TeV scale mirage mediation in NMSSM

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based on
T.K., Makino, Okumura, Shimomura, Takahashi, arXiv:1204.3561,
T.K., Makino, Okumura, Shimomura, arXiv:1502.XXXX
1. Introduction

Supersymmetric extension of the standard model is interesting, still.

Its minimal one is the minimal supersymmetric SM (MSSM).

\[
\frac{m_Z^2}{2} \approx -m_{H_u}^2 - |\mu|^2 + \cdots
\]

\(\mu\) : supersymmetric mass

\(m_{H_u}^2\) : SUSY breaking mass

It is usually of order of stop mass squared gluino mass squared due to RG effects.
mu-term

\[ \frac{m_Z^2}{2} \approx -m_{H_u}^2 - |\mu|^2 + \cdots \]

\[ W = \mu H_u H_d + \cdots \]

\( \mu \) : supersymmetric mass

\( m_{H_u}^2 \) : SUSY breaking mass

It is usually of order of stop mass squared gluino mass squared due to RG effects.

The \( \mu \) and the soft mass have origins different from each other in MSSM.

Why are those of the same order?
the mu-problem

\[ \rightarrow \text{motivation for the NMSSM} \]
Fine-tuning and little hierarchy
The stop mass and gluino mass are of $O(1)$ TeV or more.

$$\frac{m_Z^2}{2} \approx -m_{H_u}^2 - |\mu|^2 + \cdots$$

If $m_{H_u}^2$ is of order of the stop mass, e.g. 1 TeV, the fine-tuned cancelation between the $\mu$ and the soft mass is required in order to derive the Z-boss mass.

$$\frac{m_Z^2}{2} / |m_{H_u}^2| \approx 0.5\% \text{ tuning for } |m_{H_u}| = 1\text{ TeV}$$

We may need heavier stop mass to realize 125 GeV higgs mass in the MSSM.
TeV scale mirage mediation

Certain type of moduli stabilization in string theory
(Main SUSY braking source is far from the SM sector)

Mirage mediation
= modulus mediation + anomaly mediation

Choi, Falkowski, Nilles, Olechowski, Pokorski, '04
Choi, Falkowski, Nilles, Olechowski, '05
Choi, Jeong, Okumura, '05,   Endo, Yamaguchi, Yoshioka, '05

The TeV scale mirage mediation can realize
the little hierarchy at the weak scale,

| $m_{H_u}$ | << stop mass, gluino mass |

One can ameliorate the fine-tuning in the MSSM.
Choi, Jeong, TK, Okumura, '05
Next-to minimal supersymmetric standard model (NMSSM)

NMSSM = the extension of MSSM by adding the singlet $S$
We also impose the $Z3$ symmetry.

The mu-term,

\[ W = \mu H_u H_d + \cdots \]

is forbidden, but

\[ W = \lambda SH_u H_d + \cdots \]

is allowed.

The superpotential does not include dimensionful couplings, but dimensionful couplings appear only as SUSY breaking terms, e.g. soft masses, A-terms.
**NMSSM**

After $S$ develops its VEV, the effective mu-term is generated.

\[ W = \lambda \langle S \rangle H_u H_d + \cdots \]

\[ \Rightarrow \mu H_u H_d + \cdots \]

The VEV of $S$ is determined by SUSY breaking terms.

Thus, the VEV is of order of superpartner masses. the solution for the mu-problem

Other interesting aspects:

- the lightest Higgs boson mass increases,
- the Higgs sector as well as the neutralino sector has a rich structure by adding the singlet (singlino) because of their mixture.
The NMSSM also leads

\[ \frac{m_Z^2}{2} \approx -m_{H_u}^2 - |\mu|^2 + \cdots \]

If \( m_{H_u}^2 \) is of order of the stop mass, e.g. 1 TeV, the fine-tuning would be required.

We consider the TeV scale mirage mediation scenario in the NMSSM.
2. TeV-scale mirage mediation

Mixture of modulus mediation and anomaly mediation

Gaugino mass

\[ M_a = M_0 + \frac{m_{3/2}^2}{8\pi^2} b_a g_a \]

- \( M_0 \): modulus mediation
- \( m_{3/2} \): gravitino mass
- \( b_a \): beta-function coefficients of the gauge couplings
Mirage mediation

A-terms and soft scalar masses

\[ A_{ijk} = \alpha_{ijk}M_0 - \frac{m_{3/2}^3}{8\pi^2} \left( \gamma_i + \gamma_j + \gamma_k \right) \]

\[ m_i^2 = c_i M_0 - \frac{m_{3/2}^3}{8\pi^2} M_0 \sum \theta_i - \gamma_i' \left( \frac{m_{3/2}^3}{8\pi^2} \right)^2 \]

\[ \gamma_i = 2 \sum_a g_a^2 C_2^a \left( \phi^i \right) - \frac{1}{2} \sum_{jk} |y_{ijk}|^2 \]

\[ \theta_i = 4 \sum_a g_a^2 C_2^a \left( \phi^i \right) - \sum_{jk} a_{ijk} |y_{ijk}|^2 \]

\[ \gamma_i' = 8\pi^2 \frac{d\gamma_i}{d \ln \mu_R} \]

Coefficients, \( c \) and \( a \) are determined by string-derived supergravity.
Modulus mediation: simple case

Gauge kinetic function: \( T : \text{modulus} \)

\[
f = T
\]

\[
M_0 = \frac{F_T}{(T + \overline{T})}
\]

Kahler metric

\[
\int d^2 \theta \left| C \right|^2 Y_i(T + \overline{T}) \left| Q_i \right|^2
\]

\[
Y_i(T + \overline{T}) = (T + \overline{T})^{c_i}
\]

\[
m_i^2 = c_i \frac{\left| F_T \right|^2}{(T + T)^2} = c_i \left| M_0 \right|^2
\]

if Yukawa couplings is independent of \( T \),

\[
a_{ijk} = c_i + c_j + c_k
\]
Kahler metric
loop corrections

\[ Y_i(T + \bar{T}) = (T + \bar{T})^C_{i (tree)} + \Delta_{\text{one-loop}}(T, \bar{T}) \]

scalar mass

\[ m_i^2 = c_i M_0 - \frac{m_{3/2}^3}{8\pi^2} M_0 \theta_i - \gamma_i \left( \frac{m_{3/2}}{8\pi^2} \right)^2 \]

\[ c_i = c_{i (tree)} + \delta c_{i (loop)} \]

\( c_{i (tree)} \): fractional number including 0 and 1
\( \delta c_{i (loop)} \): suppressed by a loop factor

The latter is not important unless the tree-level is vanishing.
Even if \( c_{i (tree)} = 0 \), \( c_i \) is not exactly vanishing, but has ambiguity due to \( \delta c_{i (loop)} \).
Mirage scale

Choi, Jeong, Okumura, ‘05

Mirage mediation + RG effects
= (modulus mediation) + (anomaly med.) + (RG)
cancel at a certain scale

\[ \alpha = \frac{m_{3/2}}{M_0 \ln(M_P / m_{3/2})} \]

\[ M_{\text{mir}} = \left( \frac{m_{3/2}}{M_P} \right)^{\alpha/2} M_C \]

At the mirage scale, the values of gaugino masses, as well as A-terms and soft scalar masses are obtained by those corresponding to pure modulus mediation.

Mirage scale is determined by setup for moduli stabilization.
Mirage in gaugino masses

RG flow of gaugino masses

\[ M_a (M_C) = M_0 + \frac{\beta_a (M_C)}{g_a (M_C)} m_{3/2} \]

\[ M_a (\mu_R) = \frac{g_a^2 (\mu_R)}{g_a^2 (M_C)} \left( M_0 + \frac{\beta_a (M_C)}{g_a (M_C)} m_{3/2} \right) \]

\[ = \frac{g_a^2 (\mu_R)}{g_a^2 (M_C)} M_0 + \frac{\beta_a (\mu_R)}{g_a (\mu_R)} m_{3/2} \]

\[ \frac{g_a^2 (\mu_R)}{g_a^2 (M_C)} = 1 + 2 \frac{\beta_a (\mu_R)}{g_a (\mu_R)} \ln \frac{\mu_R}{M_C} \]

\[ M_a (\mu_R) = M_0 + \frac{\beta_a (\mu_R)}{g_a (\mu_R)} \left( M_0 \ln \left( \frac{\mu_R}{M_C} \right)^2 + m_{3/2} \right) \]

\[ = M_0 + \frac{\beta_a (\mu_R)}{g_a (\mu_R)} M_0 \ln \left( \frac{M_P}{m_{3/2}} \right)^\alpha \left( \frac{\mu_R}{M_C} \right)^2 \]

\[ m_{3/2} = \alpha M_0 \ln \left( \frac{M_P}{m_{3/2}} \right) \]

At \( \mu_R = M_{mir} \), the second term vanishes.
Similarly, we obtain

\[ \alpha_{ijk} = c_i + c_j + c_k = 1 \]
Similarly, we obtain

\[ M_a (M_{\text{mir}}) = M_0 \]

Similarly, we obtain

\[ A_{ijk} (M_{\text{mir}}) = (c_i + c_j + c_k) M_0 \]
\[ m_i^2 (M_{\text{mir}}) = c_i M_0^2 \]

if the Yukawa couplings are small enough or if the following conditions are satisfied for non-vanishing Yukawa couplings,

\[ a_{ijk} = c_i + c_j + c_k = 1 \]
We apply the TeV scale mirage mediation scenario to the NMSSM.

\[ M_{\text{mir}} = \left( \frac{m_{3/2}}{M_P} \right)^{\alpha/2} M_C \]
\[ \alpha = 2 \]
\[ M_C = M_{\text{GUT}}, \quad m_{3/2} = O(100) \text{TeV} \]
\[ M_{\text{mir}} \text{ is around 1 TeV.} \]

There would be a little hierarchy between \( M_0 \) and scalar mass with \( c_i \approx 0 \).
3. TeV scale mirage in NMSSM
3.1 NMSSM

\[ W_{\text{Higgs}} = -\lambda S H_u H_d + \left( \kappa / 3 \right) S^3 \]

\[ V_{\text{soft}} = m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_S^2 |S|^2 \]

\[ -\lambda A H_u H_d + \left( \kappa / 3 \right) A_S S^3 + \text{h.c.} \]

\[ V = \sum_i |W_i|^2 + V_D + V_{\text{soft}} \]

\[ V_D = (1/8)(g_1^2 + g_2^2)(|H_d^0|^2 - |H_u^0|^2)^2 \]

By the stationary conditions, the VEVs are determined,

\[ \frac{\partial V}{\partial H_u^0} = \frac{\partial V}{\partial H_d^0} = \frac{\partial V}{\partial S} = 0 \]

\[ \Rightarrow \]

\[ v^2 = \langle |H_u^0|^2 \rangle + \langle |H_d^0|^2 \rangle, \quad \tan \beta = \frac{\langle H_u^0 \rangle}{\langle H_d^0 \rangle} \]

\[ s = \langle S \rangle \]

\[ m_Z^2 = (g_1^2 + g_2^2) v^2 / 2 \]
Parameters

We usually write other (two) parameters in models in terms of Z-boson mass and tan beta by using the stationary conditions. We also obtain

\[ m_Z^2 = \frac{1 - \cos 2\beta}{\cos 2\beta} m_{Hu}^2 - \frac{1 + \cos 2\beta}{\cos 2\beta} m_{Hd}^2 - 2\mu^2 + \ldots \]

For \( \tan^2 \beta \gg 1 \),

\[ m_Z^2 = -2m_{Hu}^2 + \frac{2}{\tan^2 \beta} m_{Hd}^2 - 2\mu^2 + \ldots \]

The natural value of each term would be of \( \text{O}(100) \) GeV without cancelation. If there is a certain cancelation, some of them could be larger.
3.2 TeV scale mirage in NMSSM

We consider pure modulus mediation for the couplings, and Yukawa for the $1$ satisfy

$$
\kappa = \sum\limits_{\text{tree}} c_{i}^{(\text{tree})} = 1, \\
\lambda = \sum\limits_{\text{tree}} c_{i}^{(\text{tree})} = 0, \\
S = \sum\limits_{\text{tree}} c_{i}^{(\text{tree})} = 0,
$$

They satisfy $\sum\limits_{\text{tree}} c_{i}^{(\text{tree})} = 1$ for the top Yukawa and $\lambda$ couplings, but not for the $\kappa$ coupling.

pure modulus mediation

$$
A_{t} = A_{\lambda} = M_{0}, \quad A_{\kappa} = 0
$$

$$
m_{Hd}^{2} = M_{0}^{2}, \quad m_{\text{stop}}^{2} = M_{0}^{2} / 2
$$

$$
m_{Hu}^{2} = m_{S}^{2} = 0, \quad \text{up to } \delta c^{(\text{loop})}
$$
TeV scale mirage in NMSSM

At the TeV scale,

\[
A_t \approx A_\lambda \approx M_0, \\
m_{Hd}^2 \approx M_0^2, \\
m_{stop}^2 \approx M_0^2 / 2 \\
\text{up to } O(\kappa / 8\pi^2)
\]

Their precise values are ambiguous. What we can say is they are suppressed compared with other soft masses.

\[
m_{Hu}^2 = \delta c_{Hu}^{(loop)} M_0^2, \\
m_S^2 = \delta c_S^{(loop)} M_0^2
\]

\(A_\kappa\) is also expected to be suppressed, but its precise value is ambiguous. Thus, we use \(A_\kappa\) as a free parameter, which must be small.
At the TeV scale, we determine $m_{Hu}^2$, $m_S^2$ and $\mu = \lambda \langle S \rangle$ by the stationary conditions.

$$\mu = \lambda \langle S \rangle \approx \frac{m_{Hd}^2}{A_\lambda \tan \beta}$$

$$m_{Hu}^2 \approx \frac{m_{Hd}^2}{\tan^2 \beta} - \frac{m_{Hd}^4}{A_\lambda^2 \tan^2 \beta} - \frac{m_Z^2}{2}$$

The mu term is dynamical determined, and the first and second terms cancel each other.

$m_Z$ is almost determined by $m_{Hu}^2$, which is suppressed in our scenario.

$m_Z$ is insensitive to $\mu$, and $\mu$ can be larger.
3.3 Spectrum

\[ \alpha = 2 \]

Free parameters

\[ \lambda, \kappa, \tan\beta, M_0, A_\kappa \]

Using them, we determine all of soft SUSY breaking parameters except \( m_{Hu}^2 \) and \( m_S^2 \).

We determine \( m_{Hu}^2, m_S^2 \) and \( \mu = \lambda \langle S \rangle \) by the stationary conditions.

When \( \lambda \) and/or \( \kappa \) are large at the weak scale, they blow up below the GUT scale. Thus, we have constraints on large values of \( \lambda \) and \( \kappa \).
Parameter region

When $\lambda$ and/or $k$ are large at the weak scale, they blow up below the GUT scale. Thus, we have constraints on large values of $\lambda$ and $k$.

For (very) small $\kappa/\lambda$, the Higgs sector includes a tachyonic mode. Such a parameter region is excluded.

Also, for (very) small $\kappa/\lambda$, false vacua appear.

At the region with large $\kappa/\lambda$, $m_s$ is not suppressed, but such a large value could not be realized in our scenario. Such a region is disfavorable.
Higgs sector and fine-tuning

The value of $\mu$ is large compared with $M_z$, i.e.,
$$\mu = O(500)\text{GeV}.$$

However, $m^2_{Hu}$ is small like
$$|m^2_{Hu}| = O(200)\text{ GeV}$$
because of cancellation between $\mu$ and $m^2_{Hd}$.

The fine-tuning problem is ameliorated.
The coupling between $h_1$ ($h_2$) and $Z$ is almost the same as one in the SM when its mass is around 125 GeV, and the other is almost singlet.

- Singlet higgs mass $> 90$ GeV
- LEPII bound
- Tachyonic modes and unrealistic vauca

Kanehata, T.K., Konishi, Seto, Shimimura, ‘11
T.K. Shimomura, Takahashi, ‘12
The coupling between $h_2$ and $Z$ is almost the same as one in the SM when its mass is around 125 GeV, and the other is almost singlet.
mu-term, e.g., Higgsino mass is around 300-500 GeV, but the mu-term is insensitive to the Mz in our model.

\[ M_0 = 1.5 \text{TeV}, \quad A_\kappa = -100 \text{GeV} \]

\[ \tan \beta = 3 \]

\[ \tan \beta = 10 \]
Fine tuning

How much should we tune our parameters to derive $M_z$?

$$m_z^2 = \frac{1 - \cos 2\beta}{\cos 2\beta} m_{Hu}^2 - \frac{1 + \cos 2\beta}{\cos 2\beta} m_{Hd}^2 - 2\mu^2 + ....$$

$$m_z^2 = m_z^2(\lambda, \kappa, m_{Hu}^2, ....)$$

Fine-tuning parameters

$$\Delta_\lambda = \frac{d \ln(m_z^2)}{d \ln \lambda}, \quad \Delta_\kappa = \frac{d \ln(m_z^2)}{d \ln \kappa}, \quad .......$$

$$\Delta_\lambda = 10 \quad \Rightarrow \quad 10\% \; \text{tuning}$$

$$\Delta_\lambda = 100 \quad \Rightarrow \quad 1\% \; \text{fine tuning}$$

$$\Delta_\lambda = 1000 \quad \Rightarrow \quad 0.1\% \; \text{fine tuning}$$
Fine-tuning

\[ \tan \beta = 3, \quad M_0 = 1.5\text{TeV}, \quad A_\kappa = -100\text{GeV} \]

125GeV higgs \rightarrow 20\% tuning reagion
Fine-tuning

\[ \tan \beta = 10, \quad M_0 = 1.5 \text{TeV}, \quad A_\kappa = -100 \text{GeV} \]

125 GeV higgs \implies we don’t need fine-tuning
Fine-tuning

\[
\tan \beta = 10, 20 \quad M_0 = 1.5 \text{TeV}, 3 \text{TeV}, 5 \text{TeV} \quad A_\kappa = -100 \text{GeV}
\]

\[
M = 3 \text{ TeV} \quad \rightarrow \quad 5\text{-}10\% \text{ tuning}
\]

\[
M = 5 \text{ TeV} \quad \rightarrow \quad 3\% \text{ tuning}
\]
Higgsino and LSP

The higgsino is light compared with three gauginos, whose mass is $M_0$, 
$\mu = 300-500$ GeV for $M=1.5$ TeV.

The mass of singlino is $\left( \frac{\kappa}{\lambda} \right) \mu$.
The regions with large and very small $\kappa / \lambda$ are disfavorable.
Both masses of higgsino and singlino would be of the same order.

The lightest superparticle is a linear combination between the higgsino and singlino.
Summary

We have studied the (Z3 symmetric) NMSSM with soft SUSY breaking terms, which are induced in the TeV scale mirage mediation scenario.

We can realize the up-type Higgs soft mass of O(200) GeV, while other masses such as gaugino masses are heavy like 1 TeV or more. Cancellation between the effective $\mu$ term and the down-type Higgs soft mass amelioretes the fine-tuning even for $\mu = (500)$ GeV.

We don’t need severe fine-tuning. The higgsino and singlino are light and their linear combination is the lightest superparticle.
soft mass of s and Hu

soft mass of s and Hu must be suppressed by one-loop factor in our scenario.

\[ \tan \beta = 3, \quad M_0 = 1.5 \text{TeV}, \quad A_\kappa = -100 \text{GeV} \]
soft mass of $s$ and $Hu$

soft mass of $s$ and $Hu$ must be suppressed by one-loop factor in our scenario.

$$\tan\beta = 10, \quad M_0 = 1.5\text{TeV}, \quad A_\kappa = -100\text{GeV}$$