Partial Mass Degenerated Model and Spontaneous CP Violation in the Leptonic Sector

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Ref ; arXiv : 1211.4452 [hep-ph]

・Introduction

The Standard Model can properly predict many experimental data!

BUT some problems and unsatisfactory points have been pointed out.

For example,

- Neutrino masses
- Dark Matter candidate
- The origin of Baryon Asymmetry of the Universe
- Flavour Structure of Fermions
- etc…

⇨ The Standard Model should be extended by some way!

・Introduction

Especially, there is no guiding principle for flavour structure of fermions.

For explaining the origin of the flavour structure,
many scientists have employed Discrete Flavour Symmetries.

There are so many models of discrete flavour symmetries
but we have to be careful how to introduce such new symmetries.

For instance : A_4 , S_3 , S_4 , $\cdots$

Brief summary :
Ishimori et al. Prog.Theor.Phys.Suppl. 183 (2010)

Furthermore, flavour symmetries should be broken or hidden at low energy.

―What is the guidepost?―

• Motivation

One of the possible guide for searching such broken symmetries

⟹ 't Hooft's criterion (1980)

If a new symmetry appears in the model when arbitrary parameters set to zero,
such parameters are naturally small.

For example ; Lepton number symmetry

In the limit of massless neutrinos in the effective theory,
the lepton number symmetry appears.

$\frac{Y^2 v^2}{\Lambda_\nu} \bar{\nu^C} \nu$: this effective term violates lepton number 2 units.

• Motivation of Orthogonal Symmetries

Here, we take solar neutrino mass squared difference to be zero ; $\Delta m^2_{12} = 0$.

In this limit, the $O(2)$ symmetry appears in the neutrino sector :

$$\begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} m_D & & \\ & m_D & \\ & & m_3 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} m_D & & \\ & m_D & \\ & & m_3 \end{pmatrix} ,$$

In other words, the 1st and 2nd generation neutrinos are embedded into a doublet representation of $O(2)$, e.g., $\mathbf{2}_n$.

This type of mass spectra cannot apply to charged fermion sector, because they are strongly hierarchical between the 3rd generation and others.

For now, we assume $m^f_D \to 0$ $\quad M^f = \begin{pmatrix} 0 & & \\ & 0 & \\ & & m^f_3 \end{pmatrix}_{(f = u\,,\,d\,,\,e)}$

• Motivation of Orthogonal Symmetries

To construct realistic mass matrices, we introduce small breaking terms as

$$M^f = \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_3^f \end{pmatrix} + \delta^f \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}, \quad M^\nu = \begin{pmatrix} m_D & & \\ & m_D & \\ & & m_3 \end{pmatrix} + \delta^\nu \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$$

Small breaking parameters

<u>Charged fermion sector</u>

In this situation, both of 2-3 and 1-3 mixings depend on δ^f. $\boxed{\text{Small}}$

On the other hand, 1-2 one does not strongly depend on δ^f. $\boxed{\text{Need not so small}}$

⇨ Compatible with mixing pattern of CKM matrix

<u>Neutrino sector</u>

Because neutrino mass spectrum might be milder than charged fermion sector, the leading term is close to the identity matrix. $\boxed{\text{Weakly depend on } \delta^\nu}$

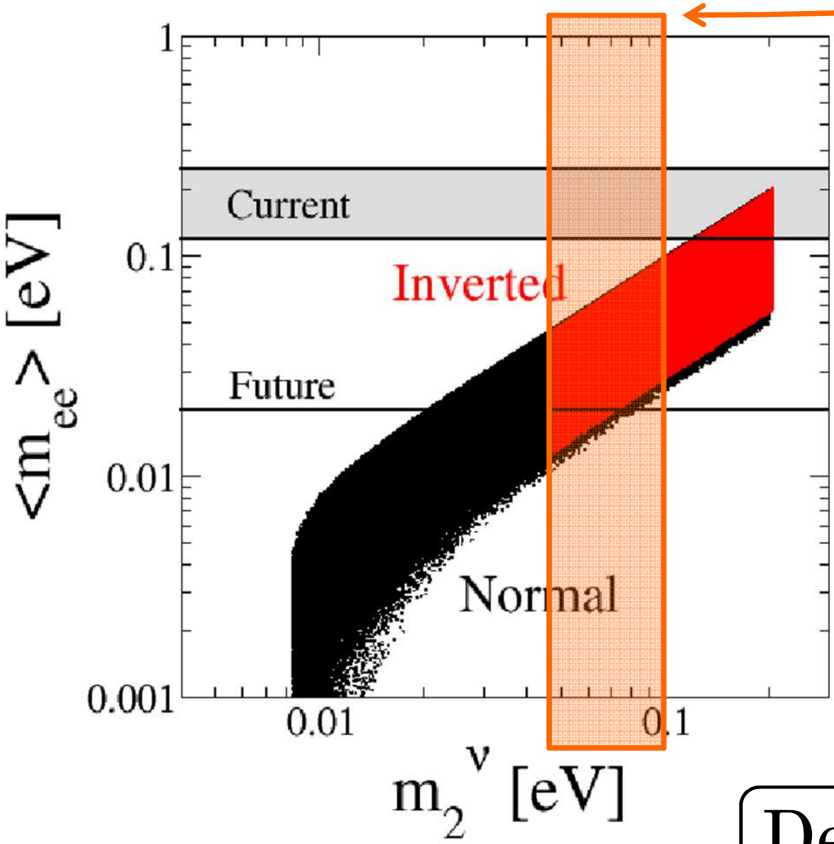
⇨ Large mixing in the PMNS matrix

Degeneracy of the 1st and 2nd generation seems to a good starting point.

• Motivation of degenerated masses

Neutrinoless double beta decay

If the neutrino masses are degenerated, e.g. $0.05\mathrm{eV} < m_2^\nu < 0.1\mathrm{eV}$,
the effective mass, $\langle m_{ee} \rangle$, can be estimated as



In both case, the effective mass is finite

$0\nu\beta\beta$ decay can be observed in near future?

OR

Such degenerated region is excluded?

$\boxed{\text{Degenerated region is very attractive!}}$

• D_N flavour symmetry

D_N flavour symmetry is a dihedral group and discrete subgroup of $O(2)$.

This symmetry has two singlet and four doublet irreducible representations.

- Multiplication rules

$$\underset{\mathbf{2}_n}{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}} \otimes \underset{\mathbf{2}_n}{\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}} = \underset{\mathbf{1}}{(x_1 y_1 + x_2 y_2)} + \underset{\mathbf{1}'}{(x_1 y_2 - x_2 y_1)} + \underset{\mathbf{2}_{2n}}{\begin{pmatrix} x_1 y_1 - x_2 y_2 \\ x_1 y_2 + x_2 y_1 \end{pmatrix}}$$

$$\underset{\mathbf{2}_n}{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}} \otimes \underset{\mathbf{2}_m}{\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}} = \underset{\mathbf{2}_{m-n}}{\begin{pmatrix} x_1 y_1 + x_2 y_2 \\ x_1 y_2 - x_2 y_1 \end{pmatrix}} + \underset{\mathbf{2}_{n+m}}{\begin{pmatrix} x_1 y_1 - x_2 y_2 \\ x_1 y_2 + x_2 y_1 \end{pmatrix}}$$

These extra charges are a little bit important for the model.

• Model

	L_I	L_3	ℓ_i	H	S_I
D_N	$\mathbf{2}_2$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}_1$

where $I = 1\,,2$ and $i = 1\,,2\,,3$ denote the indices of generations.

- L and ℓ represent the left- and right-handed leptons, respectively.
- H means SM Higgs field.
- S is SM gauge singlet scalar field.
- We denote the VEVs of scalar fields as $\langle H\rangle = v\,,\ \langle S_I\rangle = (s_1 e^{i\phi_1}\quad s_2 e^{i\phi_2})^T$.

• Model

Lagrangian invariant underD_N(NNLO)

$$\mathcal{L}_f = y_i^0\,\overline{L}_3 H \ell_i + \frac{y_i}{\Lambda_F^2}\overline{L}_I H \ell_i (S^2)_I + \frac{y_i^{'}}{\Lambda_F^2}\overline{L}_I H \ell_i (S^{*2})_I + \frac{y_i^{''}}{\Lambda_F^2}\overline{L}_I H \ell_i (|S|^2)_I$$
$$+ \frac{f_\nu}{\Lambda_\nu} L_I L_I H H + \frac{f'_\nu}{\Lambda_\nu} L_3 L_3 H H$$
$$+ \frac{g_\nu}{\Lambda_\nu \Lambda_F^2} L_3 L_I H H (S^2)_I + \frac{g'_\nu}{\Lambda_\nu \Lambda_F^2} L_3 L_I H H (S^{*2})_I + \frac{g_\nu^{''}}{\Lambda_\nu \Lambda_F^2} L_3 L_I H H (|S|^2)_I$$
$$+ \frac{h_\nu}{\Lambda_\nu \Lambda_F^4}(L_J L_K)_I H H (S^4)_I + \frac{h'_\nu}{\Lambda_\nu \Lambda_F^4}(L_J L_K)_I H H (S^{*4})_I + \frac{h_\nu^{''}}{\Lambda_\nu \Lambda_F^4}(L_J L_K)_I H H (|S|^4)_I$$
$$+ \frac{h_\nu^{'''}}{\Lambda_\nu \Lambda_F^4}(L_J L_K)_I H H (S^2|S|^2)_I + \frac{h_\nu^{''''}}{\Lambda_\nu \Lambda_F^4}(L_J L_K)_I H H (S^{*2}|S|^2)_I,$$

Froggatt-Nielsen like structure due to D_N

A typical energy scale of D_N symmetry

Dimension five Weinberg operator with an energy scale with a scale

Model

After scalar fields have vacuum expectation values,
mass matrices are given as follows：

$$\frac{1}{v}M^{\ell}=\begin{pmatrix}0&0&0\\0&0&0\\y_1^0&y_2^0&y_3^0\end{pmatrix}+\frac{1}{\Lambda_F^2}\left[\begin{pmatrix}y_1\delta_1&y_2\delta_1&y_3\delta_1\\y_1\delta_2&y_2\delta_2&y_3\delta_2\\0&0&0\end{pmatrix}+\begin{pmatrix}y_1'\delta_1^*&y_2'\delta_1^*&y_3'\delta_1^*\\y_1'\delta_2^*&y_2'\delta_2^*&y_3'\delta_2^*\\0&0&0\end{pmatrix}+\begin{pmatrix}y_1^{''}\delta_a&y_2^{''}\delta_a&y_3^{''}\delta_a\\y_1^{''}\delta_b&y_2^{''}\delta_b&y_3^{''}\delta_b\\0&0&0\end{pmatrix}\right],$$

$$\frac{\Lambda_\nu}{v^2}\mathcal{M}_\nu=\begin{pmatrix}f_\nu&0&0\\0&f_\nu&0\\0&0&f_\nu'\end{pmatrix}+\frac{1}{\Lambda_F^2}\begin{pmatrix}0&0&g_\nu\delta_1+g_\nu'\delta_1^*+g_\nu^{''}\delta_a\\0&0&g_\nu\delta_2+g_\nu'\delta_2^*+g_\nu^{''}\delta_b\\g_\nu\delta_1+g_\nu'\delta_1^*+g_\nu^{''}\delta_a&g_\nu\delta_2+g_\nu'\delta_2^*+g_\nu^{''}\delta_b&0\end{pmatrix}$$

$$+\frac{1}{\Lambda_F^4}\begin{pmatrix}h_\nu\epsilon_1+h_\nu'\epsilon_1^*+h_\nu^{''}\epsilon_a+h_\nu^{'''}\epsilon_x+h_\nu^{''''}\epsilon_x^*&h_\nu\epsilon_2+h_\nu'\epsilon_2^*+h_\nu^{''}\epsilon_b+h_\nu^{'''}\epsilon_y+h_\nu^{''''}\epsilon_y^*&0\\h_\nu\epsilon_2+h_\nu'\epsilon_2^*+h_\nu^{''}\epsilon_b+h_\nu^{'''}\epsilon_y+h_\nu^{''''}\epsilon_y^*&-(h_\nu\epsilon_1+h_\nu'\epsilon_1^*+h_\nu^{''}\epsilon_a+h_\nu^{'''}\epsilon_x+h_\nu^{''''}\epsilon_x^*)&0\\0&0&0\end{pmatrix},$$

where, δs and ϵs are some functions of s_i and ϕ_i . ($\langle S_I\rangle=(s_1e^{i\phi_1}\quad s_2e^{i\phi_2})^T$.)

There are so many parameters = No prediction.

But we note that there is no strong hierarchy among each couplings.

• Feature of mass matrices

This model has a relationship between electron mass and $\theta_{13}^{\mathrm{PMNS}}$.

When $\phi_2 = \phi_1 + \delta\phi \ (\delta\phi \ll 1)$,

the charged lepton mass matrix can be rewritten as

$$\begin{pmatrix} \left[y_1 e^{2i\phi_1} + y_1' e^{-2i\phi_1} + y_1''\right](s_1^2 - s_2^2) & \cdots & \cdots \\ \left[y_1 e^{2i\phi_1} + y_1' e^{-2i\phi_1} + y_1''\right] 2 s_1 s_2 & \cdots & \cdots \\ 0 & \cdots & \cdots \end{pmatrix} + i\delta\phi \begin{pmatrix} -\left[y_1 e^{2i\phi_1} - y_1' e^{-2i\phi_1}\right] 2 s_2^2 & \cdots & \cdots \\ \left[y_1 e^{2i\phi_1} - y_1' e^{-2i\phi_1}\right] 2 s_1 s_2 & \cdots & \cdots \\ 0 & \cdots & \cdots \end{pmatrix} ,$$

Each term is rank = 1 matrix.

Correspond to muon mass　　　Correspond to electron mass

Small electron mass is induced by spontaneous CP violation, that is, non-vanishing $\delta\phi$.

• Feature of mass matrices

Recall the mass matrix for the charged lepton.

$$M^\ell \supset \begin{pmatrix} Y_1(s_1^2 - s_2^2) & Y_2(s_1^2 - s_2^2) & Y_3(s_1^2 - s_2^2) \\ Y_1 2 s_1 s_2 & Y_2 2 s_1 s_2 & Y_3 2 s_1 s_2 \\ 0 & 0 & 0 \end{pmatrix} + i\delta\phi \begin{pmatrix} -Y_1' 2 s_2^2 & -Y_2' 2 s_2^2 & -Y_3' 2 s_2^2 \\ Y_1' 2 s_1 s_2 & Y_2' 2 s_1 s_2 & Y_3' 2 s_1 s_2 \\ 0 & 0 & 0 \end{pmatrix},$$

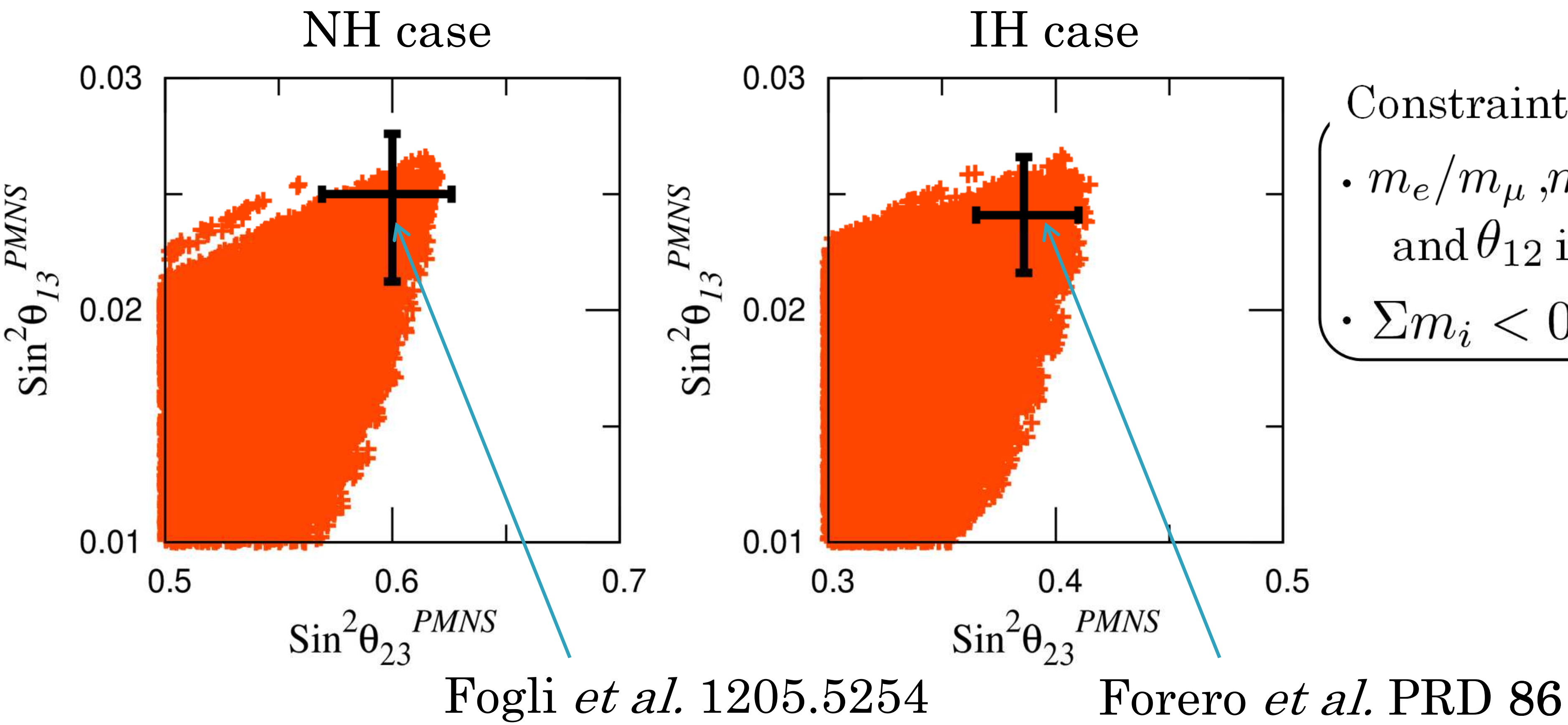
Let's move onto the basis in which upper-left 2×2 elements of $M^\ell M^{\ell\dagger}$ are diagonalized.

In this basis, the 1-3 element of the neutrino mass matrix can be rewritten as

$$(\mathcal{M})_{13} \propto i\delta\phi \left\{ \left[g_\nu e^{2i\phi_1} - g_\nu' e^{-2i\phi_1} \right] 2 s_1 s_2^3 \cdots \right\}.$$

Therefore, $\theta_{13}^{\mathrm{PMNS}}$ is controlled by $\delta\phi$.

• Numerical Analysis (θ_{13}-θ_{23} plane)



Constraints

- m_e/m_μ ,m_μ/m_τ and θ_{12} in 1σ range
- $\Sigma m_i < 0.6\mathrm{eV}$

Nearly maximal θ_{23} and relatively large θ_{13} are successfully obtained.

• Possibility of Spontaneous CP Violation

We assume that the singlet scalar fields were completely decoupled from the theory at high energy.

➡ We investigate only the potential of the singlet scalar

Scalar potential

$$
\begin{aligned}
V_S = {} & \alpha_S(s_1^2+s_2^2)+\alpha_S'(s_1^2\cos 2\phi_1+s_2^2\cos 2\phi_2)+\beta_S^a(s_1^2+s_2^2)^2 \\
& -4\beta_S^b s_1^2 s_2^2\sin^2(\phi_1-\phi_2)+\beta_S^c\{(s_1^2-s_2^2)^2+4s_1^2s_2^2\cos^2(\phi_1-\phi_2)\} \\
& +\beta_S'\left[s_1^4\cos 4\phi_1+s_2^4\cos 4\phi_2+2s_1^2s_2^2\cos[2(\phi_1+\phi_2)]\right] \\
& +\gamma_S(s_1^2+s_2^2)(s_1^2\cos 2\phi_1+s_2^2\cos 2\phi_2)\,.
\end{aligned}
$$

• Possibility of Spontaneous CP Violation

The minimization conditions respect with the phases are given as

$$\frac{\partial V_S}{\partial \phi_1} = -2s_1^2 \left[\ \left\{\alpha'_S + \gamma_S \left(s_1^2 + s_2^2\right)\right\} \sin 2\phi_1 + 2(\beta_S^b + \beta_S^c) s_2^2 \sin\left[2\left(\phi_1 - \phi_2\right)\right] \right.$$

$$\left. + 2\beta'_S \left(s_1^2 \sin 4\phi_1 + s_2^2 \sin\left[2\left(\phi_1 + \phi_2\right)\right]\right) \ \right] = 0\,,$$

$$\frac{\partial V_S}{\partial \phi_2} = -2s_2^2 \left[\ \left\{\alpha'_S + \gamma_S \left(s_1^2 + s_2^2\right)\right\} \sin 2\phi_2 - 2(\beta_S^b + \beta_S^c) s_1^2 \sin\left[2\left(\phi_1 - \phi_2\right)\right] \right]$$

$$\left. + 2\beta'_S \left(s_2^2 \sin 4\phi_2 + s_1^2 \sin\left[2\left(\phi_1 + \phi_2\right)\right]\right) \ \right] = 0\,.$$

For simplicity, we set $\alpha'_S = \beta'_S = \gamma_S = 0$.
Then we get,

$$2(\beta_S^b + \beta_S^c) s_2^2 \sin[2(\phi_1 - \phi_2)] = 0, \quad 2(\beta_S^b + \beta_S^c) s_1^2 \sin[2(\phi_1 - \phi_2)] = 0.$$

These condition suppose $\phi_1 = \phi_2$.

Conclusions

- We discuss a model in which 1st and 2nd generations are degenerated motivated by $\Delta m^2_{12} = 0$.
  - In this model, mixing patterns of quarks and leptons are realized naturally due to dependence on small breaking parameter δ.
- To illustrate such a degenerate model, we consider a D_N flavor symmetric model which is a discrete subgroup of $O(2)$.
  - We introduce one new D_N doublet and SM gauge singlet scalar, S_I .
- Due to spontaneous CP violation, small electron mass and relatively large $\theta_{13}^{\mathrm{PMNS}}$ are obtained.
- From numerical analysis, we find that it is possible to construct this model with only one D_N doublet scalar in both neutrino mass hierarchy cases and without strongly hierarchical Yukawa couplings.

• Future Works

- Applying to phenomenological problems
  - CP violation
    - Baryon asymmetry of the Universe
    - Unitarity triangle in the quark sector …and so on.

- Including a realistic neutrino mass generation mechanism in this type of model

- Searching the possible energy scale of the D_N symmetry
  - If the D_N scale is around $\mathcal{O}(\mathrm{TeV})$, the extra scalar fields might contribute to FCNC.