Phi^2\lambda_1 - M_\Delta^2}$$

$$\tan \beta = \frac{\sqrt{2}v_\Delta}{v_\Phi} \quad \tan \beta' = \frac{2v_\Delta}{v_\Phi}$$

CP-even Higgs

Singly-charged Higgs

CP-odd Higgs

Mass formulae ($v_\Delta/v_\Phi \ll 1$)

Triplet-like

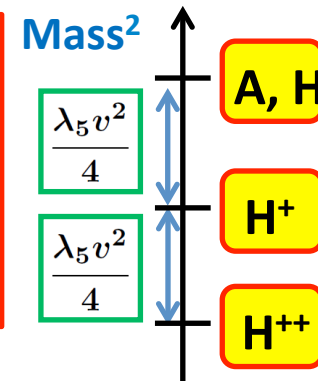
$$m_{H^{++}}^2 \simeq M_\Delta^2 - \frac{v^2}{2}\lambda_5, \quad m_{H^+}^2 \simeq M_\Delta^2 - \frac{v^2}{4}\lambda_5, \quad m_A^2 \simeq m_H^2 = M_\Delta^2$$

SM-like

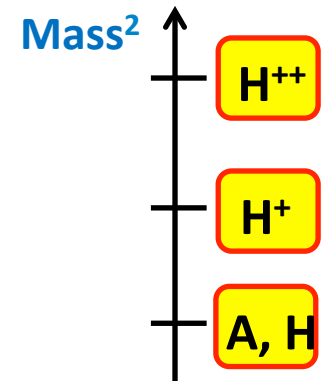
$$m_h^2 \simeq 2\lambda_1 v^2$$

$$M_\Delta^2 \equiv \frac{v_\Phi^2 \mu}{\sqrt{2}v_\Delta}$$

Case I ($\lambda_5 > 0$)



Case II ($\lambda_5 < 0$)



Constraint from EW precision data

- Renormalization in models with $\rho_{\text{tree}} \neq 1$ is different from that in models with $\rho_{\text{tree}} = 1$.
- We need **four** (not **three** such as α_{em} , G_F and m_Z) input parameters to describe the EW parameters.

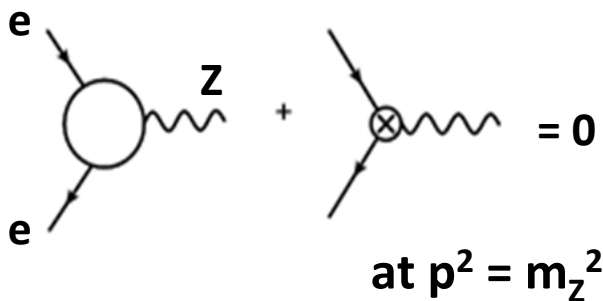
*Blanck, Hollik (1997);
Kanemura, KY (2012)*

Scheme I

Inputs:

α_{em} , G_F , m_Z and **$\sin\theta_{\text{eff}}$**

Reno.
condition:



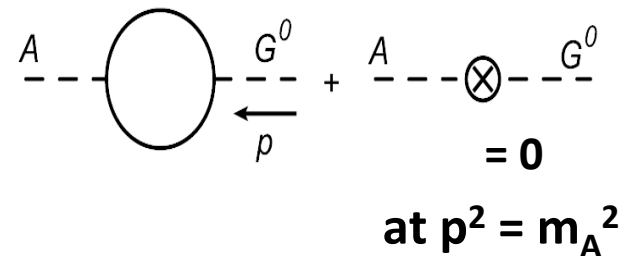
Electroweak

Reno.
Scale:

*Chankowski, Pokorski, Wagner (2007);
Chen, Dawson, Jackson (2008)*

Scheme II

α_{em} , G_F , m_Z and **β'**



Decoupling parameter

Constraint from EW precision data

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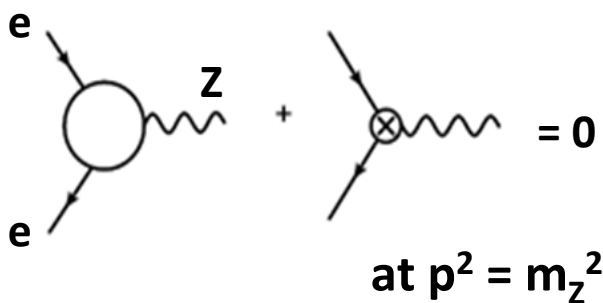
*Blanck, Hollik (1997);
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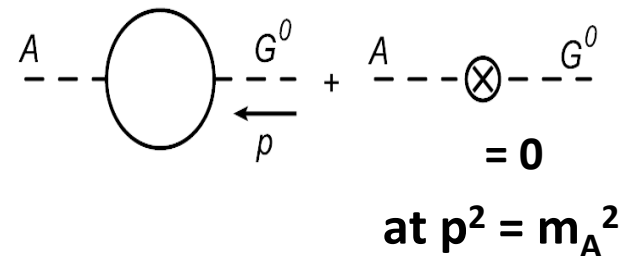
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Scale:

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Scheme II

α_{em} , G_F , m_Z and **β'**

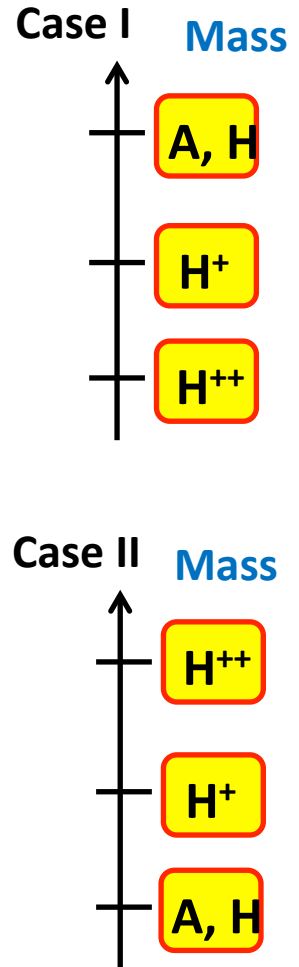
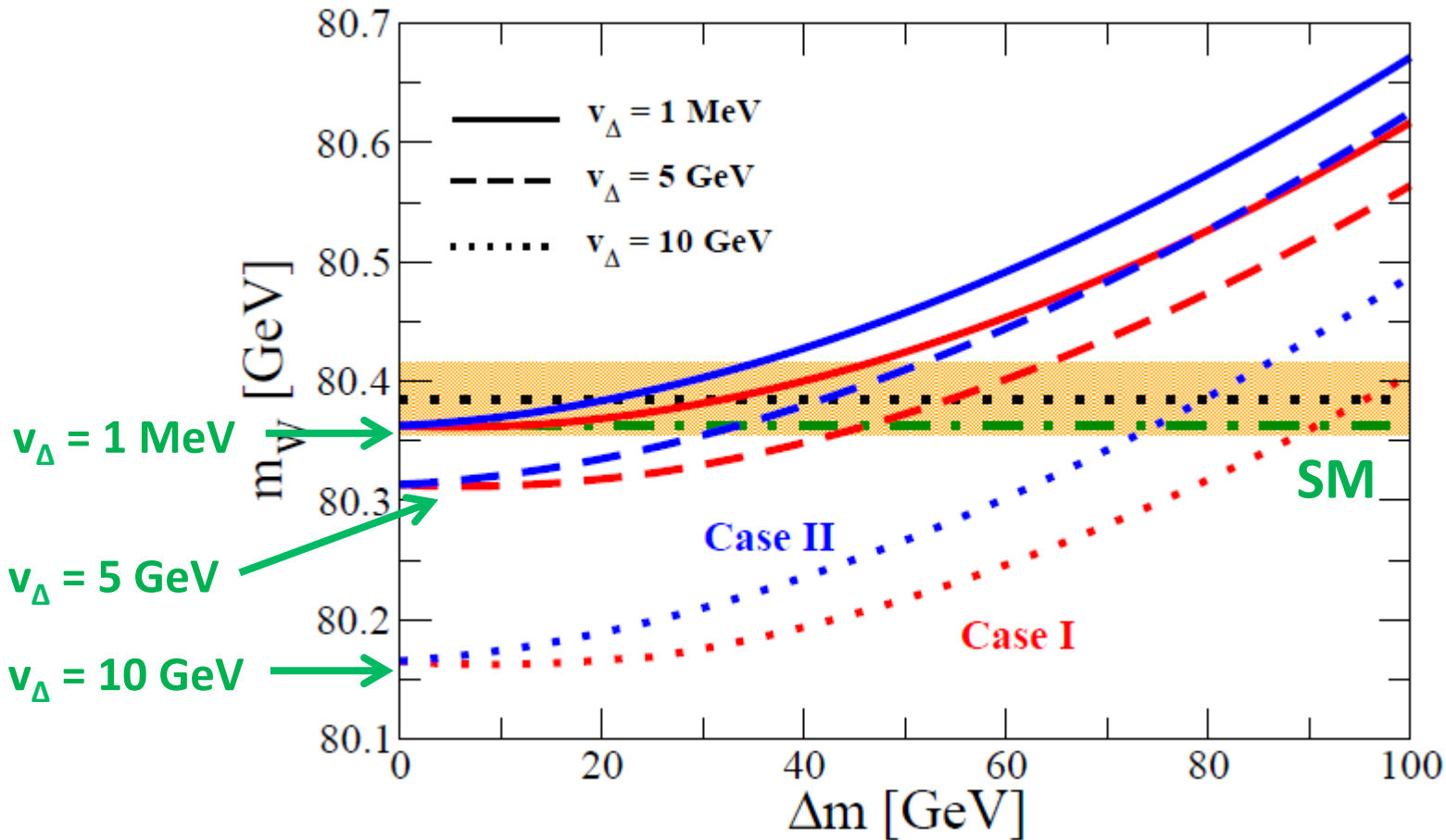


Decoupling parameter

1-loop corrected W mass

$$\Delta m = m_{H^+} - m_{\text{lightest}}$$

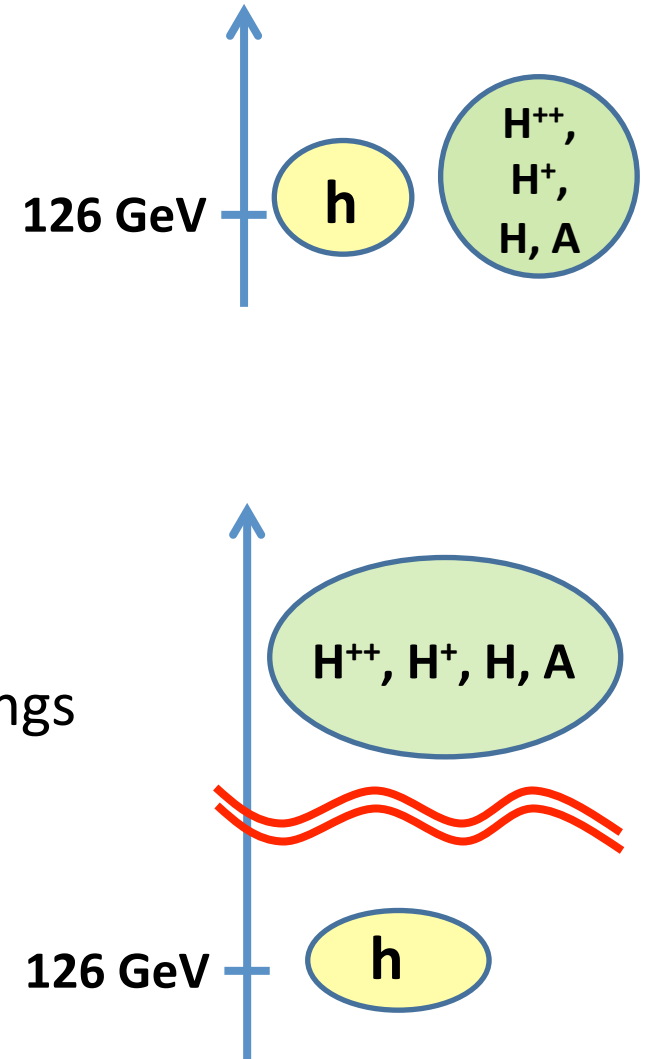
Kanemura, Kikuchi, KY, PRD87 (2012)



For large triplet VEV case, large mass difference is favored.

Testing the Higgs Triplet Model at colliders

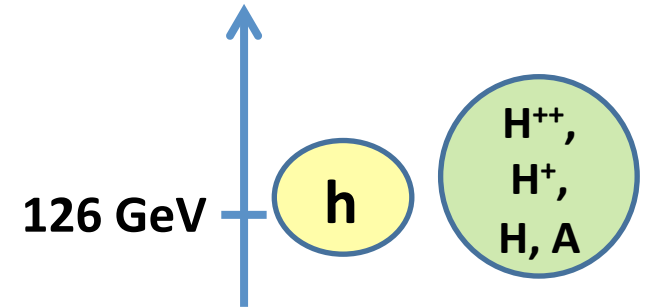
- Direct way
 - Discovery of extra Higgs bosons, e.g., $H^{\pm\pm}$, $H^\pm$, ...
 - Testing the mass spectrum among the triplet like Higgs bosons.
- Indirect way
 - Precise measurement for the Higgs couplings
Ex. $h\gamma\gamma$, hWW , hZZ , hhh



Testing the Higgs Triplet Model at colliders

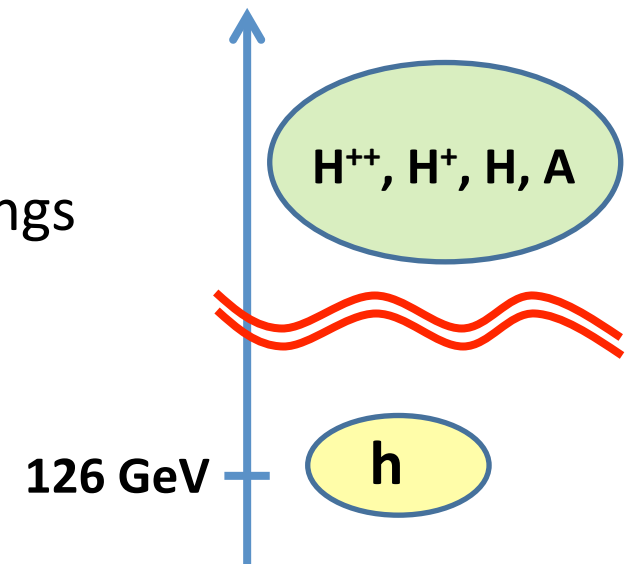
- **Direct way**

- Discovery of extra Higgs bosons, e.g., $H^{\pm\pm}$, $H^\pm$, ...
- Testing the mass spectrum among the triplet like Higgs bosons.



- Indirect way

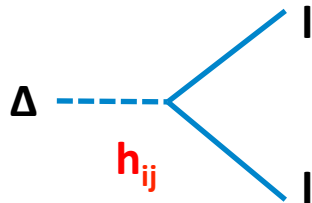
- Precise measurement for the Higgs couplings
Ex. $h\gamma\gamma$, hWW , hZZ , hhh



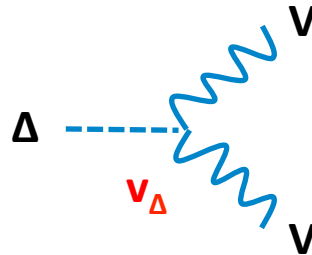
Decay of the triplet-like Higgs bosons

The decay of Δ -like Higgs bosons can be classified into 3 modes.

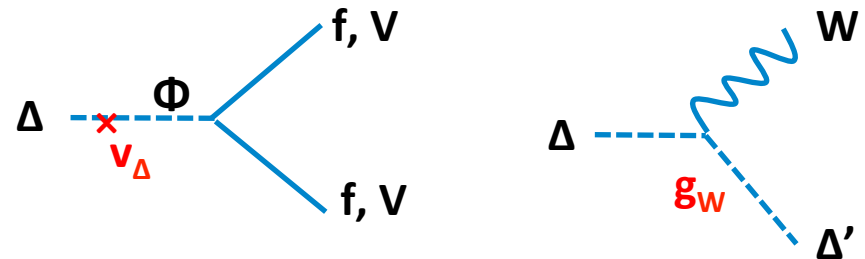
1. Decay via h_{ij}



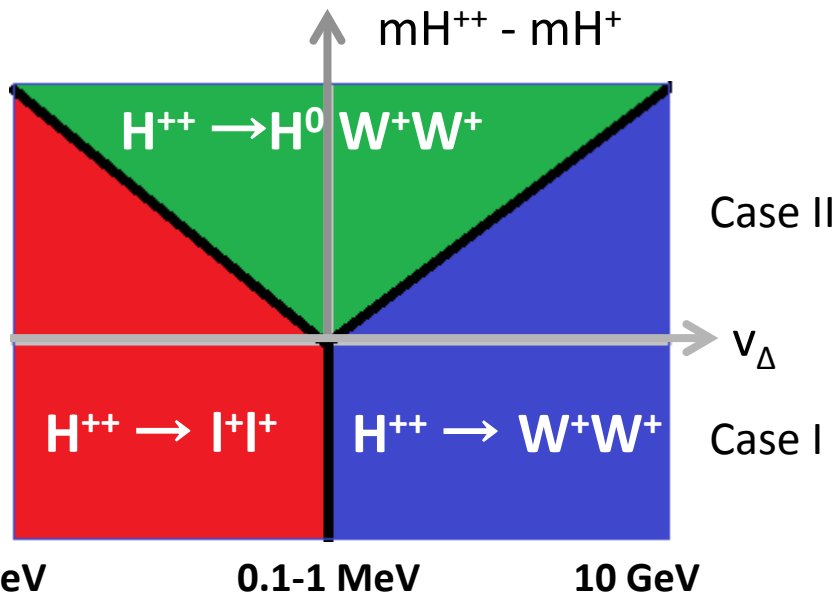
2. Decay via v_Δ



3. Decay via g



Decay of H^{++}

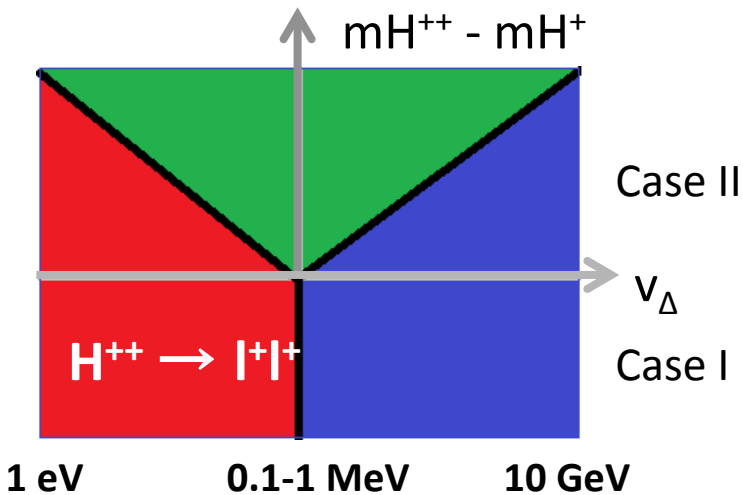


Decay modes of 1 and 2 are related to each other by the relation:

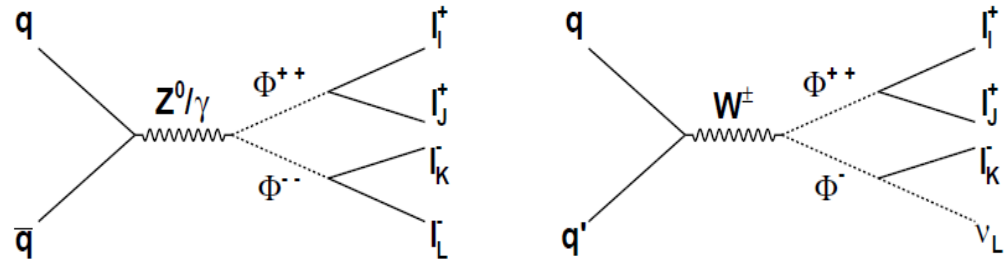
$$(m_\nu)_{ij} = h_{ij} v_\Delta$$

Decay of the triplet like scalar bosons strongly depend on v_Δ and $\Delta m (\equiv m_{H^{++}} - m_{H^+})$.

Dilepton decay scenario



Signal: Same-sign dilepton

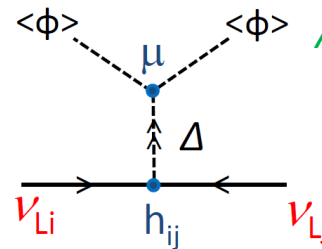
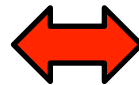


$m_{H^{++}} > \sim 400 \text{ GeV}$ by the LHC data

By measuring the pattern of leptonic decay, we can directly test the neutrino mass matrix.

$$\text{BR} \left[H^{++} \rightarrow l_i^+ l_j^+ \right]$$

$$\propto |h_{ij}|^2$$



Akeroyd, Aoki, Sugiyama, 2008

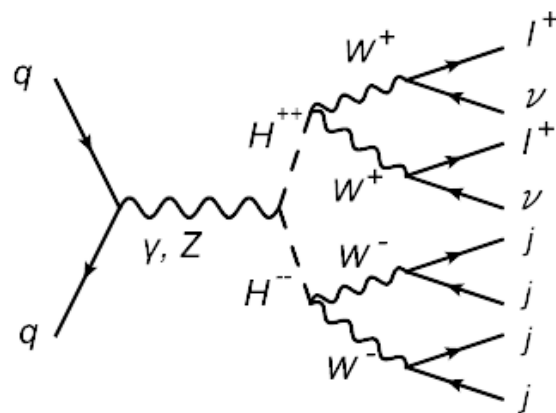
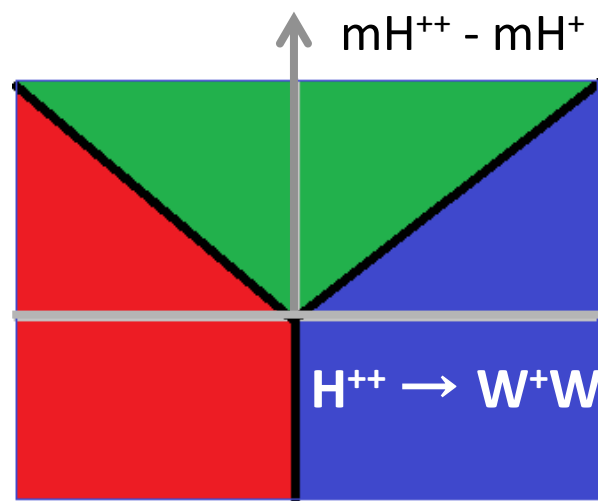
$$h_{ij} v_\Delta$$

LHC phenomenology

- 4 lepton, 3 lepton signature *Perez, Han, Huang, Li, 2008 ; Akeroyd, Chiang, Gaur, 2010, ...*
- Using tau polarization *Sugiyama, Tsumura, Yokoya (2012)*
- Discrimination of chiral structure of the Yukawa coupling
- Same sign tetra-lepton signature *Chun, Sharma (2012)*

Diboson decay scenario

►Signal: Same-sign dilepton + Jets + Missing



Case II

Case I

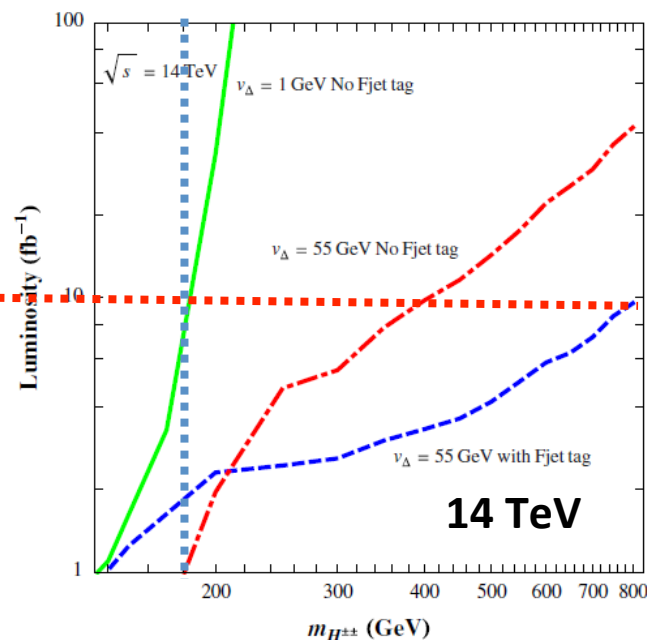
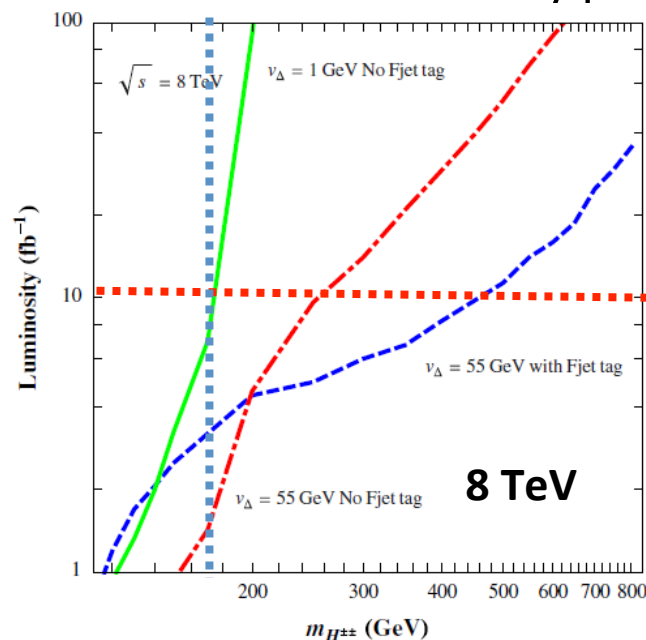
1 eV

0.1-1 MeV

10 GeV

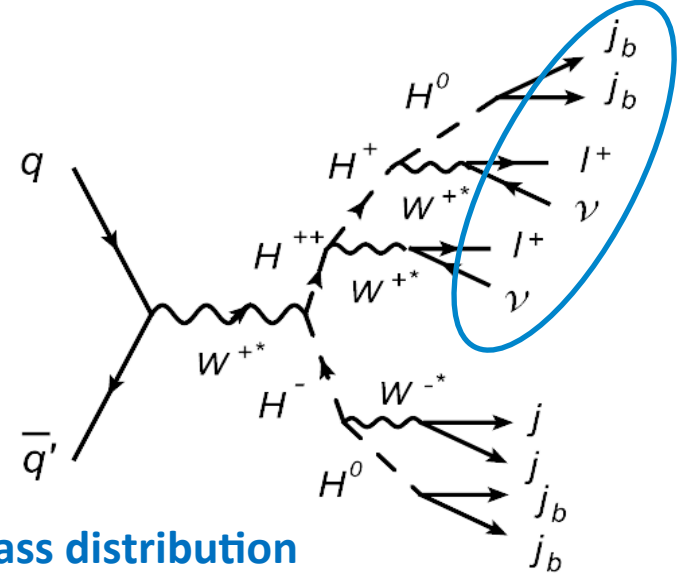
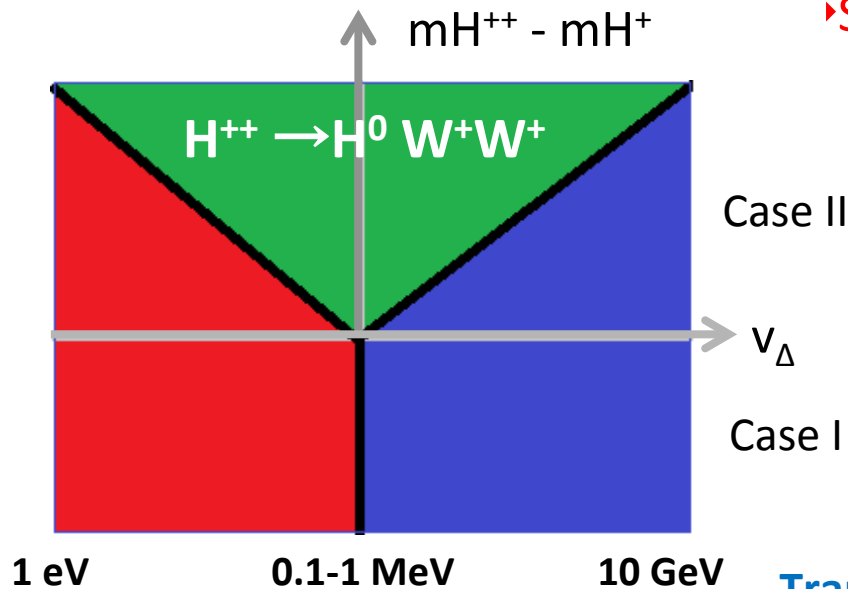
5 σ discovery potential of H^{++}

*Chiang, Nomura,
Tsumura, PRD85 (2012)*

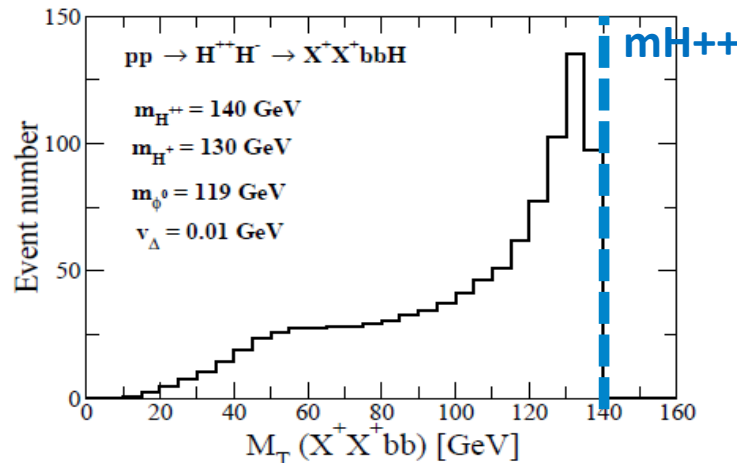
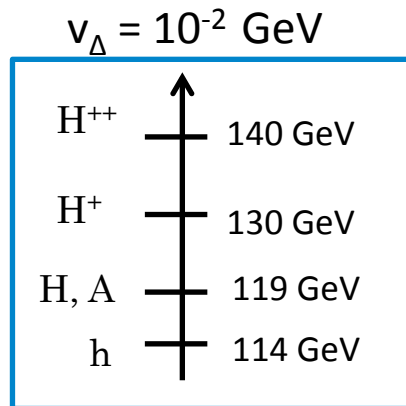


Cascade decay scenario

Signal: Same-sign dilepton + Jets + Missing



Transverse mass distribution



Aoki, Kanemura, KY, PRD85 (2012)

8.0 fb (14 TeV)

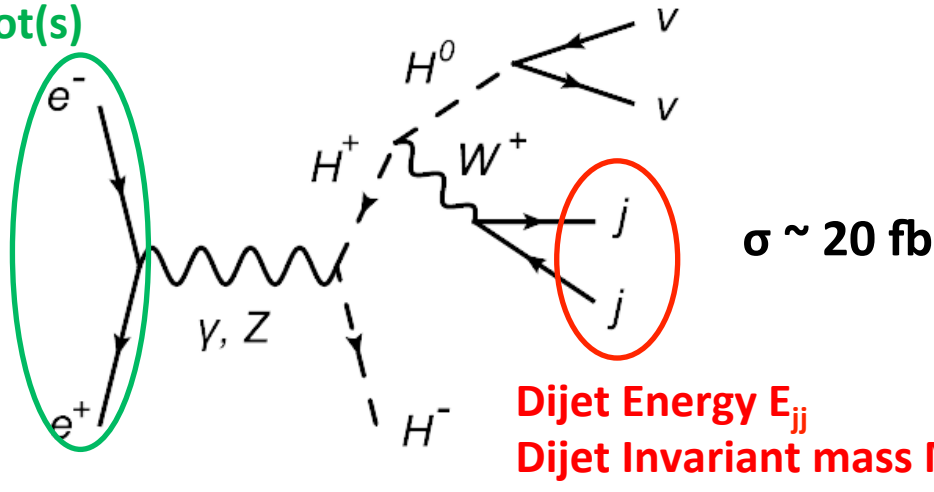
2.8 fb (7 TeV)

$$m_{H^{++}}^2 - m_{H^+}^2 \approx m_{H^+}^2 - m_H^2$$

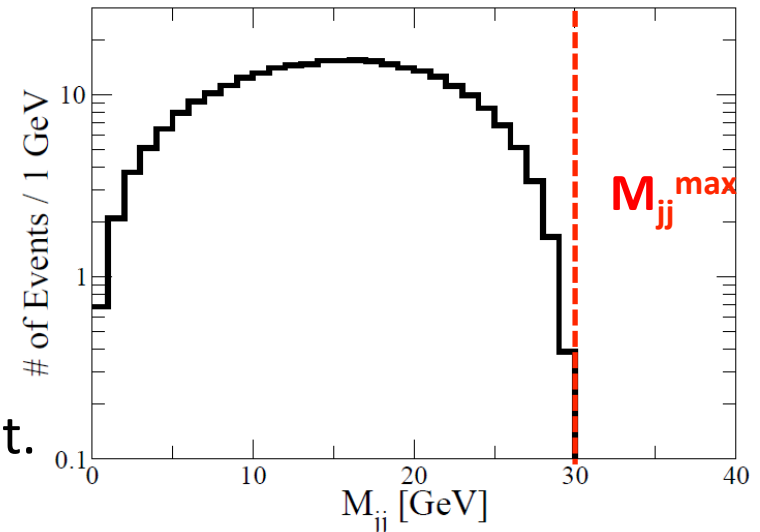
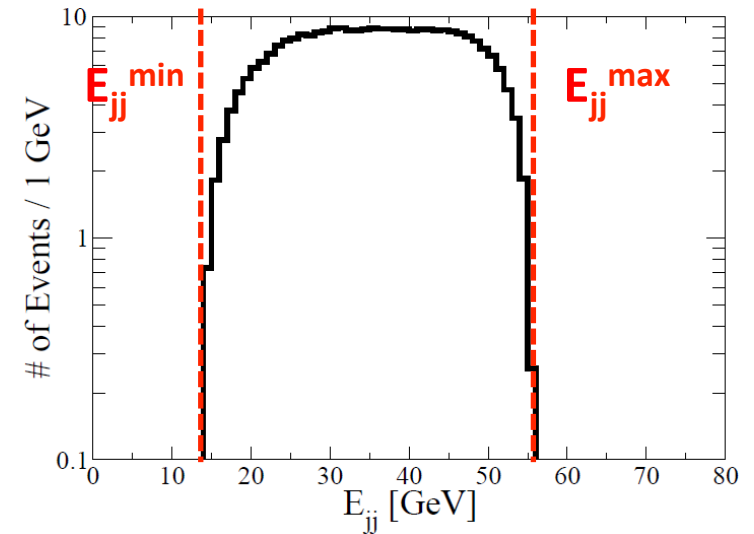
We may test the characteristic mass spectrum of Δ -like Higgs bosons

Mass reconstruction at ILC

Root(s)



$m_{H^+} = 200 \text{ GeV}$, $m_H = 170 \text{ GeV}$,
Root(s) = 500 GeV, 100 fb^{-1}



$$m_{H^+}^2 \simeq \frac{s}{4} \left[1 - \left(\frac{E_{jj}^{\max} - E_{jj}^{\min}}{E_{jj}^{\max} + E_{jj}^{\min}} \right)^2 \right]$$

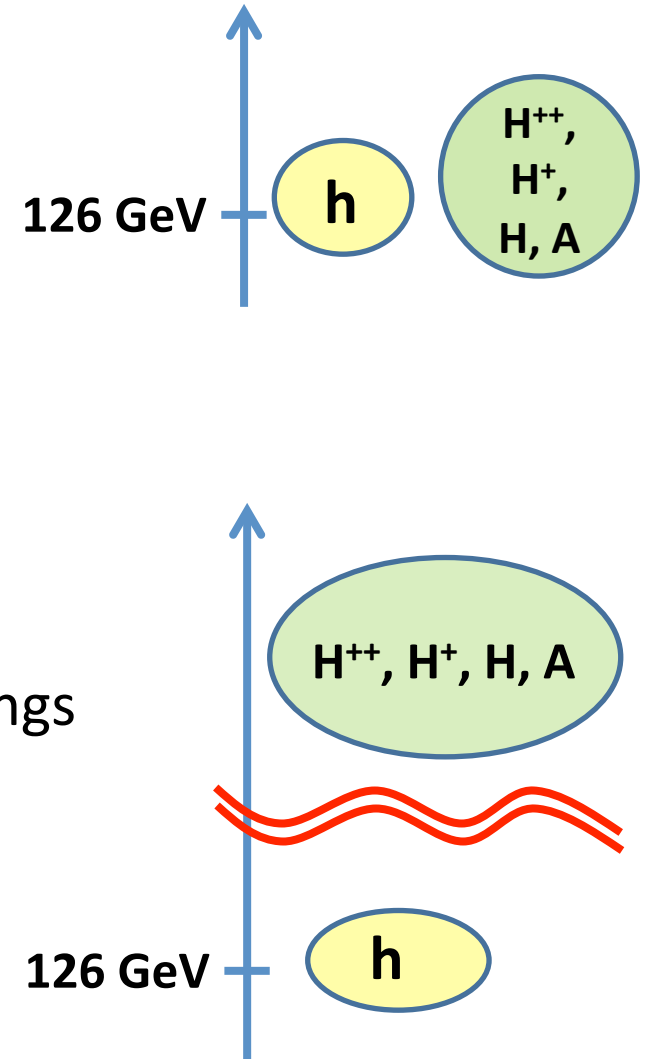
$$m_{H^0}^2 \simeq \frac{s}{4} \left[1 - \left(\frac{E_{jj}^{\max} - E_{jj}^{\min}}{E_{jj}^{\max} + E_{jj}^{\min}} \right)^2 \right] - \frac{2\sqrt{s}E_{jj}^{\max}E_{jj}^{\min}}{E_{jj}^{\max} + E_{jj}^{\min}}$$

$$M_{jj}^{\max} \simeq m_{H^+} - m_H$$

H^{++} can be measured by looking at the excess of the SS dilepton + jets + missing event.

Testing the Higgs Triplet Model at colliders

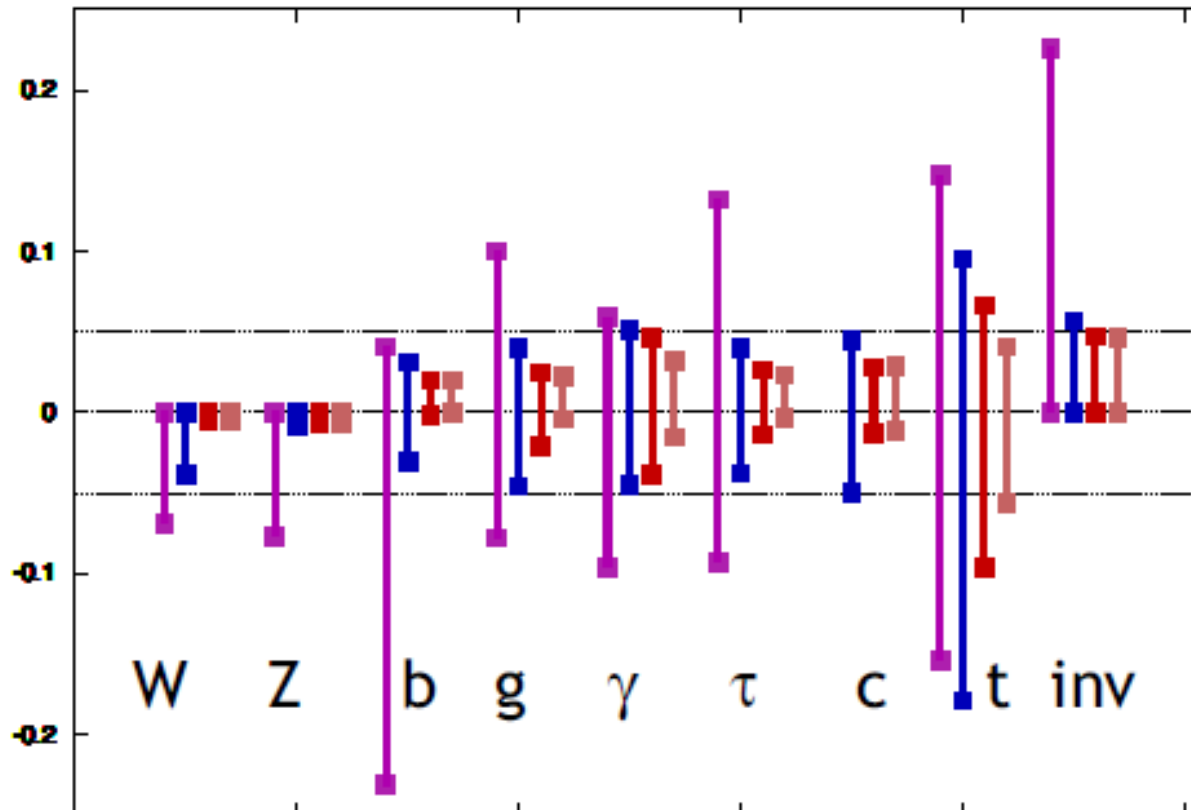
- Direct way
 - Discovery of extra Higgs bosons, e.g., $H^{\pm\pm}$, $H^{\pm}$, ...
 - Testing the mass spectrum among the triplet like Higgs bosons.
- Indirect way
 - Precise measurement for the Higgs couplings
Ex. $h\gamma\gamma$, hWW , hZZ , hhh



Higgs coupling measurements

Peskin, arXiv: 1207.2516

$g(hAA)/g(hAA)|_{SM} - 1$ LHC/ILC1/ILC/ILCTeV



LHC: 14 TeV, 300 fb⁻¹,

ILC1: 250 GeV, 250 fb⁻¹,

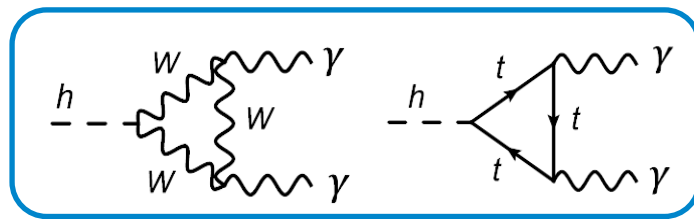
ILC: 500 GeV, 500 fb⁻¹,

ILCTeV: 1 TeV, 1000 fb⁻¹

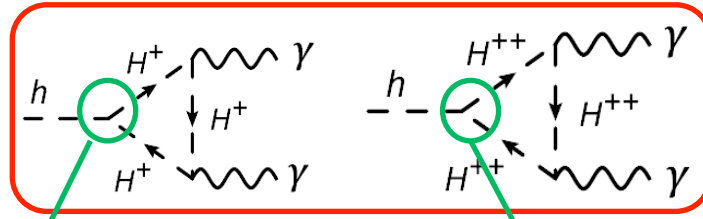
Higgs boson couplings can be measured accurately at future colliders.

Higgs $\rightarrow$ diphoton

Arhrib, et al. JHEP1204; Kanemura, KY, PRD85;
Akeroyd, Moretti PRD86; Chun, Lee, Sharma JHEP1211



SM contribution



+ Triplet-like Higgs loop contribution

$$\lambda_{hH^+H^-} \approx -(\lambda_4 + \lambda_5/2)v$$

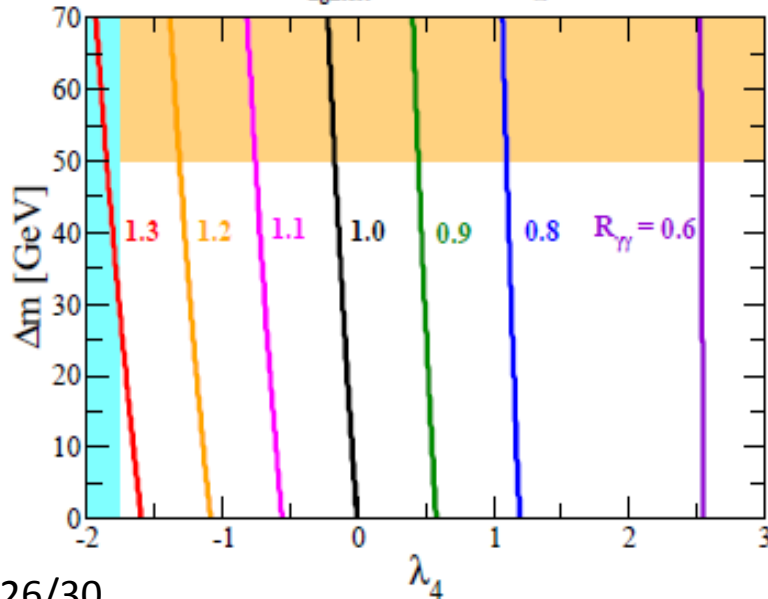
$$\lambda_{hH^{++}H^{--}} \simeq -\lambda_4 v$$

★ Sign of λ_4 is quite important! If $\lambda_4 < 0 \rightarrow$ Constructive (Destructive) contribution.

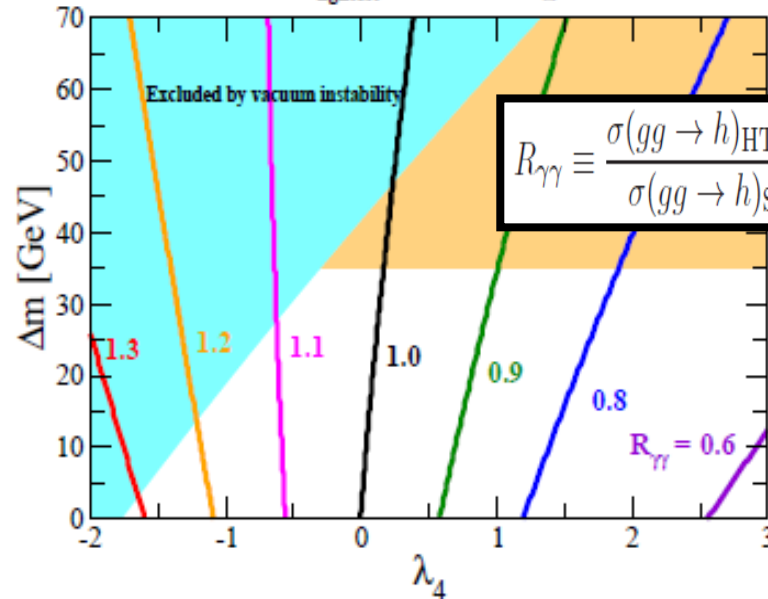
Signal strength ($\sigma_{\text{obs}}/\sigma_{\text{SM}}$) is 1.56 ± 0.43 (CMS) and 1.9 ± 0.5 (ATLAS).

Kanemura, Kikuchi,
KY (2012)

Case I, $m_{\text{lightest}} = 300 \text{ GeV}$, $v_\Delta = 1 \text{ MeV}$



Case II, $m_{\text{lightest}} = 300 \text{ GeV}$, $v_\Delta = 1 \text{ MeV}$



$$R_{\gamma\gamma} \equiv \frac{\sigma(gg \rightarrow h)_{\text{HTM}} \times \text{BR}(h \rightarrow \gamma\gamma)_{\text{HTM}}}{\sigma(gg \rightarrow h)_{\text{SM}} \times \text{BR}(h \rightarrow \gamma\gamma)_{\text{SM}}}$$

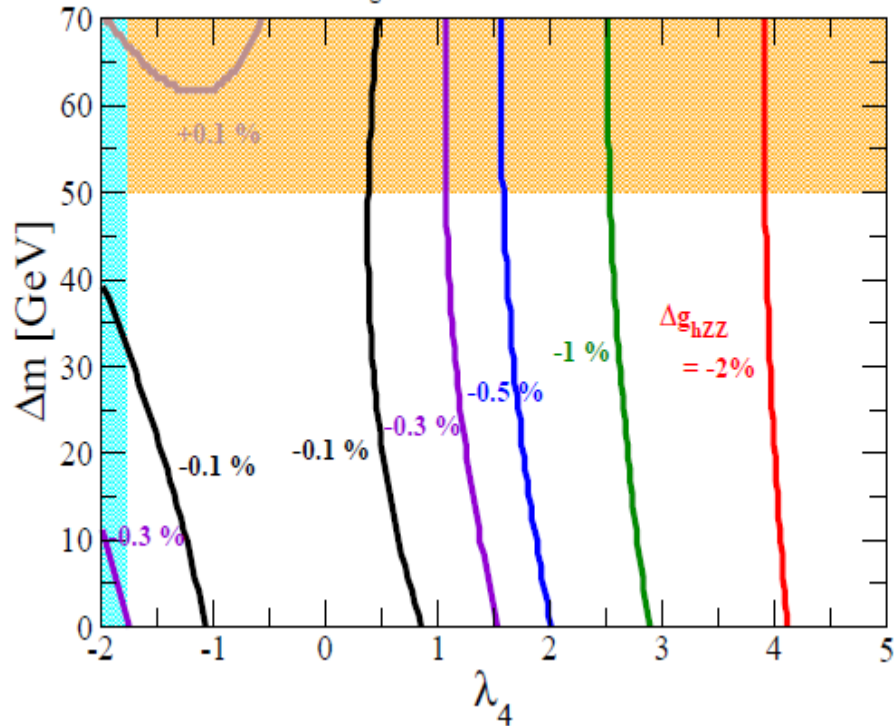
Renormalized hZZ

Kanemura, Kikuchi,
KY, PRD87 (2012)

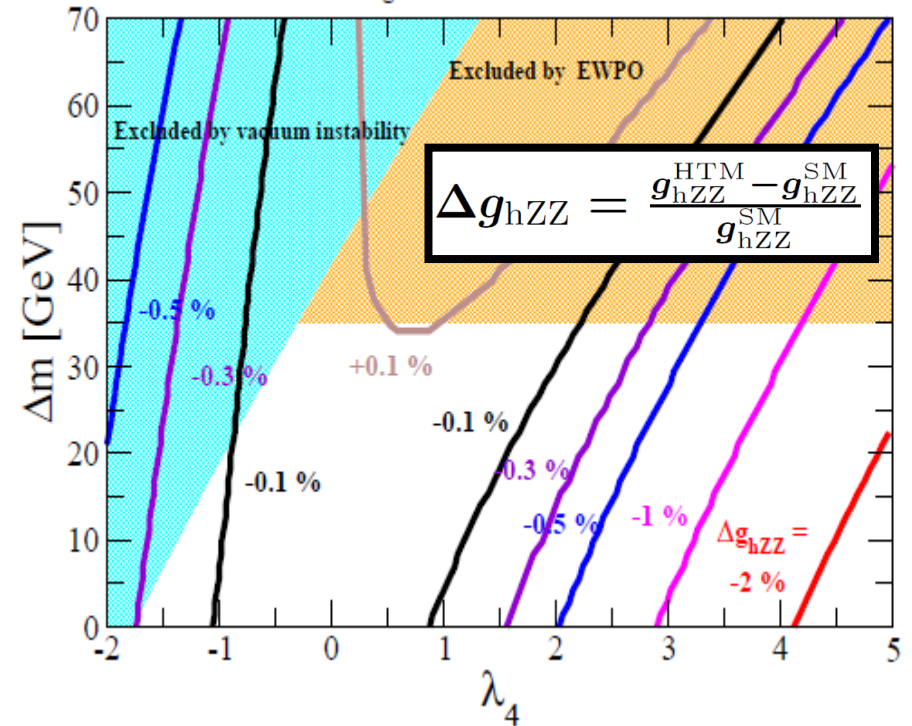
$$g_{hVV} \simeq \frac{2m_V^2}{v} \left[1 - \frac{1}{32\pi^2} \left(\frac{\lambda_{H^{++}H^{--}h}^2}{6m_{H^{++}}^2} + \frac{\lambda_{H^+H^-h}^2}{6m_{H^+}^2} + \frac{\lambda_{AAh}^2}{3m_A^2} + \frac{\lambda_{HHh}^2}{3m_H^2} \right) + \frac{1}{4\pi^2} \frac{c_W^2 - s_W^2}{s_W^2} \left(\frac{F_5(m_{H^{++}}, m_{H^+})}{v^2} + \frac{F_5(m_{H^+}, m_A)}{v^2} \right) \right]$$

$$\simeq \frac{2m_V^2}{v} \left[1 - \frac{1}{32\pi^2} \left(\frac{\lambda_4^2}{6m_{H^{++}}^2} + \frac{(\lambda_4 + \frac{\lambda_5}{2})^2}{6m_{H^+}^2} + \frac{(\lambda_4 + \lambda_5)^2}{12m_A^2} + \frac{(\lambda_4 + \lambda_5)^2}{12m_H^2} \right) + \frac{1}{4\pi^2} \frac{2(c_W^2 - s_W^2)}{3s_W^2} \left(\frac{(m_{H^{++}} - m_{H^+})^2}{v^2} + \frac{(m_{H^+} - m_A)^2}{v^2} \right) \right]$$

Case I, $m_{\text{lightest}} = 300 \text{ GeV}$, $v_\Delta = 1 \text{ MeV}$



Case II, $m_{\text{lightest}} = 300 \text{ GeV}$, $v_\Delta = 1 \text{ MeV}$



Δg_{hZZ} is predicted -0.3 % ~ -2 % when $R_{\gamma\gamma}$ is +30% ~ -40%

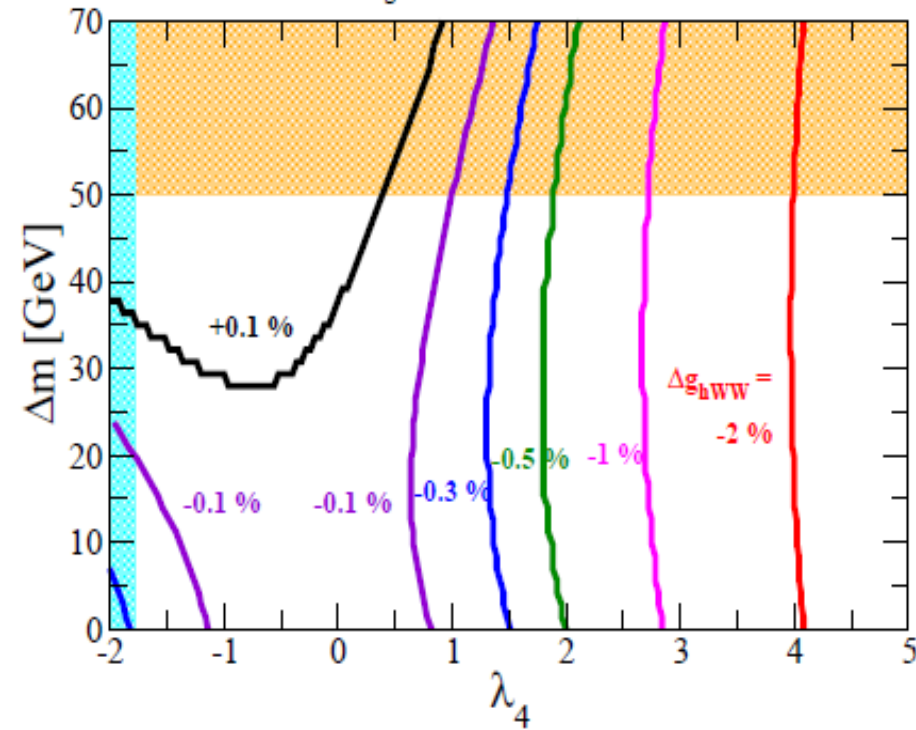
Renormalized hWW

Kanemura, Kikuchi,
KY, PRD87 (2012)

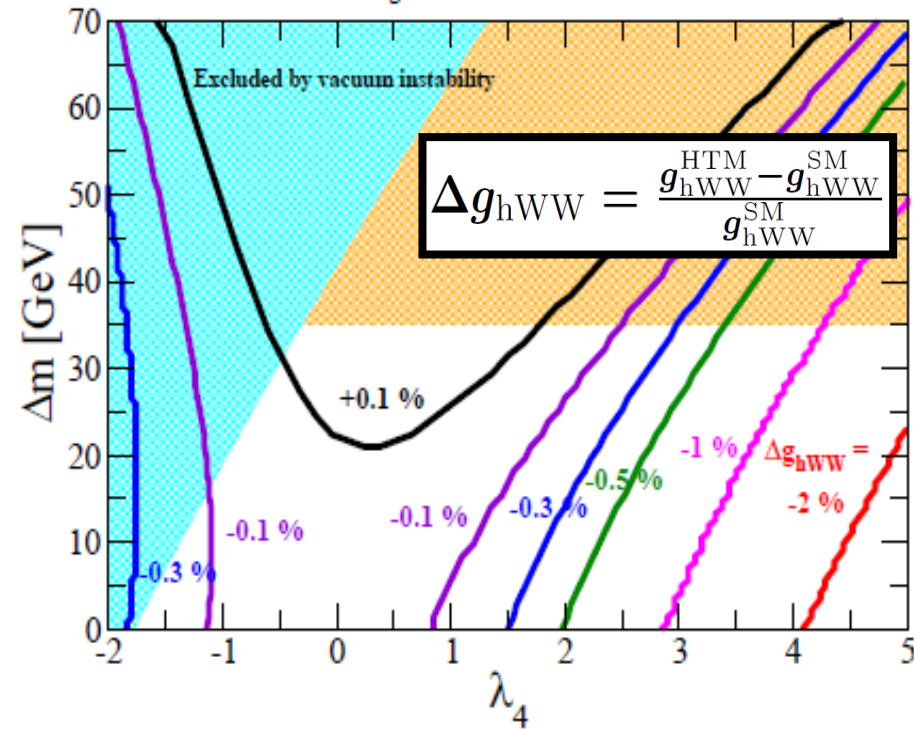
$$g_{hVV} \simeq \frac{2m_V^2}{v} \left[1 - \frac{1}{32\pi^2} \left(\frac{\lambda_{H^{++}H^{--}h}^2}{6m_{H^{++}}^2} + \frac{\lambda_{H^+H^-h}^2}{6m_{H^+}^2} + \frac{\lambda_{AAh}^2}{3m_A^2} + \frac{\lambda_{HHh}^2}{3m_H^2} \right) + \frac{1}{4\pi^2} \frac{c_W^2 - s_W^2}{s_W^2} \left(\frac{F_5(m_{H^{++}}, m_{H^+})}{v^2} + \frac{F_5(m_{H^+}, m_A)}{v^2} \right) \right]$$

$$= \frac{2m_V^2}{v} \left[1 - \frac{1}{32\pi^2} \left(\frac{\lambda_4^2}{6m_{H^{++}}^2} + \frac{(\lambda_4 + \frac{\lambda_5}{2})^2}{6m_{H^+}^2} + \frac{(\lambda_4 + \lambda_5)^2}{12m_A^2} + \frac{(\lambda_4 + \lambda_5)^2}{12m_H^2} \right) + \frac{1}{4\pi^2} \frac{2(c_W^2 - s_W^2)}{3s_W^2} \left(\frac{(m_{H^{++}} - m_{H^+})^2}{v^2} + \frac{(m_{H^+} - m_A)^2}{v^2} \right) \right]$$

Case I, $m_{\text{lightest}} = 300 \text{ GeV}$, $v_\Delta = 1 \text{ MeV}$



Case II, $m_{\text{lightest}} = 300 \text{ GeV}$, $v_\Delta = 1 \text{ MeV}$



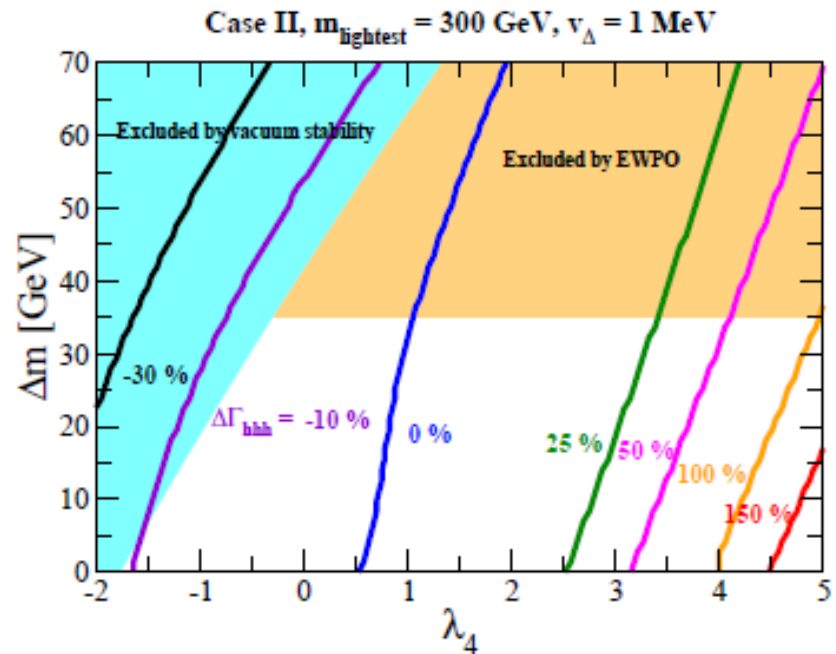
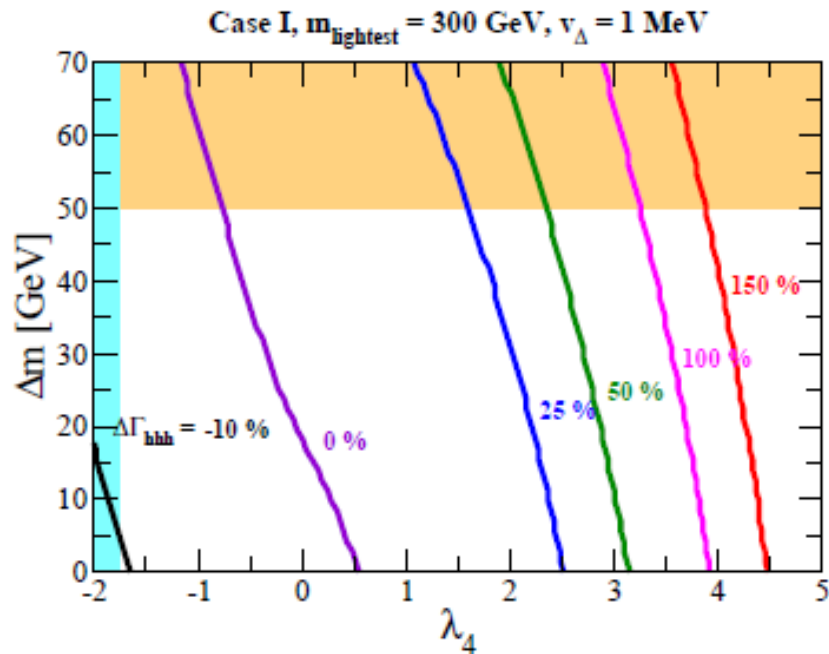
Δg_{hWW} is predicted -0.3 % ~ -2 % when $R_{\gamma\gamma}$ is +30% ~ -40%

Renormalized hhh coupling

Kanemura, Kikuchi,
KY, PRD87 (2012)

$$\Gamma_{hhh} \simeq -\frac{3m_h^2}{v} \left[1 - \frac{v}{48\pi^2 m_h^2} \left(\frac{\lambda_{H^{++}H^{--}h}^3}{m_{H^{++}}^2} + \frac{\lambda_{H^+H^-h}^3}{m_{H^+}^2} + \frac{4\lambda_{AAh}^3}{m_A^2} + \frac{4\lambda_{HHh}^3}{m_H^2} \right) + \dots \right]$$

$$\simeq -\frac{3m_h^2}{v} \left\{ 1 + \frac{v^4}{48\pi^2 m_h^2} \left[\frac{\lambda_4^3}{m_{H^{++}}^2} + \frac{(\lambda_4 + \frac{\lambda_5}{2})^3}{m_{H^+}^2} + \frac{(\lambda_4 + \lambda_5)^3}{2m_A^2} + \frac{(\lambda_4 + \lambda_5)^3}{2m_H^2} \right] + \dots \right\},$$



**hhh coupling can deviate about -10% ~ +150 %,
when $R_{\gamma\gamma}$ is +30% ~ -40%.**

Summary

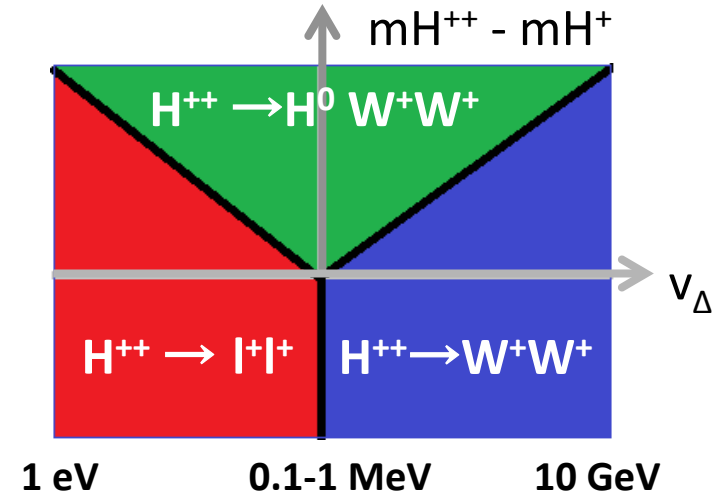
► Probing exotic Higgs sectors :

There are several characteristic natures in exotic Higgs sectors,
e.g., ρ parameter, $H^\pm W^\mp Z$ vertex, hVV vertex
and multi-charged scalar bosons.

► Direct way of testing the HTM :

-Decay of H^{++} can be classified into 4 regions
in the $v\Delta - \Delta m$ plane.

If small $v\Delta$ and large $m_{H^{++}} - m_{H^+}$ is realized,
we need ILC to identify H^{++} .

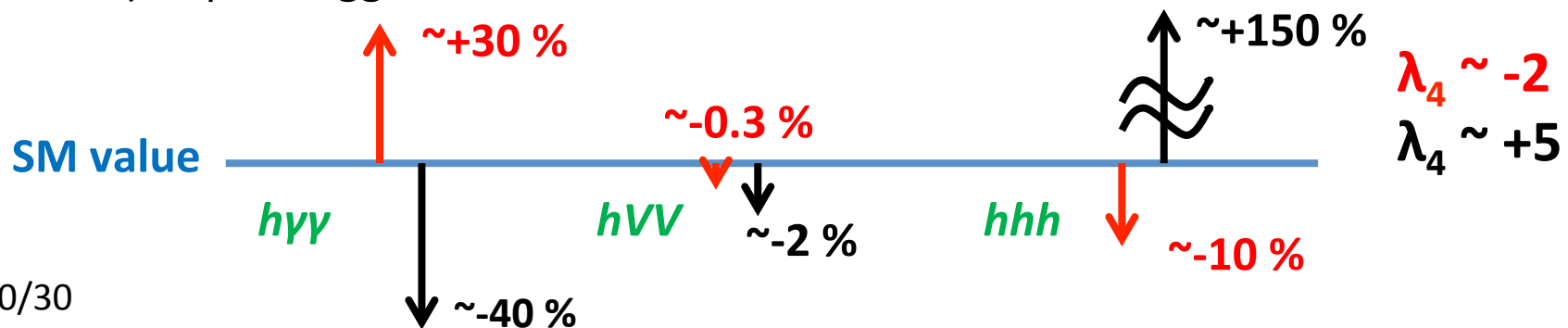


► Indirect way of testing the HTM :

-Precise measurements of the Higgs couplings

-Precise calculations of the deviation in the Higgs couplings from the SM prediction.

Ex) Triplet Higgs masses = 300 GeV case



Back up slides

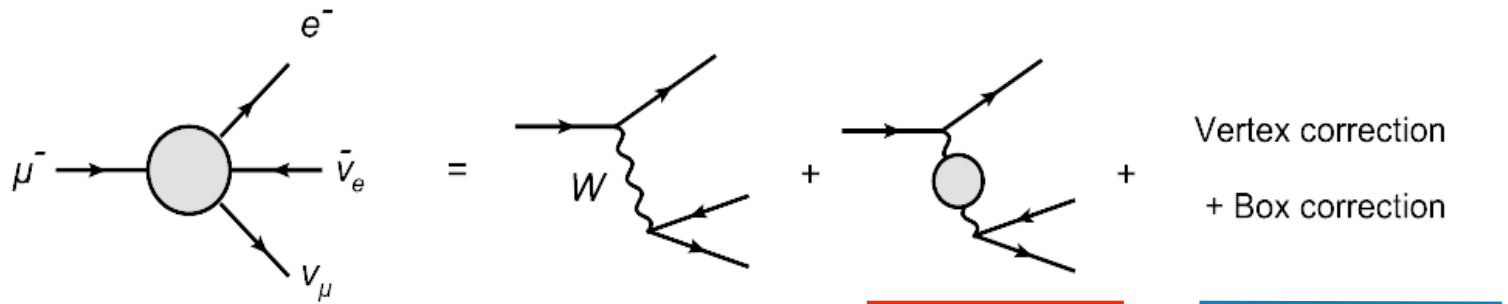
Radiative corrections to the EW parameters

Hollik, Fortsch, Phys. (1990)

★ One-loop corrections to the W boson mass can be calculated via Δr :

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2(1-\Delta r)} = \frac{\pi\alpha_{\text{em}}}{2m_W^2 s_W^2(1-\Delta r)}$$

Muon decay



$$\Delta r = \frac{\hat{\Pi}_{WW}(0)}{m_W^2} + \delta_{VB}$$

$$= \Delta\alpha_{\text{em}} - \frac{c_W^2}{s_W^2} \Delta\rho + \Delta r_{\text{rem}}$$

$[\rho_{\text{tree}} = 1]$

$\Delta\alpha_{\text{em}}$: shift of α_{em} from 0 to m_Z .

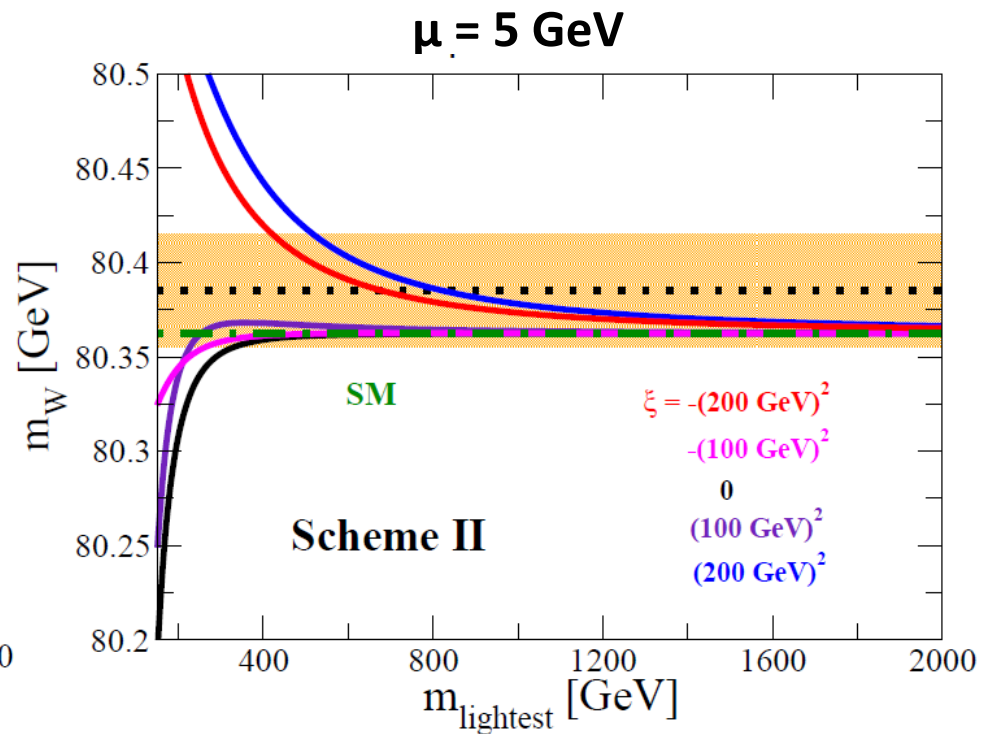
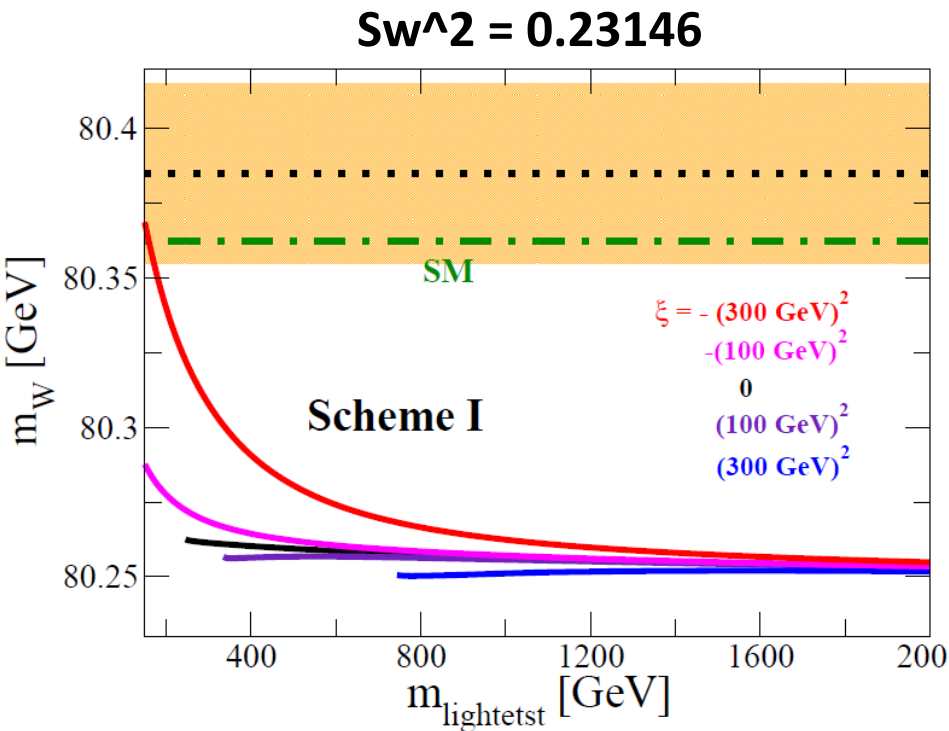
$\Delta\rho$: corrections to the rho parameter

Δr_{rem} : The other terms

Decoupling

$$\xi = m_{H^{++}}^2 - m_{H^+}^2 (= m_{H^+}^2 - m_A^2)$$

$$v_{\Delta} \simeq \mu \frac{v^2}{m_A^2}$$



In Scheme II, we can take the decoupling limit.

Theoretical bounds

▸ Vacuum stability bound (Bounded from below) *Arhrib, et al., PRD84, (2011)*

$$\lim_{r \rightarrow \infty} V(rv_1, rv_2, \dots, rv_n) > 0$$

$$\lambda_2 = \lambda_3 = \lambda_\Delta$$

$$\lambda_1 > 0, \lambda_\Delta > 0,$$

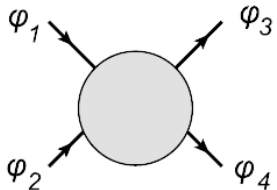
$$2\sqrt{2\lambda_1\lambda_\Delta} + \lambda_4 + \text{MIN}[0, \lambda_5] > 0$$

$$V^{(4)} = \lambda_1(\Phi^\dagger\Phi)^2 + \lambda_2[\text{Tr}(\Delta^\dagger\Delta)]^2 + \lambda_3\text{Tr}[(\Delta^\dagger\Delta)^2] \\ + \lambda_4(\Phi^\dagger\Phi)\text{Tr}(\Delta^\dagger\Delta) + \lambda_5\Phi^\dagger\Delta\Delta^\dagger\Phi$$

Lee, Quigg, Thacker, PRD16, (1977)

Aoki, Kanemura, PRD77, (2008); Arhrib, et al., PRD84, (2011)

▸ Unitarity bound



j_i : Longitudinal modes of weak gauge bosons and physical Higgs bosons

$$|\langle \varphi_3 \varphi_4 | a^0 | \varphi_1 \varphi_2 \rangle| < \frac{1}{2} \text{ or } 1$$

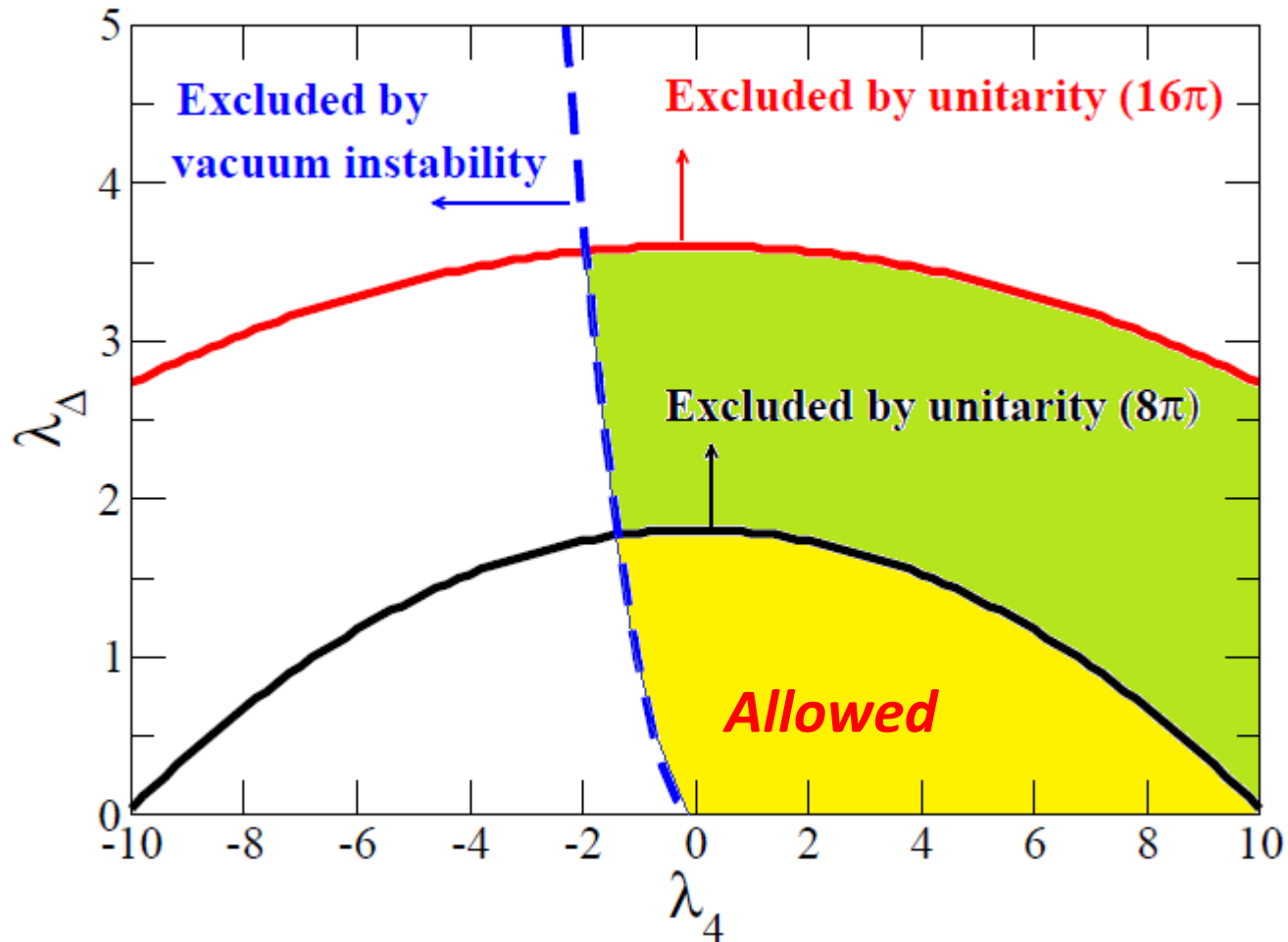
Eigenvalues
of the matrix

$$\begin{cases} x_1 = 3\lambda_1 + 7\lambda_\Delta + \sqrt{(3\lambda_1 - 7\lambda_\Delta)^2 + \frac{3}{2}(2\lambda_4 + \lambda_5)^2}, \\ x_2 = \frac{1}{2}(2\lambda_4 + 3\lambda_5), \\ x_3 = \frac{1}{2}(2\lambda_4 - \lambda_5) \end{cases}$$

$$|x_i| < 8\pi \text{ or } 16\pi$$

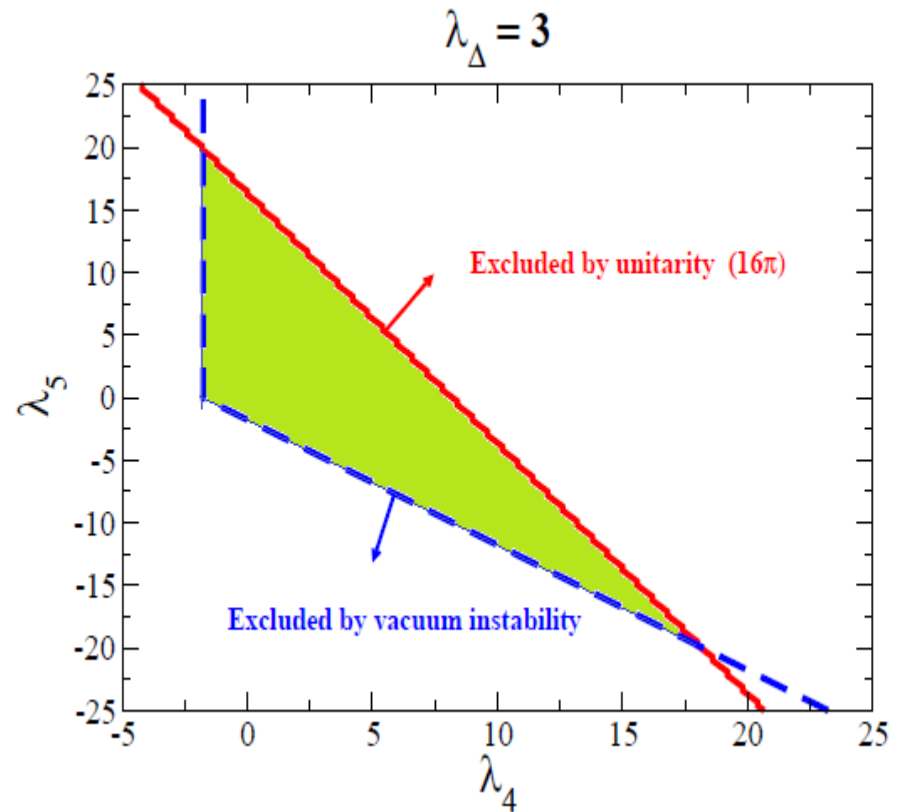
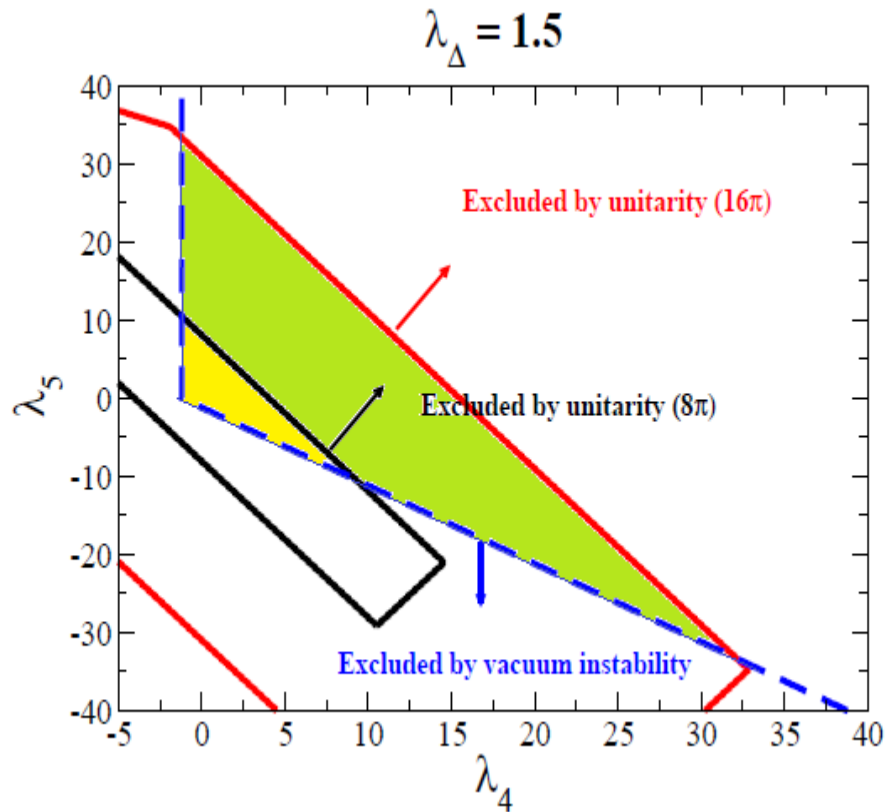
Theoretical bounds

Case for $\lambda_5 = 0$ ($\Delta m = 0$)



Negative values of λ_4 are constrained by the vacuum stability.

Theoretical bounds



When $\lambda_4 < 0$, negative values of λ_5 (Case of $m_{H^{++}} > m_{H^+} > m_A$) is highly constrained by the vacuum stability bound.

Custodial Symmetry

- The kinetic term can be written by the 2×2 matrix form of the Higgs doublet:

$$\Sigma = (\tilde{\Phi}, \Phi) = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^- & \phi^0 \end{pmatrix}$$

$$\mathcal{L}_{\text{kin}} = \frac{1}{2} \text{Tr} \left[(\tilde{D}_\mu \Sigma)^\dagger (\tilde{D}^\mu \Sigma) \right]$$

$$\tilde{D}_\mu \Sigma = \partial_\mu \Sigma + i \frac{g}{2} \tau \cdot W_\mu \Sigma - i \frac{g'}{2} B_\mu \Sigma \tau_3$$

- When g' is zero,

$\mathcal{L}_{\text{kin}}$ is invariant under the $SU(2)_L \times SU(2)_R$ transformation: $\Sigma \rightarrow U_L \Sigma U_R^\dagger$

- After the EWSB: $\Sigma \rightarrow \langle \Sigma \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

this symmetry is reduced to $SU(2)_L = SU(2)_R = SU(2)_V$ (custodial symmetry).

- Rho parameter: $\rho = m_W^2 / m_Z^2 \cos^2 \theta_W = 1$

$$\rho_{\text{exp}} = 1.0008^{+0.0017}_{-0.0007}$$

Custodial symmetry in the kinetic term maintains the electroweak rho parameter to be unity at the tree level.

Custodial Symmetry

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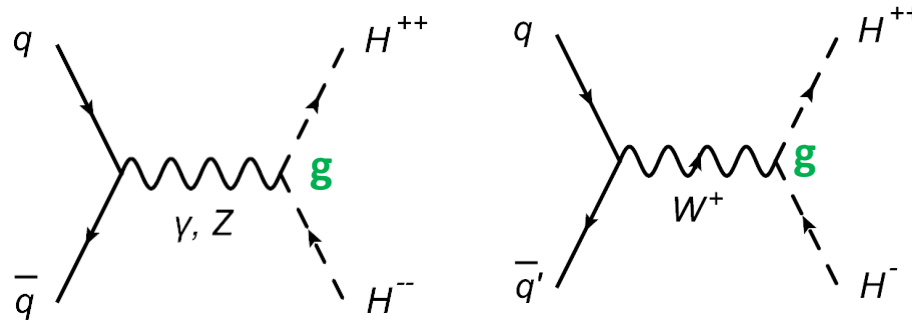
The data suggests that the Higgs sector should be custodial symmetric!

Production processes for H^{++}

Main production process

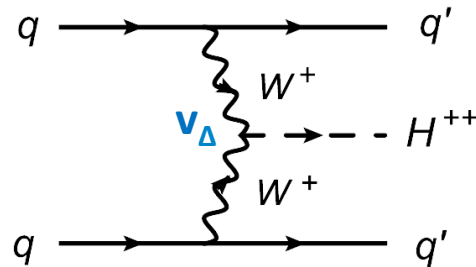
Drell-Yan

- depends on the gauge coupling



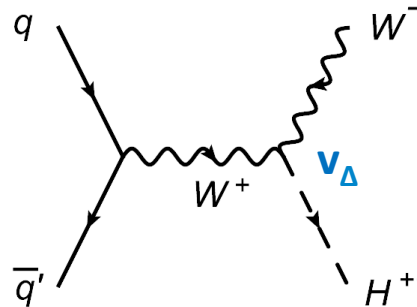
Vector Boson Fusion

- depends on $v_{\Delta} \rightarrow$ **Suppressed**



W associate

- depends on $v_{\Delta} \rightarrow$ **Suppressed**



Root(s) = 7 TeV, $v_\Delta = 8$ GeV, $\Delta m = 0$

