

Collider Signatures of Gauge-Higgs Unification at LHC



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2/11/2015 HPNP2015@Toyama

References

- Diphoton and Z photon Decays of Higgs Boson
in Gauge-Higgs Unification:
A Snowmass white paper, arXiv: 1307:8181
- $H \rightarrow Z\gamma$ in Gauge-Higgs Unification,
PRD88 037701 (2013)
- Diphoton Decay Excess and 125 GeV Higgs Mass
in Gauge-Higgs Unification,
PRD87 095019 (2013)
- Gauge-Higgs Unification at CERN LHC,
PRD77 055010 (2008)

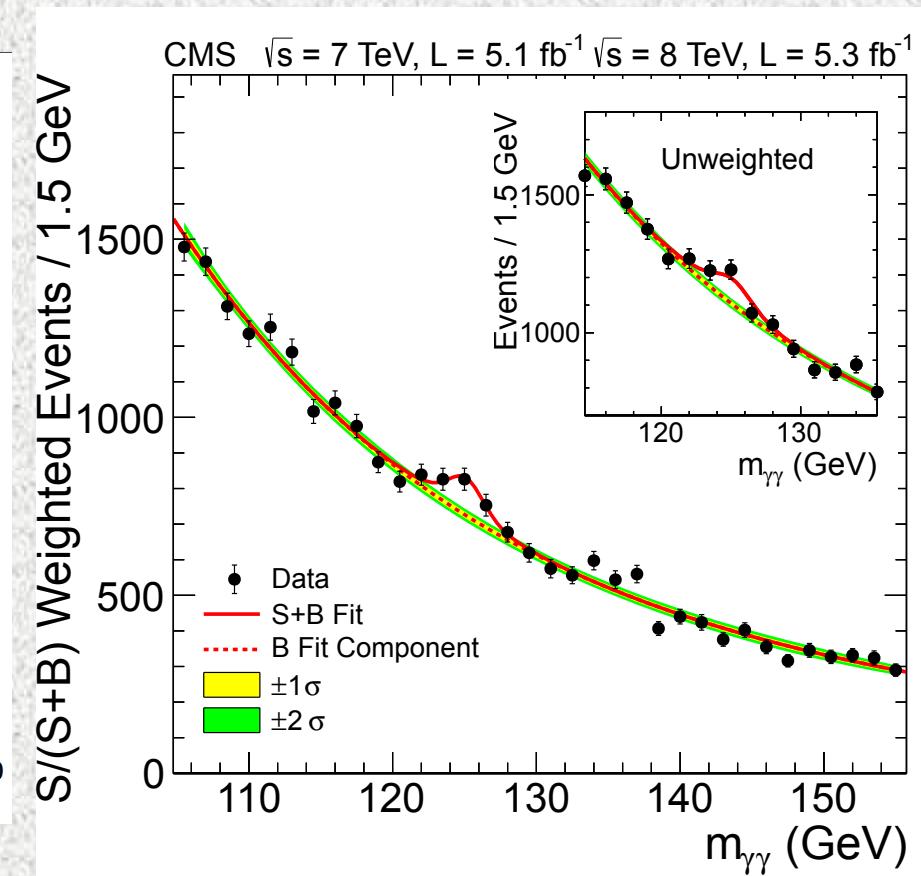
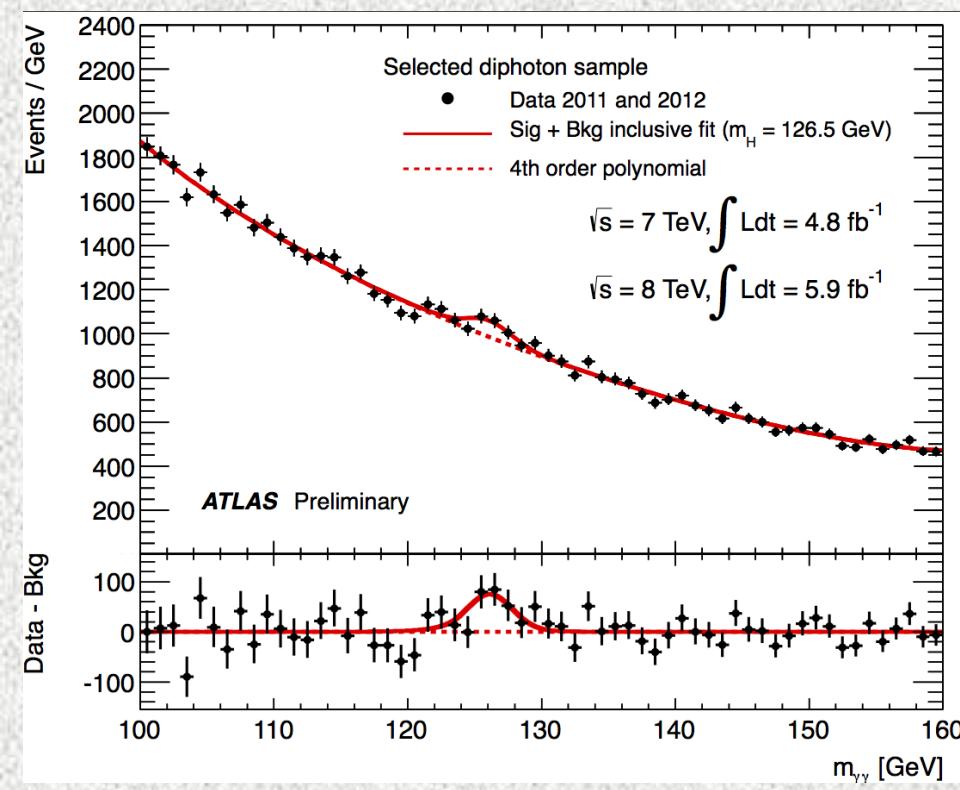
All papers with Nobuchika Okada

PLAN

- Introduction
- A Model of GHU
- $gg \rightarrow H \not\rightarrow H \rightarrow \gamma\gamma$ in GHU
- $H \rightarrow Z\gamma$ in GHU
- Summary

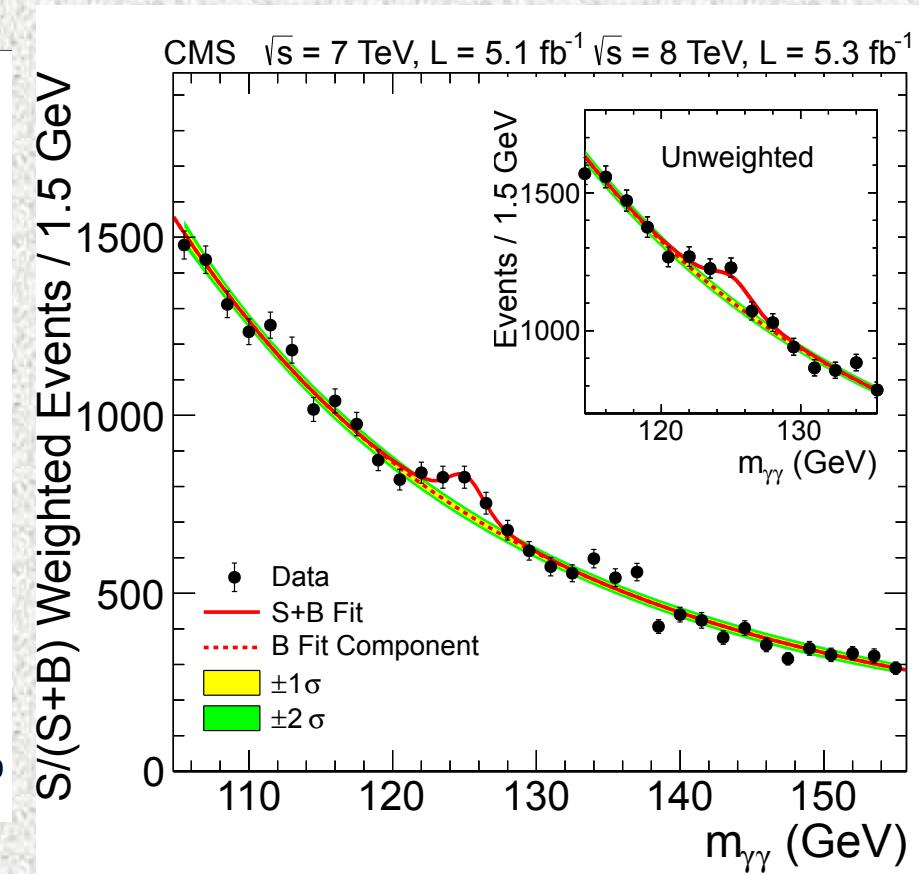
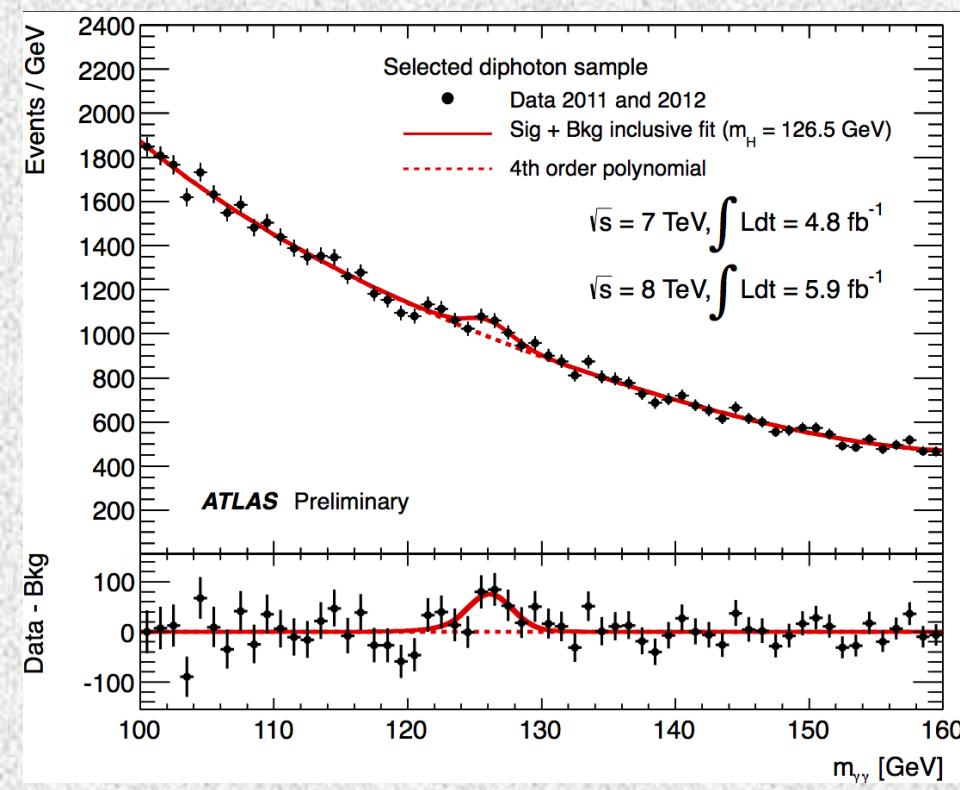
Introduction

A Higgs was discovered, but...



Introduction

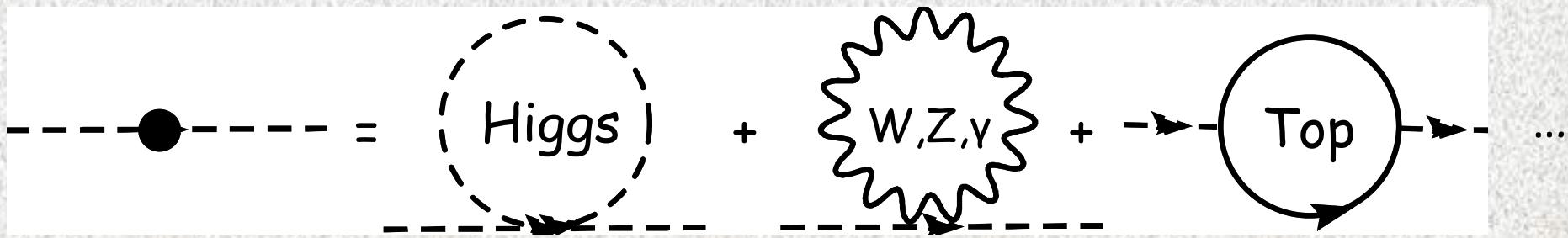
Still unclear, the origin of Higgs ??



Motivation to consider GHU

One of the problems in the Standard Model:
Hierarchy Problem

Quantum corrections to Higgs mass
is sensitive to the cutoff scale of the theory



$$\delta m_H^2 \approx \frac{\Lambda^2}{16\pi^2}$$

Too large!
(Natural cutoff scale is
Planck scale or GUT scale)

In GHU, the SM Higgs is identified with the 5th component of the 5D gauge field

Gauge trf.

$$\therefore A_5 \rightarrow A_5 + \partial_5 \varepsilon(x, x_5) + i [\varepsilon(x, x_5), A_5]$$

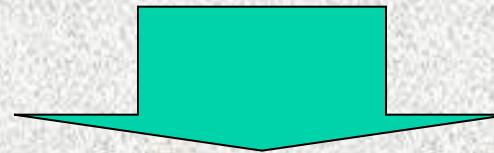
forbids a local Higgs mass term $(A_5)^2$

⇒ No quadratic divergence, finite regardless of the nonrenormalizability

$$m_{A_5}^2 \simeq \frac{1}{16\pi^2} \frac{1}{R^2}$$

In the gauge-Higgs unification,

- 1: New structure in the Higgs sector
- 2: Coupling of new particles to Higgs boson controlled by higher dimensional gauge invariance



Deviations from the SM predictions &
Collider signatures specific to GHU
are expected!!

A Model of GHU

Lagrangian

5D $SU(3) \times U(1)'$ model on S^1/Z_2

Scrucca, Serone, Silvestrini (2003), Cacciapaglia, Csaki, Park (2006)

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} \text{Tr} \left(F_{MN} F^{MN} \right) + \bar{\Psi}_3^{i=1,2,3} \left(i \Gamma^M D_M - M_d^i \mathcal{E}(y) \right) \Psi_3^{i=1,2,3} \\ & + \bar{\Psi}_{\bar{6}}^{i=1,2} \left(i \Gamma^M D_M - M_u^i \mathcal{E}(y) \right) \Psi_{\bar{6}}^{i=1,2} + \bar{\Psi}_{\bar{15}} i \Gamma^M D_M \Psi_{\bar{15}} \\ & + \bar{\Psi}_{10}^{i=1,2,3} \left(i \Gamma^M D_M - M_l^i \mathcal{E}(y) \right) \Psi_{10}^{i=1,2,3} \quad \Gamma^M = (\gamma^\mu, i \gamma^5) \end{aligned}$$

Boundary conditions: S^1 : $\Psi(y+2\pi R) = \psi(y)$, Z_2 : $\Psi(-y) = \pm \Psi(y)$

$$A_\mu = \begin{pmatrix} (+,+) & (+,+) & (-,-) \\ (+,+) & (+,+) & (-,-) \\ (-,-) & (-,-) & (+,+) \end{pmatrix}, A_5 = \begin{pmatrix} (-,-) & (-,-) & (+,+) \\ (-,-) & (-,-) & (+,+) \\ (+,+) & (+,+) & (-,-) \end{pmatrix}$$

Lagrangian

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Boundary conditions:

(+,+) only has
massless mode

(+,+): $\cos(ny/R)$
(-,-): $\sin(ny/R)$

$$A_\mu = \begin{pmatrix} (+,+) & (+,+) & (-,-) \\ (+,+) & (+,+) & (-,-) \\ (-,-) & (-,-) & (+,+) \end{pmatrix}, A_5 = \begin{pmatrix} (-,-) & (-,-) & (+,+) \\ (-,-) & (-,-) & (+,+) \\ (+,+) & (+,+) & (-,-) \end{pmatrix}$$

$SU(3) \times U(1)' \rightarrow SU(2) \times U(1)_Y \times U(1)_X$

Lagrangian

5D $SU(3) \times U(1)'$ model on S^1/Z_2

Scrucca, Serone, Silvestrini (2003), Cacciapaglia, Csaki, Park (2006)

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} \text{Tr} \left(F_{MN} F^{MN} \right) + \bar{\Psi}_3^{i=1,2,3} \left(i \Gamma^M D_M - M_d^i \mathcal{E}(y) \right) \Psi_3^{i=1,2,3} \\ & + \bar{\Psi}_{\bar{6}}^{i=1,2} \left(i \Gamma^M D_M - M_u^i \mathcal{E}(y) \right) \Psi_{\bar{6}}^{i=1,2} + \bar{\Psi}_{\bar{15}} i \Gamma^M D_M \Psi_{\bar{15}} \\ & + \bar{\Psi}_{10}^{i=1,2,3} \left(i \Gamma^M D_M - M_l^i \mathcal{E}(y) \right) \Psi_{10}^{i=1,2,3} \quad \Gamma^M = (\gamma^\mu, i \gamma^5) \end{aligned}$$

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0 mode of A_5 = SM Higgs

Lagrangian

5D $SU(3) \times U(1)'$ model on S^1/Z_2

Scrucca, Serone, Silvestrini (2003), Cacciapaglia, Csaki, Park (2006)

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} \text{Tr} \left(F_{MN} F^{MN} \right) + \bar{\Psi}_3^{i=1,2,3} \left(i \Gamma^M D_M - M_d^i \mathcal{E}(y) \right) \Psi_3^{i=1,2,3} \\ & + \bar{\Psi}_{\bar{6}}^{i=1,2} \left(i \Gamma^M D_M - M_u^i \mathcal{E}(y) \right) \Psi_{\bar{6}}^{i=1,2} + \bar{\Psi}_{\bar{15}} i \Gamma^M D_M \Psi_{\bar{15}} \\ & + \bar{\Psi}_{10}^{i=1,2,3} \left(i \Gamma^M D_M - M_l^i \mathcal{E}(y) \right) \Psi_{10}^{i=1,2,3} \quad \Gamma^M = (\gamma^\mu, i\gamma^5) \end{aligned}$$

Boundary conditions:

(+,+) only has massless mode

(+,+): $\cos(ny/R)$
(-,-): $\sin(ny/R)$

$$A_\mu^{(0)} = \frac{1}{2} \begin{pmatrix} W_\mu^3 + B_\mu^3 / \sqrt{3} & \sqrt{2} W_\mu^+ & 0 \\ \sqrt{2} W_\mu^- & -W_\mu^3 + B_\mu^3 / \sqrt{3} & 0 \\ 0 & 0 & -2B_\mu / \sqrt{3} \end{pmatrix}, A_5^{(0)} = \frac{1}{2} \begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H^0 \\ H^- & H^{0*} & 0 \end{pmatrix}$$

$SU(2) \times U(1)$ gauge fields

Higgs doublet

Fermion matter content

Scrucca, Serone, Silvestrini (2003), Cacciapaglia, Csaki, Park (2006)

$$3 = \mathbf{2}_{L1/6}(Q) + \mathbf{1}_{L-1/3} \\ \mathbf{2}_{R1/6} + \mathbf{1}_{R-1/3}(d_R)$$

Down quark sector
(d, s, b)

$$6^* = \mathbf{3}_{L-1/3} + \mathbf{2}_{L1/6}(Q) + \mathbf{1}_{L2/3} \\ \mathbf{3}_{R-1/3} + \mathbf{2}_{R1/6} + \mathbf{1}_{R2/3}(u_R)$$

Up quark sector
(u, c)

$$10 = \mathbf{4}_{L1/2} + \mathbf{3}_{L0} + \mathbf{2}_{L-1/2}(L) + \mathbf{1}_{L-1} \\ \mathbf{4}_{R1/2} + \mathbf{3}_{R0} + \mathbf{2}_{R-1/2} + \mathbf{1}_{R-1}(e_R)$$

Charged lepton
sector (e, μ, τ)

$$15^* = \mathbf{5}_{L-4/3} + \mathbf{4}_{L-5/6} + \mathbf{3}_{L-1/3} + \mathbf{2}_{L1/6}(Q) + \mathbf{1}_{L2/3} \\ \mathbf{5}_{R-4/3} + \mathbf{4}_{R-5/6} + \mathbf{3}_{R-1/3} + \mathbf{2}_{R1/6} + \mathbf{1}_{R2/3}(t_R)$$

Top
quark

Unwanted massless exotics (blue reps) & a half of extra Qs
must be massive by brane localized mass terms

Fermion matter content

Scrucca, Serone, Silvestrini (2003), Cacciapaglia, Csaki, Park (2006)

$$3 = \mathbf{2}_{L1/6}(Q) + \mathbf{1}_{L-1/3}$$

$$2_{R1/6} + \mathbf{1}_{R-1/3}(d_R)$$

Down quark sector
(d, s, b)

$$6^* = \mathbf{2}_{L1/6}(Q) + \mathbf{1}_{L2/3}$$

$$\mathbf{3}_{R-1/3} + \mathbf{2}_{R1/6} + \mathbf{1}_{R2/3}(u_R)$$

Up quark sector
(u, c)

$$10 = \mathbf{4}_{L1/2} + \mathbf{3}_{R0} + \mathbf{2}_{L-1/2}(L) + \mathbf{1}_{L+1}$$

$$4_{R1/2} + \mathbf{3}_{R0} + \mathbf{2}_{R-1/2} + \mathbf{1}_{R-1}(e_R)$$

Charged lepton
sector (e, μ, τ)

$$15^* = \mathbf{5}_{L-4/3} + \mathbf{4}_{L-5/6} + \mathbf{3}_{L-1/3} + \mathbf{2}_{L1/6}(Q) + \mathbf{1}_{L2/3}$$

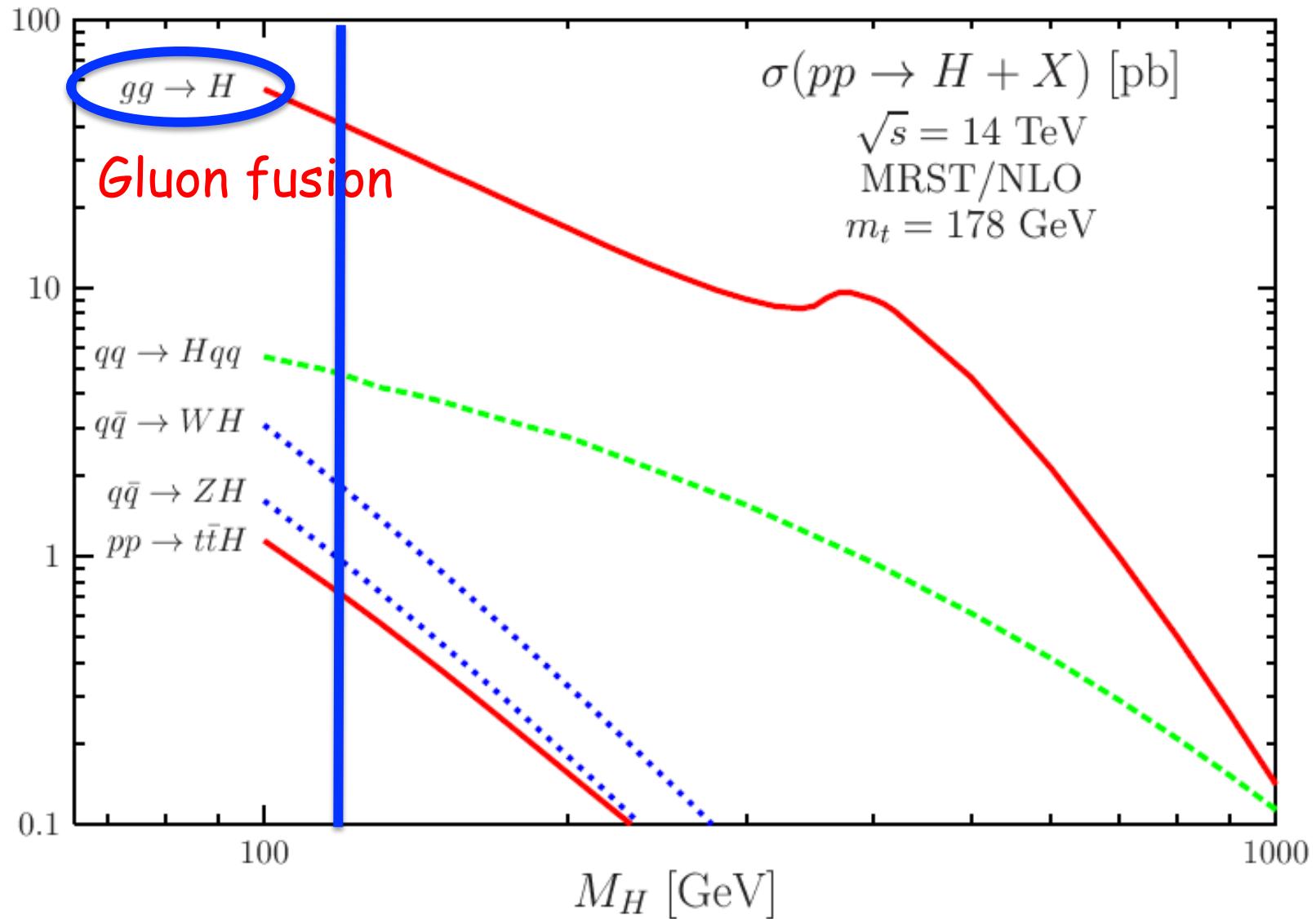
$$5_{R-4/3} + 4_{R-5/6} + \mathbf{3}_{R-1/3} + 2_{R1/6} + \mathbf{1}_{R2/3}(t_R)$$

Top
quark

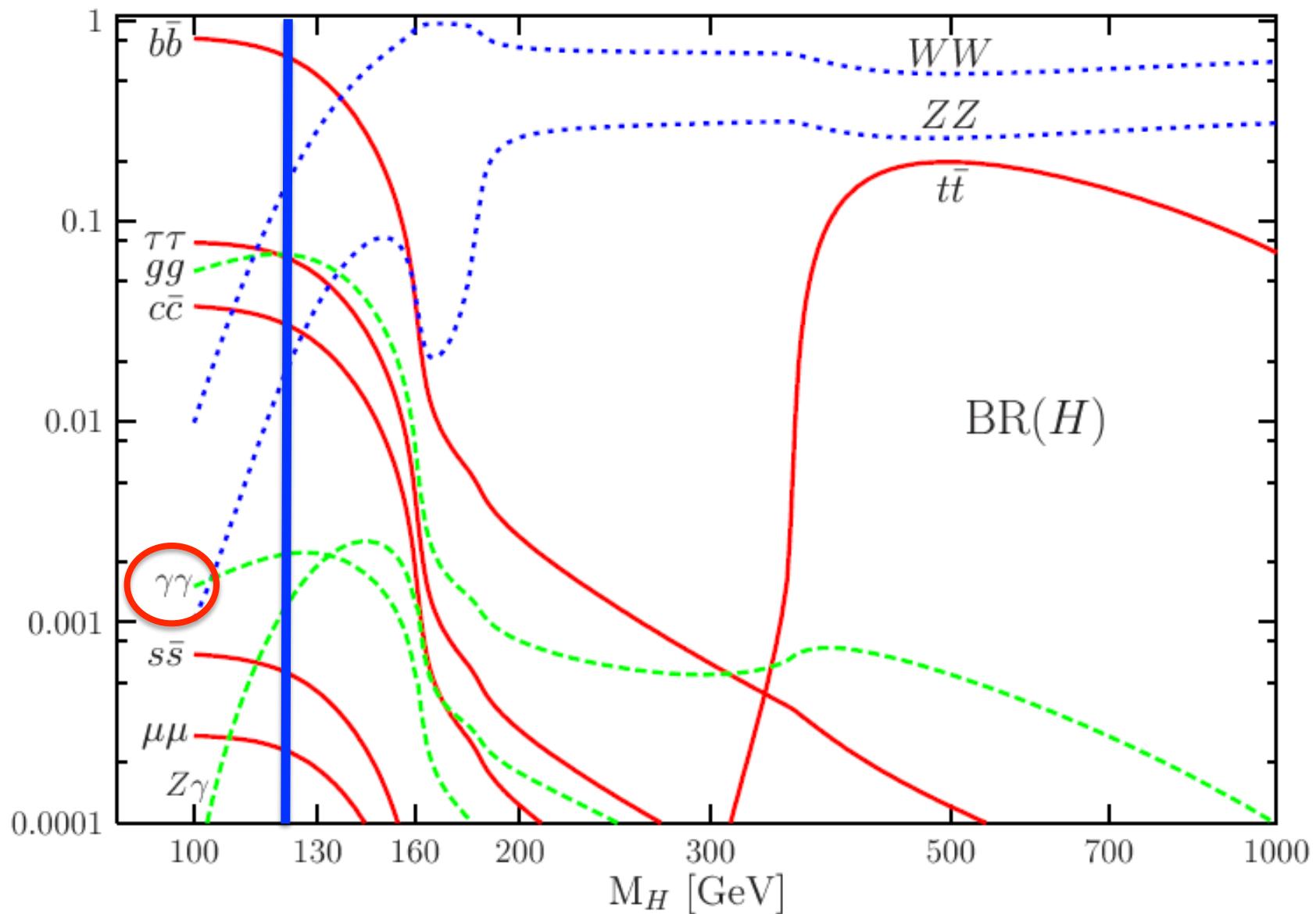
Unwanted massless exotics (blue reps) & a half of extra Qs
must be massive by brane localized mass terms

$gg \rightarrow H \rightarrow \gamma\gamma$ in GHU

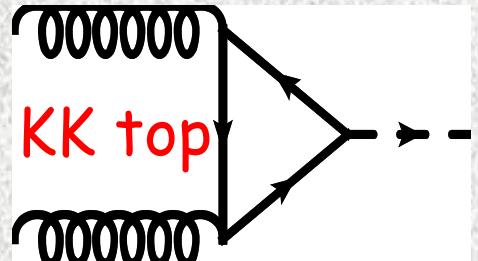
Higgs production



Decay rate of Higgs boson



KK mode contributions: $gg \rightarrow H$



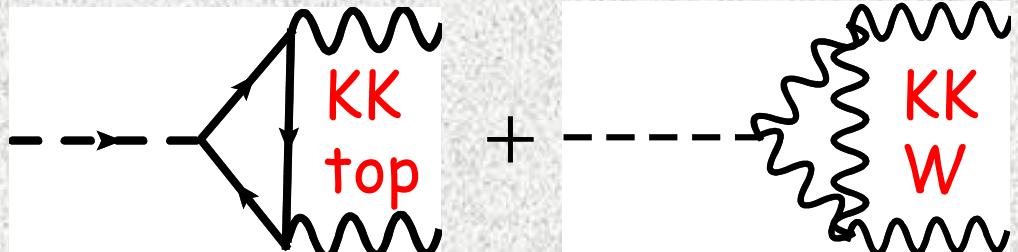
$$\mathcal{L}_{eff} = C_{gg}^{KK} h G^{a\mu\nu} G^a_{\mu\nu}$$

$$\begin{aligned}
 C_{gg}^{KKtop} &= \frac{\alpha_s}{8\pi\nu} \frac{2}{3} \sum_{n=1}^{\infty} \frac{\partial}{\partial \ln \nu} \left[\ln(m_n + m_t) + \ln(m_n - m_t) \right] \\
 &= \frac{\alpha_s}{12\pi\nu} \sum_{n=1}^{\infty} \left[\frac{m_t}{m_n + m_t} - \frac{m_t}{m_n - m_t} \right] \quad \text{log}^{\infty} - \text{log}^{\infty} \\
 &\approx -\frac{\alpha_s}{12\pi\nu} 2 \sum_{n=1}^{\infty} \frac{m_t^2}{m_n^2} \left(m_t^2 \ll m_n^2 \right) = -\frac{\alpha_s}{12\pi\nu} \frac{1}{3} (\pi m_t R)^2
 \end{aligned}$$

Opposite sign to SM \Rightarrow destructive

KK mode contributions: $H \rightarrow \gamma\gamma$

$$\mathcal{L}_{eff} = C_{\mathcal{W}}^{KK} h F^{\mu\nu} F_{\mu\nu}$$



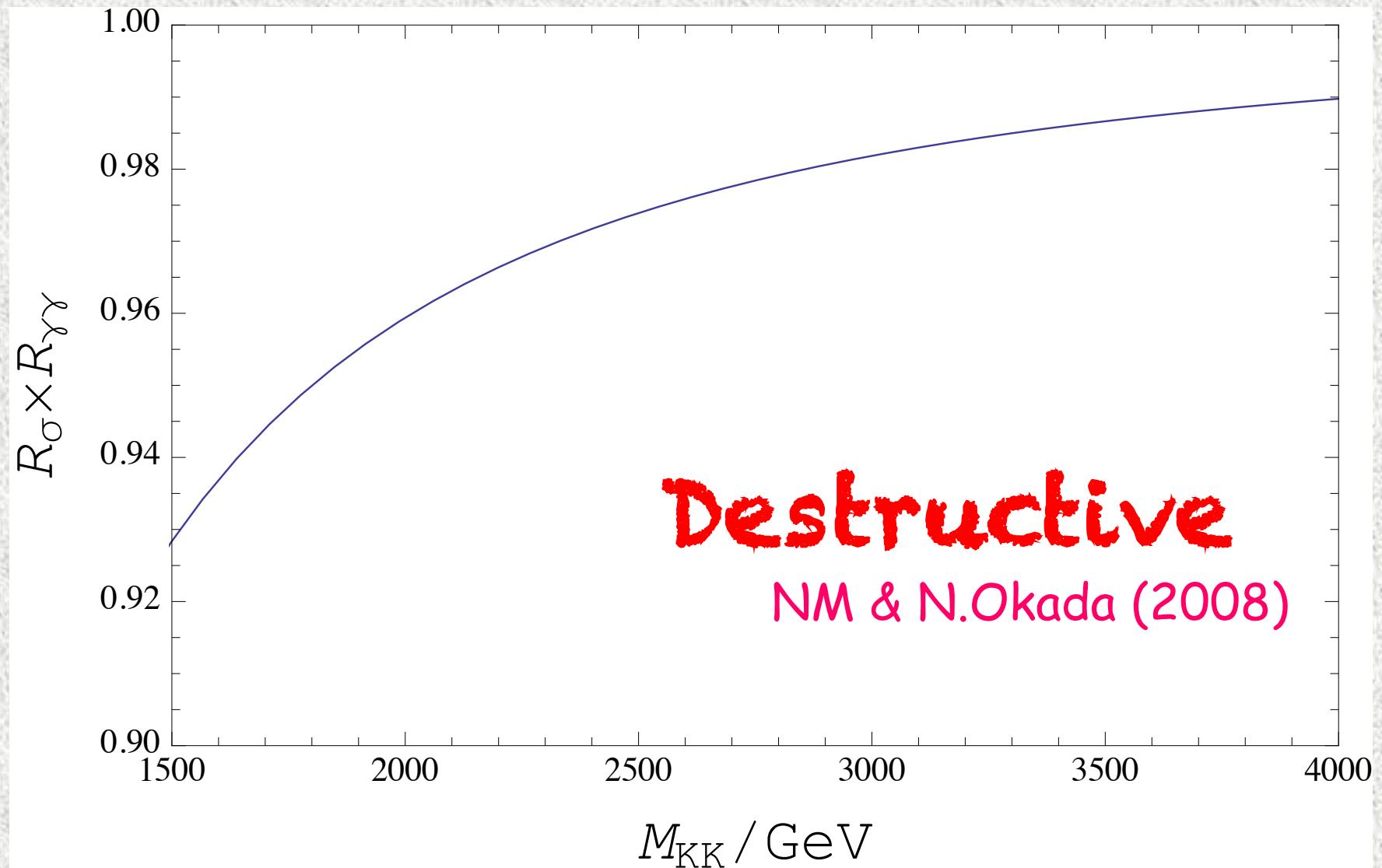
$$C_{\mathcal{W}}^{KKtop} = \frac{\alpha_{em}}{6\pi\nu} \frac{4}{3} \sum_{n=1}^{\infty} \frac{\partial}{\partial \ln \nu} \left[\ln(m_n + m_t) + \ln(m_n - m_t) \right]$$

$$\approx -\frac{2\alpha_{em}}{9\pi\nu} \frac{1}{3} (\pi m_t R)^2 \quad \text{Opposite sign to SM}$$

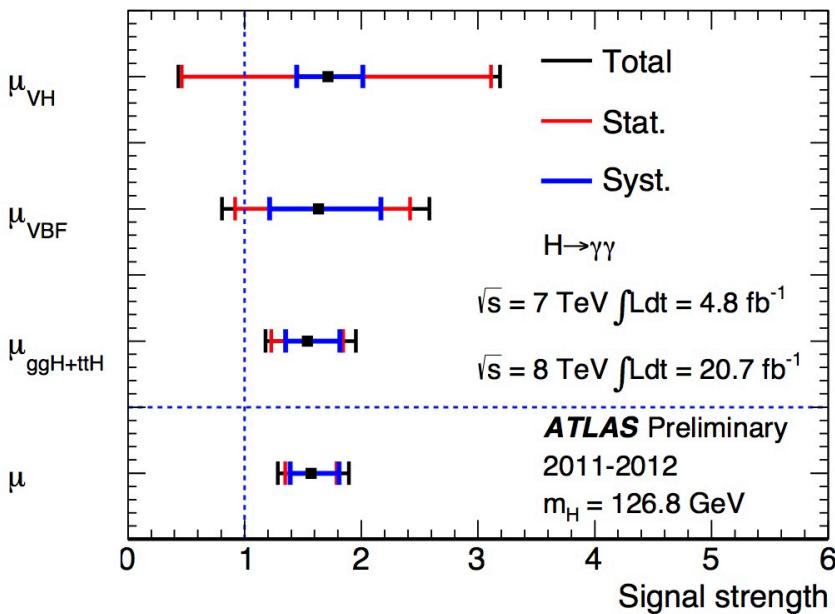
$$C_{\mathcal{W}}^{KKW} = \frac{\alpha_{em}}{8\pi\nu} (-7) \sum_{n=1}^{\infty} \frac{\partial}{\partial \ln \nu} \left[\ln(m_n + m_W) + \ln(m_n - m_W) \right]$$

$$\approx +\frac{7\alpha_{em}}{8\pi\nu} \frac{1}{3} (\pi m_W R)^2 \quad \text{Opposite sign to SM}$$

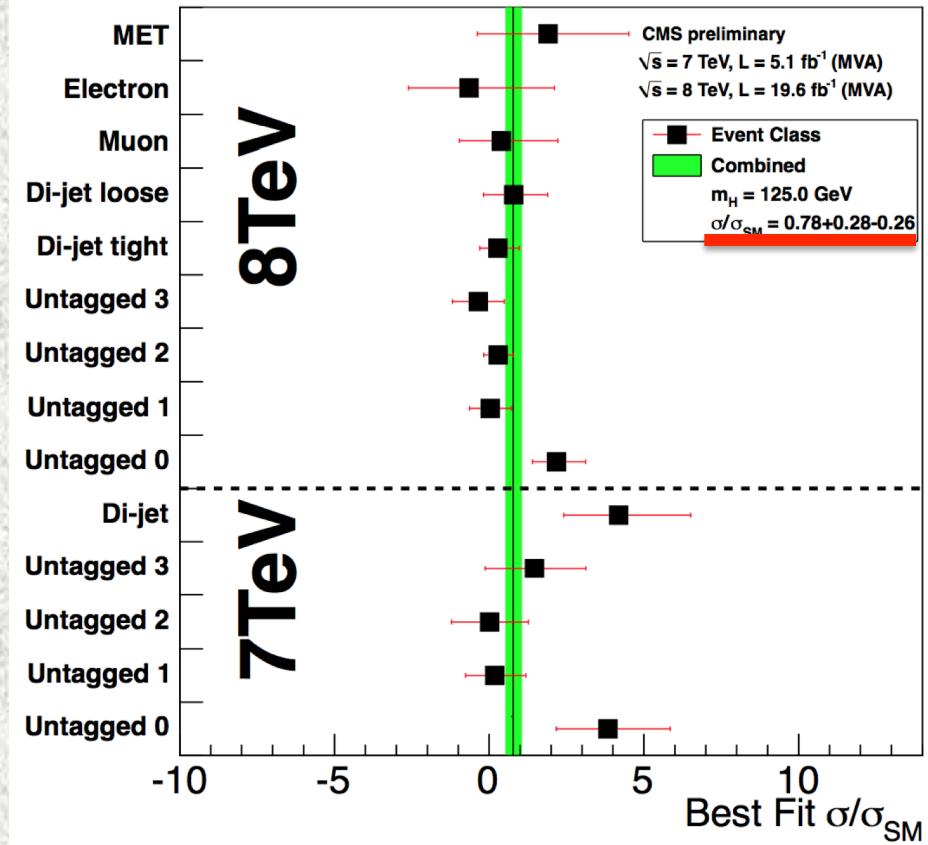
$$(gg \rightarrow H \rightarrow \gamma\gamma)_{GHU} / (gg \rightarrow H \rightarrow \gamma\gamma)_{SM}$$



Diphoton decay excess



$$\sigma/\sigma_{SM} = 1.57 \pm 0.22(\text{stat}) + 0.24 - 0.18(\text{syst})$$



Extension is required

One of the simplest extensions:

“extra leptons”

(colored particles greatly affect the gluon fusion)

NM & N.Okada, PRD87 (2013) 095019

Contributions of KK fermions to $H \rightarrow \gamma\gamma$ constructive

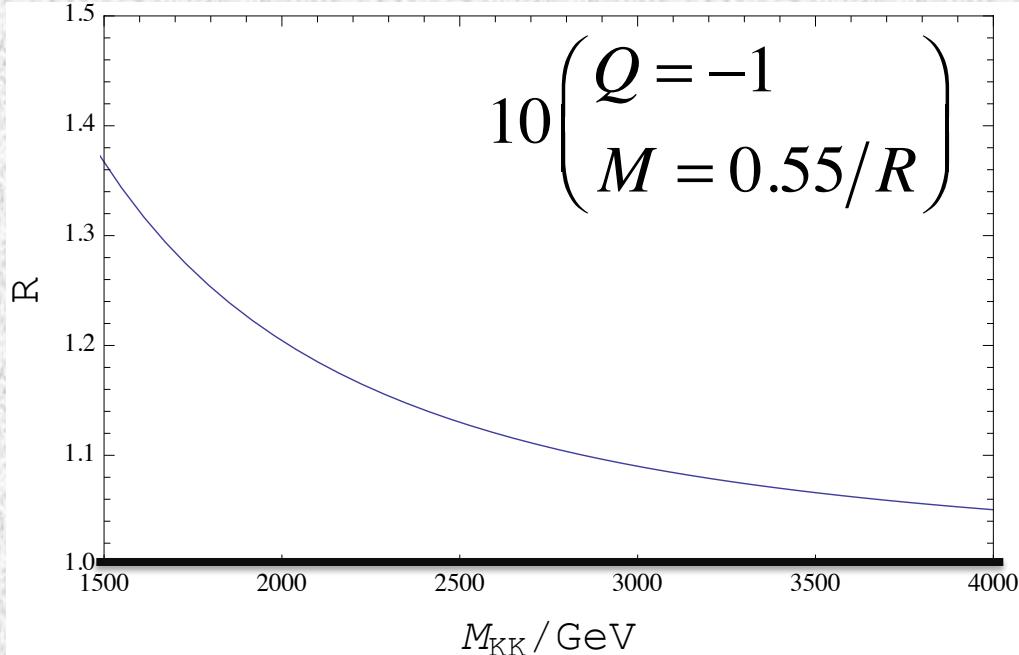
$$\therefore C_{\gamma\gamma}^{\text{SM}} < 0 \text{ & } C_{\gamma\gamma}^{\text{KK fermions}} < 0 \Rightarrow C_{\gamma\gamma}^{\text{KK fermions}} / C_{\gamma\gamma}^{\text{SM}} > 0$$

Two examples: **10, 15** plets of SU(3) with bulk mass
& half-periodic BC $\psi(y+2\pi R) = -\psi(y)$

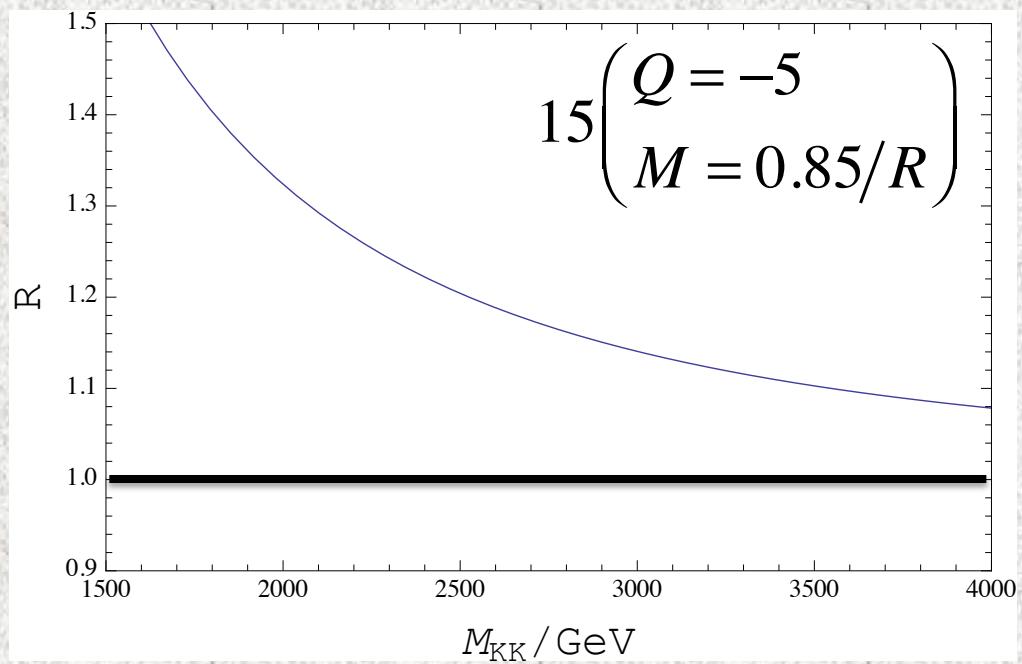
No unwanted massless fermions

$$1^{\text{st}} \text{ KK mass} = 1/(2R) \Rightarrow \text{Higgs mass}$$

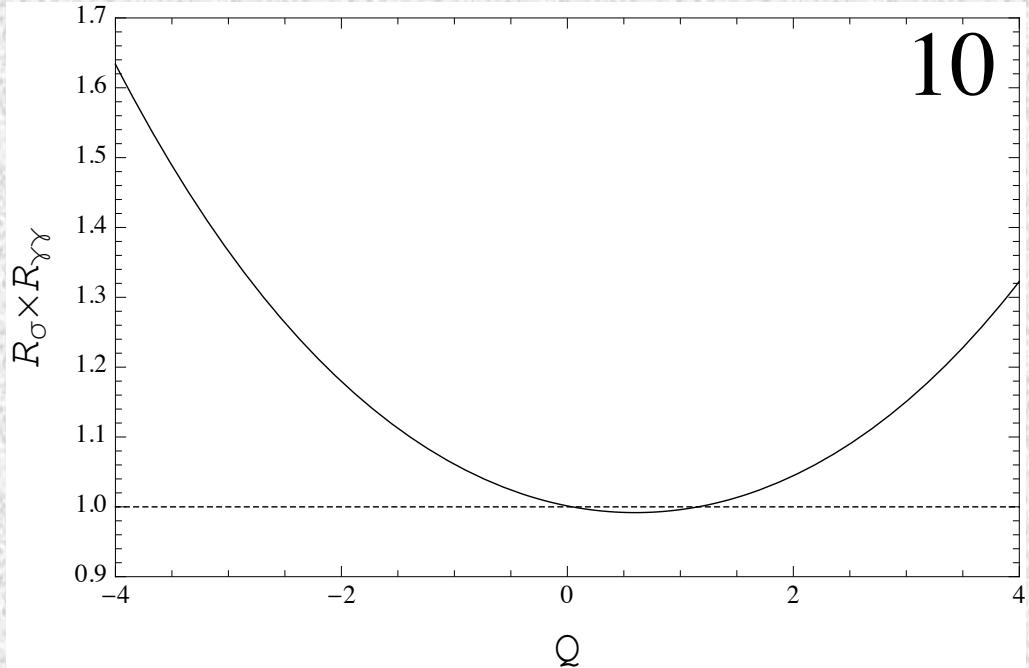
Helpful to adjust
126 GeV Higgs



$$R = \frac{\sigma(gg \rightarrow H)_{GHU+10} \times BR(H \rightarrow \gamma\gamma)_{GHU+10}}{\sigma(gg \rightarrow H)_{SM} \times BR(H \rightarrow \gamma\gamma)_{SM}}$$



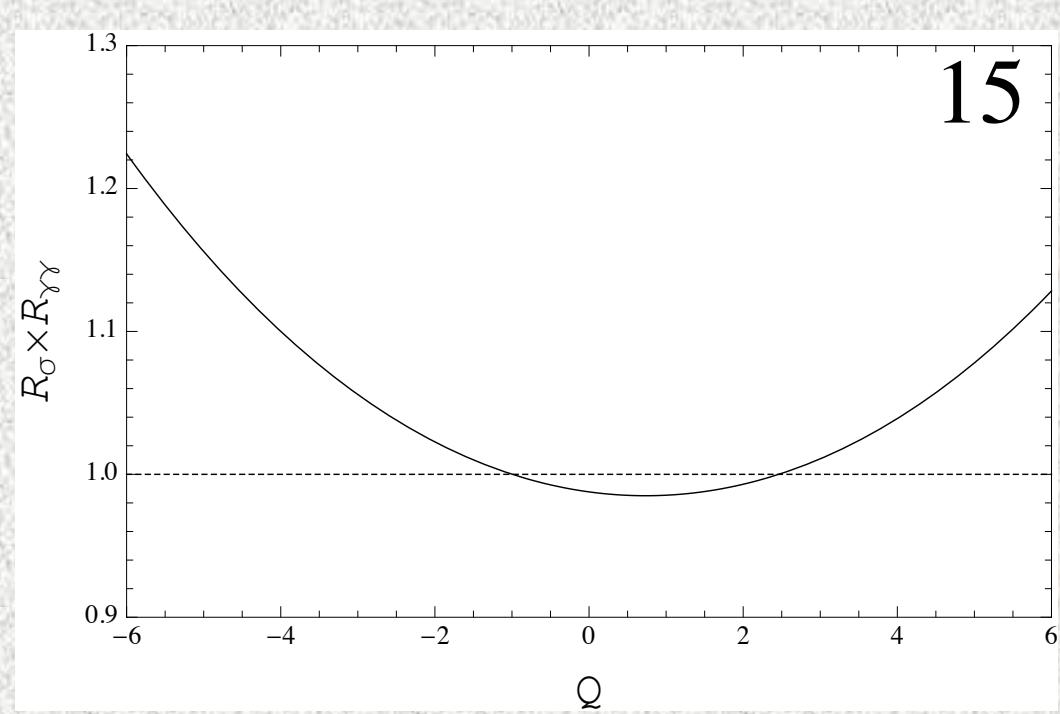
$$R = \frac{\sigma(gg \rightarrow H)_{GHU+15} \times BR(H \rightarrow \gamma\gamma)_{GHU+15}}{\sigma(gg \rightarrow H)_{SM} \times BR(H \rightarrow \gamma\gamma)_{SM}}$$



10

$$R = \frac{\sigma(gg \rightarrow H)_{GHU} \times BR(H \rightarrow \gamma\gamma)_{GHU}}{\sigma(gg \rightarrow H)_{SM} \times BR(H \rightarrow \gamma\gamma)_{SM}}$$

$1/R = 3\text{TeV}$ fixed

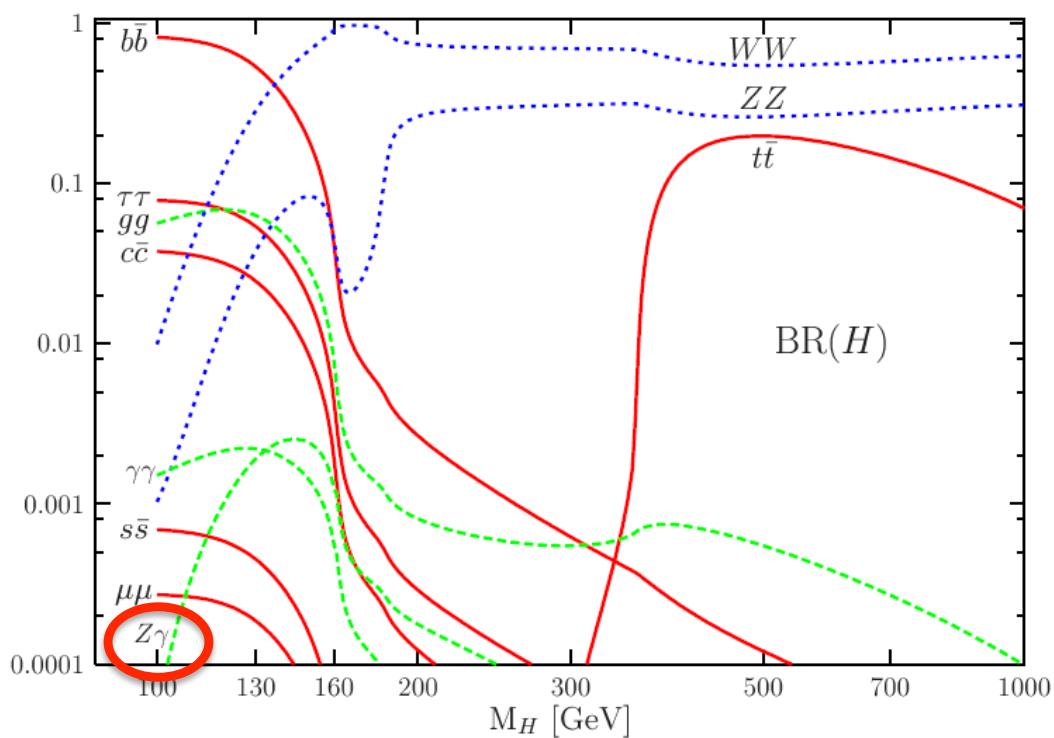


15

$H \rightarrow Z \gamma$ in GHU

$H \rightarrow Z\gamma$ in GHU

NM & N.Okada, PRD88 (2013) 037701



KK modes have
electroweak charges



Naturally expect deviation
of $Z\gamma$ decay from the SM
prediction

Model dep. Correlation
btw $\gamma\gamma$ & $Z\gamma$ is interesting

Our result is very striking!!

No KK mode contributions to $Z\gamma$ decay@1-loop

Simple reason: Higgs & photon couples to KK modes
with **same** mass eigenstates,

Z does NOT

Fermion coupling

$$\left(\bar{\psi}_0^{(n)}, \bar{\psi}_+^{(n)}, \bar{\psi}_-^{(n)}\right) \begin{pmatrix} 2\gamma_\mu/\sqrt{3} & W_\mu^+ & W_\mu^+ \\ W_\mu^- & -\gamma_\mu/\sqrt{3} & -Z_\mu \\ W_\mu^- & -Z_\mu & -\gamma_\mu/\sqrt{3} \end{pmatrix} \gamma^\mu \begin{pmatrix} \psi_0^{(n)} \\ \psi_+^{(n)} \\ \psi_-^{(n)} \end{pmatrix}, \psi_{0,\pm}^{(n)} : \frac{n}{R}, \frac{n}{R} \pm m_f$$

ZWⁿWⁿ
coupling

$$Z_\mu \left(W_{\mu\nu+}^{\mp(n)}, W_{\mu\nu-}^{\mp(n)} \right) \begin{pmatrix} 0 & \mp i \\ \pm i & 0 \end{pmatrix} \left(W_{\nu+}^{\pm(n)}, W_{\nu-}^{\pm(n)} \right)$$

$$W_{\mu\pm}^{(n)} : n/R \pm m_W, W_{\mu\nu} \equiv \partial_\mu W_\nu - \partial_\nu W_\mu$$

ZγWⁿWⁿ
coupling

$$Z^\mu \gamma_\nu \left(W_{\mu+}^{\mp(n)}, W_{\mu-}^{\mp(n)} \right) \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} W_+^{\pm\nu(n)} \\ W_-^{\pm\nu(n)} \end{pmatrix} + 2Z_\mu \gamma^\mu \left(W_{\nu+}^{\mp(n)}, W_{\nu-}^{\mp(n)} \right) \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \begin{pmatrix} W_+^{\pm\nu(n)} \\ W_-^{\pm\nu(n)} \end{pmatrix}$$

NO H-Z-γ coupling@1-Loop found

Summary

- We calculated KK mode contributions to
 $gg \rightarrow H$ & $H \rightarrow \gamma\gamma$ @LHC in 5D $SU(3) \times U(1)'$ GHU
- Minimal model cannot explain the data (NM & Okada)
- Extra leptons can enhance $H \rightarrow \gamma\gamma$ as we like
by adjusting $U(1)'$ charges
 - ex. 10 & 15 plets of $SU(3)$
w/ bulk mass & half-periodic BC
- Lightest KK lepton with a mass 1-3 TeV
 \Rightarrow DM candidate in case of zero $U(1)_{em}$ charge
- No KK mode contributions to $H \rightarrow Z\gamma$ @1-loop

Thank you very much
for your attention

Backup Slides

Summary

- 1-loop RGE analysis of Higgs quartic coupling with GH condition $\lambda=0 @ M_{KK}$
 - ⇒ 10 with $M=0.55/R$, 15 with $M=0.85/R$
 - ⇒ 125 GeV Higgs
 - ⇒ No instability
- Extra leptons are (some kind of) Z_2 odd & stable due to the half-periodic BC
 - ⇒ lightest KK leptons of $1 \sim 3$ TeV mass can be DM candidate in case of vanishing electric charge
- No KK mode contribution to $H \rightarrow Z\gamma @ 1\text{-loop}$

Higgs mass analysis by 4D EFT approach

In GHU, m_H likely to be small \therefore loop generated

Instead of 5D Higgs potential minimization,
solve 1-loop RGE for Higgs quartic coupling λ
by imposing BC $\lambda=0@1/R$ "gauge-Higgs condition"

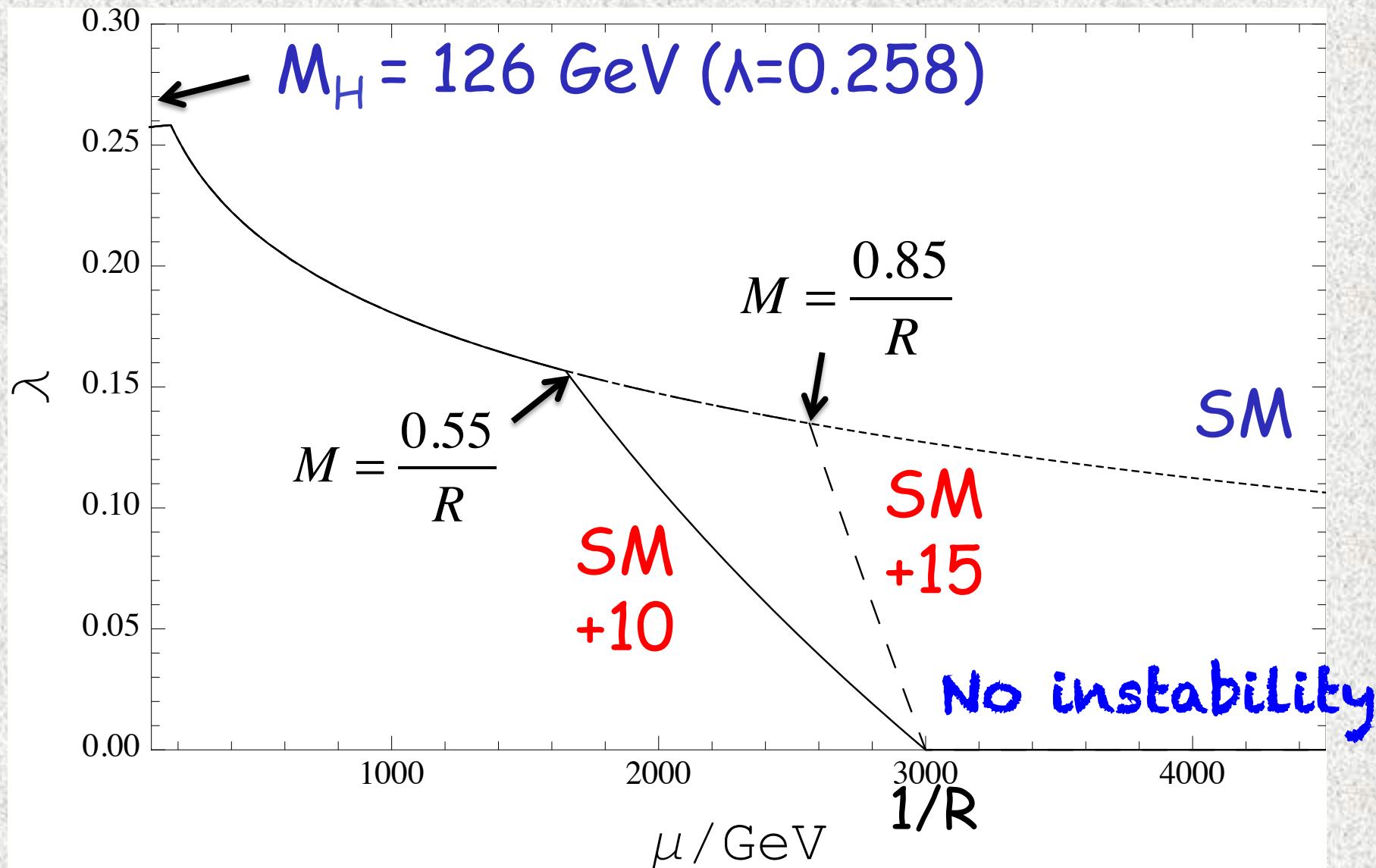
Haba, Matsumoto, Okada & Yamashita (2006, 2008)

Natural realization of GHU in 4D viewpoint:
 $V_H = 0$ above $1/R$ by 5D gauge invariance

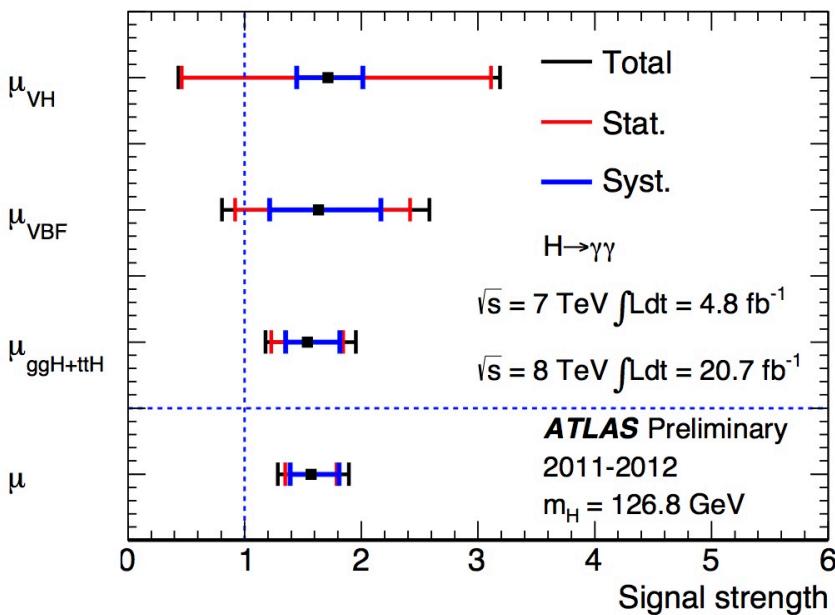
Furthermore, NO vacuum instability

This approach greatly simplifies Higgs mass study

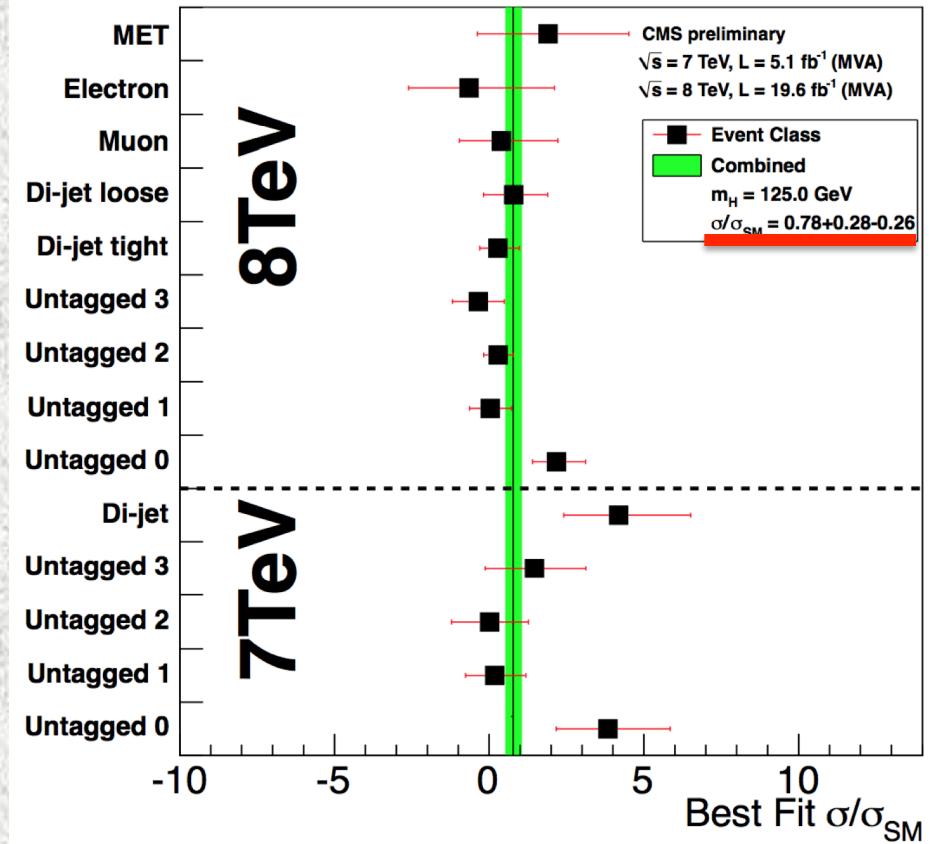
Numerical results for 1-loop RGE of λ



Diphoton decay excess



$$\sigma/\sigma_{SM} = 1.57 \pm 0.22(\text{stat}) + 0.24 - 0.18(\text{syst})$$



Hint for New Physics??

	$gg \rightarrow H$	$H \rightarrow \gamma\gamma$
Top	$\frac{\alpha_s}{12\pi\nu}$	$\frac{2\alpha_{em}}{9\pi\nu}$
W		$-\frac{7\alpha_{em}}{8\pi\nu}$
KK Top	$-\frac{\alpha_s}{12\pi\nu} \frac{1}{3}(\pi m_t R)^2$	$-\frac{2\alpha_{em}}{9\pi\nu} \frac{1}{3}(\pi m_t R)^2$
KK W		$\frac{7\alpha_{em}}{8\pi\nu} \frac{1}{3}(\pi m_W R)^2$
GHU/SM	$1 - \frac{1}{3}(\pi m_t R)^2$	$1 + \frac{1}{141}(\pi m_W R)^2$

KK mode contributions: opposite sign!!

KK mass mixing after EW symmetry breaking

$$A_\mu^{(0)} = \frac{1}{2} \begin{pmatrix} \frac{2}{\sqrt{3}} \gamma_\mu & \sqrt{2} W_\mu^+ & 0 \\ \sqrt{2} W_\mu^- & -Z_\mu - \frac{\gamma_\mu}{\sqrt{3}} & 0 \\ 0 & 0 & Z_\mu - \frac{\gamma_\mu}{\sqrt{3}} \end{pmatrix}, A_\mu^{(n)} = \frac{1}{2} \begin{pmatrix} \frac{2}{\sqrt{3}} \gamma_\mu^{(n)} & \sqrt{2} W_\mu^{+(n)} & \sqrt{2} A_\mu^{+(n)} \\ \sqrt{2} W_\mu^{-(n)} & -Z_\mu^{(n)} - \frac{\gamma_\mu^{(n)}}{\sqrt{3}} & A_\mu^{6(n)} - i A_\mu^{7(n)} \\ \sqrt{2} A_\mu^{-(n)} & A_\mu^{6(n)} + i A_\mu^{7(n)} & Z_\mu^{(n)} - \frac{\gamma_\mu^{(n)}}{\sqrt{3}} \end{pmatrix}$$

No mixing

$W^{\pm(n)} \Leftrightarrow A^{\pm(n)}$

$Z^{(n)} \Leftrightarrow A^7(n)$

$$\left(\bar{\psi}_{1L}^{(0)}, \bar{\psi}_{2L}^{(0)}, \bar{\psi}_{3R}^{(0)} \right) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -m \\ 0 & -m & 0 \end{pmatrix} \begin{pmatrix} \psi_{1L}^{(0)} \\ \psi_{2L}^{(0)} \\ \psi_{3R}^{(0)} \end{pmatrix} + \left(\bar{\psi}_1^{(n)}, \bar{\psi}_2^{(n)}, \bar{\psi}_3^{(n)} \right) \begin{pmatrix} m_n & 0 & 0 \\ 0 & m_n & -m \\ 0 & -m & m_n \end{pmatrix} \begin{pmatrix} \psi_1^{(n)} \\ \psi_2^{(n)} \\ \psi_3^{(n)} \end{pmatrix},$$

chiral \Rightarrow no mass splitting

Vector-like \Rightarrow mass splitting