Towards systematic exploration of multi-Higgs-doublet models

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Quest for NHDM phenomenology

Conservative picture of the TeV-scale physics:

- LHC Run 1 results point to a very SM-like visible sector.
- Yet, it is compatible with a potentially very rich but hidden Higgs sector (e.g. decoupling/alignment limit, dark sector, portals, etc.).
- $\Rightarrow$ what specific signals could provide the best short-cuts towards the complicated Higgs sector?

The answer depends on the specific Higgs sector, and hundrends of paper studied it in specific circumstances. Fully systematic investigation of the entire parameter space is very challenging beyond the simplest bSM scalar sectors.

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Quest for NHDM phenomenology

Phenomenology of 2HDM has been analyzed in exquisite detail.

Multi-Higgs-doublet models (NHDM) are nowhere near in the detail of their phenomenological investigations, despite offering a multitude of opportunities. Only very specific models with $N > 2$ doublets have been analyzed

Aggravating factors:

- highly multi-dimensional parameter space (not even clear how to describe it efficiently!),
- lack of spectacular phenomenological differences as compared with 2HDM.

Quest for NHDM phenomenology

Phenomenological motivations for NHDM ($N = 3$ example):

- *CP*-violation from scalar sector compatible with NFC [Weinberg, 1976; Sato, 1979; Branco, 1980; Deshpande, 1981; Sanda, 1981; ...]
- attack on the flavour puzzle based on groups S_n [Pakvasa, Sugawara, 1978; Derman, 1979; Wyler, 1979; Ma, 1980; ...] or other groups;
- various novel flavour physics effects via charged Higgs exchanges [Grossman, 1994; ...]
- possibility to combine several 2HDM effects such as inert doublets and spontaneous *CP*-violation [Grzadkowski, Ogreid, Osland, 2009; ...];
- Higgs doublets plus other scalars for neutrino and scalar DM models.

In the absence of direct experimental clues, it makes sense to explore NHDM possibilities in fine details, and to do it cleverly.

Quest for NHDM phenomenology

The big task is to have a systematic coverage of the NHDM phenomenology, at a level comparable to the 2HDM case.

A natural starting point is to explore, in general terms and in systematic fashion, NHDMs in the vicinity of large discrete symmetry groups.

- Highly symmetric NHDMs are usually unrealistic (bad CKM matrix) or very SM-like but are (relatively) easy to explore. They provide the firm background of what to expect.
- Small soft breaking terms should produce small effects beyond SM or beyond 2HDM, at least what concerns the 125 GeV Higgs and flavour physics observables $\to$ can safely fit experimental constraints or possible bSM signals.
- Convenient playground for efficient work with highly multidimensional parameter spaces. We need a versatile tool beyond numerical scans or repeating Taylor expansion on model-to-model basis.

Discrete symmetry groups

Basic facts about discrete symmetries in the Higgs sector:

- For a given field content and interaction type, the freedom of imposing symmetries is limited. Finding full lists of allowed symmetry groups G is a non-trivial task in each case.
- For the 3HDM scalar sector, it was recently solved [Ivanov, Vdovin, 2013]:

$$\mathbb{Z}_2, \quad \mathbb{Z}_3, \quad \mathbb{Z}_4, \quad \mathbb{Z}_2 \times \mathbb{Z}_2, \quad S_3, \quad \Delta(54)/\mathbb{Z}_3,$$
$$D_4, \quad A_4, \quad S_4, \quad \Sigma(36).$$

- Some symmetry groups automatically lead to explicit CP-conservation in the scalar sector.

Spontaneous breaking of discrete symmetries

Basic facts about discrete symmetry breaking:

- A scalar potential symmetric under G can have the global minimum of lower symmetry, $G_v \subseteq G$. Three options *a priori*: no breaking, partial breaking, complete breaking.
- In NHDM, symmetry breaking cannot be arbitrarily strong! Typically, a large non-abelian G always breaks to a non-trivial subgroup G_v (3HDM examples known since Derman, 1979).
- Incomplete symmetry breaking has phenomenological consequences.
  Quark sector: G-symmetric NHDM can lead to viable quark masses and CKM only if G is broken completely (in the space of active Higgs doublets) [Leurer, Nir, Seiberg, 1993; Gonzalez Felipe et al, 2014].
  Astroparticle: DM candidates are often protected by residual G_v.
- Spontaneous CP-violation in the scalar sector also depends on G.

Spontaneous symmetry breaking in 3HDM

Results on strongest and weakest breaking of discrete symmetries in 3HDM, as well as on spontaneous *CP*-violation [Ivanov, Nishi, 2015]:

group	$\|G\|$	$\|G_v\|_{min}$	$\|G_v\|_{max}$	sCPv possible?
abelian	2, 3, 4, 8	1	$\|G\|$	yes
$\mathbb{Z}_3 \rtimes \mathbb{Z}_2^*$	6	1	6	yes
S_3	6	1	6	—
$\mathbb{Z}_4 \rtimes \mathbb{Z}_2^*$	8	2	8	no
$S_3 \times \mathbb{Z}_2^*$	12	2	12	yes
$D_4 \times \mathbb{Z}_2^*$	16	2	16	no
$A_4 \rtimes \mathbb{Z}_2^*$	24	4	8	no
$S_4 \times \mathbb{Z}_2^*$	48	6	16	no
CP-violating $\Delta(27)$	18	6	6	—
CP-conserving $\Delta(27)$	36	6	12	yes
$\Sigma(36)$	72	12	12	no

CP-violation comes in pairs: if horizontal symmetry forbids explicit CPV, it also forbids spontaneous CPV.

Soft breaking

Consider now a Higgs sector in the vicinity of a large symmetry group G. Assume that

- G is broken softly, via terms quadratic in Higgs fields:

$$-m^2 \sum_i \phi_i^\dagger \phi_i + V_{4,G} \quad \Rightarrow \quad -m_{ij}^2(\phi_i^\dagger \phi_j) + V_{4,G}$$

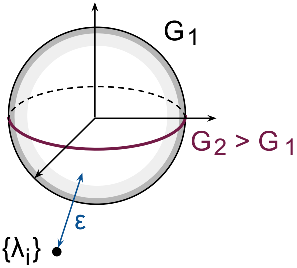
- whether the Yukawa sector respects G or NFC is left open.

The goal is to find generic and robust analytic quantities which would allow for systematic investigation of the huge parameter space.

Critical exponents

A possible example of such quantities: critical exponents.

Consider a model with large multi-dimensional parameter space $\{\lambda_i\} \subset R^n$. Symmetric models occupy low-dimensional manifolds and lead to certain quantities $x = 0$ (e.g. quark masses, CP-violation, Higgs mass splitting, etc). Let's call them order parameters.



A generic point $\{\lambda_i\}$ does not correspond to exact symmetry, and has non-zero order parameters $x \neq 0$. But if $\{\lambda_i\}$ lies close to the symmetric manifold, at small distance ϵ, then $x \propto \epsilon^\nu$. ν is the critical exponent for the quantity x.

Critical exponents

My claim is that

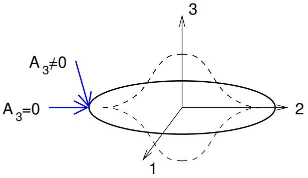
- critical exponents are robust upon variation of free parameters and are, therefore, calculable;
- critical exponents can depend on the symmetry group and other structural properties of the model;
- knowing them should boost quantitative understanding of multi-parametric models in the vicinity of symmetries (e.g. NHDM with large softly broken non-abelian groups). For example, $\nu = 3$ or $\nu = 1/3$ would make a huge difference!

Critical exponents

A 2HDM-like example from [Ivanov, 2009].
The phase diagram of the tree-level 2HDM potential contains a cusped manifold separating potentials with one or two minima.

The rim corresponds to continous symmetry $\to m_h = 0$. In its vicinity, $m_h \propto \epsilon^\nu$ with

- symmetric approach: $\nu = 1/2$
- generic approach: $\nu = 1/3$



Critical exponents

Another example: algebraic symmetry breaking.

- Start with a NHDM with high discrete symmetry G which, upon minimization, is broken to G_v.
- Instead of adding soft terms which explicitly break G, add higher-order G-invariant terms.
- The symmetry group is still G, but it can now break down completely upon minimization due to of presence of new terms and the higher algebraic degree of the potential.
- Phenomenological consequences will follow, and small effects can again be studied in terms of critical exponents.

Note differences from soft and radiative symmetry breaking.

Conclusions

- Starting point: a hidden, but potentially complex, Higgs sector with a very SM-like h, e.g. NHDM. Quest: systematically investigate phenomenology.
- Obstacles: huge parameter space and mathematical challenges $\rightarrow$ ideas needed on how to move beyond numerical scans or case-by-case study.
- Claim: much can be learned, in a very general manner, about the NHDMs in the vicinity of large discrete symmetry. Rigorous group-theoretic results together with innovative algebraic or geometric methods could provide insights into what multi-doublet models are capable of.