$hhh$ Coupling in SUSY models after LHC run I

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Outline

- Introduction
- hhh coupling in MSSM & NMSSM under current constraints
- hhh coupling relation to the measurements in future colliders
- Conclusion
In order to describe the full Higgs sector, we need to know the Higgs self-coupling.

Multi-Higgs New Physics models always have very non-SM-like Higgs self-coupling

It’s important to show how large this deviation will be and whether the future colliders could detect such deviation
Motivation and $hhh$ Coupling in SM

SM Higgs sector:

$$\mathcal{L} = (D_\mu \Phi)\dagger (D^\mu \Phi) - V(\Phi)$$  \hspace{1cm} (1)

$$V(\Phi) = \frac{\lambda}{4} (\Phi\dagger \Phi)^2 - \mu^2 \Phi\dagger \Phi, \Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$  \hspace{1cm} (2)

In Unitary gauge we have:

$$\Phi = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + h) \end{pmatrix}, v = 2\frac{\mu}{\sqrt{\lambda}}$$ \hspace{1cm} (3)

Then the tree-level potential will yield:

$$m_h = \sqrt{\frac{\lambda}{2}} v, \ \lambda_{hhh} = \frac{3}{2} \lambda v = \frac{3m_h^2}{v}$$ \hspace{1cm} (4)

Consider the one loop top correction, we get:

$$\lambda_{hhh} = \frac{3m_h^2}{v} - \frac{3m_t^4}{\pi^2v^3}$$ \hspace{1cm} (5)
Effective Potential Method

\[ \mathcal{V}_{\text{tree}} = m_{11}^{2} \Phi_{1}^\dagger \Phi_{1} + m_{22}^{2} \Phi_{2}^\dagger \Phi_{2} - \left[ m_{12}^{2} \Phi_{1}^\dagger \Phi_{2} + h.c. \right] \]
\[ + \frac{1}{8} (g_{2}^{2} + g_{1}^{2}) (\Phi_{1}^\dagger \Phi_{1})^{2} + \frac{1}{8} (g_{2}^{2} + g_{1}^{2}) (\Phi_{2}^\dagger \Phi_{2})^{2} \]
\[ + \frac{1}{4} (g_{2}^{2} - g_{1}^{2}) (\Phi_{1}^\dagger \Phi_{1})(\Phi_{2}^\dagger \Phi_{2}) - \frac{1}{2} g_{2}^{2} (\Phi_{1}^\dagger \Phi_{2})(\Phi_{2}^\dagger \Phi_{1}) \] (6)

\[ \Phi_{1} = \begin{pmatrix} H_{1}^{+} \\ \frac{1}{\sqrt{2}} (v_{1} + H_{1}^{0} + iA_{1}^{0}) \end{pmatrix}, \, \Phi_{2} = \begin{pmatrix} H_{2}^{+} \\ \frac{1}{\sqrt{2}} (v_{2} + H_{2}^{0} + iA_{2}^{0}) \end{pmatrix} \] (7)

In SUSY models, for example the MSSM, \( m_{H} \) is not an input parameter but a calculable quantity that mainly controlled by top and stop sector, and the tree-level couplings are fixed by gauge couplings constants. So one may expect a very different \( hhh \) coupling in SUSY. Let’s perform a simple estimation first.
Effective Potential Method

Correction to triple coupling is (simplified stop sector):

\[ \Gamma_{\tilde{t}h\tilde{h}h}^{t,\tilde{t}} = \frac{3m_t^4}{2\pi^2v^3} \left( 3\ln \frac{m_t^2}{m_{\tilde{t}}^2} - 2 \right) \]  

(8)

So it seems we get a very large corrections from top/stop sector. But the top/stop correction to Higgs mass is:

\[ \Delta m_h^2 = \frac{3m_t^4}{2\pi^2v^2} \ln \frac{m_t^2}{m_{\tilde{t}}^2} \]  

(9)

So if we shift the tree level Higgs mass to the one-loop level Higgs mass, the relationship we found in SM still hold.

Effective Potential Method

As we’ve seen, the loop correction to $hhh$ coupling may have a corresponding part in the loop correction to Higgs mass. So the key point in here is to treat the $hhh$ coupling and Higgs mass in a consistent way.

A convenient way to perform such a consistent calculation is the effective potential method. The effective potential method can absorb the dominant loop contribution into the effective couplings in low energy scale and deal with the Higgs mass and coupling in the same accuracy.
Effective Potential Method

These infinity Feynman graphs series can be summed to a elegant expression:

\[ \mathcal{V}^{(1)} = \frac{3}{16\pi^2} \left\{ m_t^4 \left( \ln \frac{m_t^2}{\mu^2} - \frac{3}{2} \right) - m_t^4 \left( \ln \frac{m_t^2}{\mu^2} - \frac{3}{2} \right) \right\} \]  

\[ \{\text{expand the effective potential in SUSY scale}\} \Rightarrow \{\text{effective couplings in low energy scale}\} \]


We use too loop result.  

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In order to illustrate the genuine SUSY effect on $hhh$ coupling, it’s convenient to define:

\[
\delta_{hhh} = \frac{\lambda_{hhh}^{SM} - \lambda_{hhh}^{SUSY}}{\lambda_{hhh}^{SM}} = \left( \frac{3m_h^2}{\sqrt{2}v} - \lambda_{hhh}^{SUSY} \right) / \frac{3m_h^2}{\sqrt{2}v}
\]  

(11)

We will call the quantity ”triple coupling deviation”. This triple coupling deviation is more suitable than the absolute value of triple coupling, because the uncertainty in Higgs mass calculation have little impact to this deviation.

\[
\left\{ \text{all non-SM particles mass} \to \infty \right\} \Rightarrow \left\{ \delta_{hhh} \to 0 \right\}
\]

(for conciseness, we don’t write $m_t^4$ term in here)
MSSM Higgs sector dominantly decided by $m_A$ and $\tan \beta$. $\delta_{hhh}$ dependency on $m_A$ and $\tan \beta$ is the under graph (other parameters: all the other soft mass and $A_t$ are equal to $m_{q3}$, and other $A$-terms are zero):

![Graph showing dependency of $\delta_{hhh}$ on $m_A$ and $\tan \beta$. Red and green lines represent different values of $m_{q3}$. The graph suggests that small $\tan \beta$ and light $m_A$ are wanted.](image)

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$hhh$ Coupling in MSSM and NMSSM

Scan range: $1 < \tan \beta < 50, 100 \text{GeV} < m_A < 1 \text{TeV}, |\mu| < 1 \text{TeV}, |A_t = A_b| < 6 \text{TeV}, 500 \text{GeV} < m_{u3} = m_{d3} = mq3 < 2.5 \text{TeV}$. Other soft mass are equal to 2TeV and other A-terms are equal to zero.
**hhh Coupling in MSSM and NMSSM**

Similar to:
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NMSSM give a solution of the $\mu$-problem in MSSM by introducing a Singlet Field. The Higgs part super potential of NMSSM is:

$$W_{Higgs} = \lambda \hat{S} \hat{H}_u \cdot \hat{H}_d + \frac{\kappa}{3} \hat{S}^3$$

and the corresponding Higgs soft term is:

$$-\mathcal{L}_{soft} = m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_S^2 |S|^2$$

$$+ (\lambda A_\lambda H_u \cdot H_d S + \frac{1}{3} \kappa A_\kappa S^3 + h.c.)$$

In order to describe the NMSSM Higgs sector, we need six parameters:

$\lambda, \kappa, A_\lambda, A_\kappa, \tan \beta, \mu_{eff} = \lambda s.$ ($s$ is the vev of singlet Higgs field)

Such couplings can be very large:

$$\lambda_{sss} = \sqrt{2} \kappa \left( \frac{6 \kappa \mu_{eff}}{\lambda} + \sqrt{2} A_\kappa \right), \lambda_{uds} = -\frac{\lambda A_\lambda}{\sqrt{2}} - \sqrt{2} \kappa \mu_{eff}. \quad (14)$$

and the mixing between singlet and SM-like Higgs – $O_{HS}$ is the most relevant quantity.
The new couplings in NMSSM are not fixed by gauge coupling and can vary in a large range, so significant mixing $O_{HS}$ will induce a very different Higgs triple coupling.

We use NMSSMTools to scan the parameter space.

Scan range:

- $0 < \lambda < 0.7$, $-0.7 < \kappa < 0.7$, $200 \text{GeV} < A_\lambda < 1 \text{TeV}$, $-500 \text{GeV} < A_\kappa < 500 \text{GeV}$
- $1 < \tan \beta < 20$, $100 \text{GeV} < \mu < 500 \text{GeV}$
- $500 \text{GeV} < m_{q3} = m_{u3} = m_{d3} < 2.5 \text{TeV}$, $|A_t = A_b| < 6 \text{TeV}$

Other soft mass are equal to 2TeV and other A-terms are equal to zero.

Most relevant constrains: Higgs mass and Higgs signal strength in $2\sigma$ region. Higgs mass will strongly limit Higgs sector parameters and Higgs signal strength will restrict $O_{HS}$ directly.
**hhh Coupling in MSSM and NMSSM**

huge deviation is a hint of another scalar in the neighbor

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Coupling Measurement in Future Colliders

\[
\begin{align*}
\text{mixing between SM-like Higgs} & \quad \text{and non-SM Higgs} \\
& \implies C_{hbb}, C_{h\gamma\gamma} \ldots \text{deviation}
\end{align*}
\]

\[\lambda_{hhh} \text{ deviation}
\]

So there is a close relationship between $C_{hbb}, C_{h\gamma\gamma} \ldots$ and $\lambda_{hhh}$. Precise measurement on Higgs couplings will confine $\lambda_{hhh}$.

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**Table 1-20.** Expected precisions on the Higgs couplings and total width from a constrained 7-parameter fit assuming no non-SM production or decay modes. The fit assumes generation universality ($\kappa_u \equiv \kappa_t = \kappa_c$, $\kappa_d \equiv \kappa_b = \kappa_s$, and $\kappa_\ell \equiv \kappa_\tau = \kappa_\mu$). The ranges shown for LHC and HL-LHC represent the conservative and optimistic scenarios for systematic and theory uncertainties. ILC numbers assume $(e^-, e^+)$ polarizations of $(-0.8, 0.3)$ at 250 and 500 GeV and $(-0.8, 0.2)$ at 1000 GeV, plus a 0.5% theory uncertainty. CLIC numbers assume polarizations of $(-0.8, 0)$ for energies above 1 TeV. TLEP numbers assume unpolarized beams.

<table>
<thead>
<tr>
<th>Facility</th>
<th>LHC</th>
<th>HL-LHC</th>
<th>ILC500</th>
<th>ILC500-up</th>
<th>ILC1000</th>
<th>ILC1000-up</th>
<th>CLIC</th>
<th>TLEP (4 IPs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{s}$ (GeV)</td>
<td>14,000</td>
<td>14,000</td>
<td>250/500</td>
<td>250/500</td>
<td>250/500/1000</td>
<td>250/500/1000</td>
<td>350/1400/3000</td>
<td>240/350</td>
</tr>
<tr>
<td>$\int L dt$ (fb$^{-1}$)</td>
<td>300/expt</td>
<td>3000/expt</td>
<td>250+500</td>
<td>1150+1600</td>
<td>250+500+1000</td>
<td>1150+1600+2500</td>
<td>500+1500+2000</td>
<td>10,000+2600</td>
</tr>
<tr>
<td>$\kappa_\gamma$</td>
<td>5 – 7%</td>
<td>2 – 5%</td>
<td>8.3%</td>
<td>4.4%</td>
<td>3.8%</td>
<td>2.3%</td>
<td>$-5.5/\leq5.5$</td>
<td>1.45%</td>
</tr>
<tr>
<td>$\kappa_g$</td>
<td>6 – 8%</td>
<td>3 – 5%</td>
<td>2.0%</td>
<td>1.1%</td>
<td>1.1%</td>
<td>0.67%</td>
<td>3.6/0.79/0.56%</td>
<td>0.79%</td>
</tr>
<tr>
<td>$\kappa_W$</td>
<td>$\pm 6%$</td>
<td>2 – 5%</td>
<td>0.39%</td>
<td>0.21%</td>
<td>0.21%</td>
<td>0.2%</td>
<td>1.5/0.15/0.11%</td>
<td>0.10%</td>
</tr>
<tr>
<td>$\kappa_Z$</td>
<td>$\pm 6%$</td>
<td>2 – 4%</td>
<td>0.49%</td>
<td>0.24%</td>
<td>0.50%</td>
<td>0.3%</td>
<td>0.49/0.33/0.24%</td>
<td>0.05%</td>
</tr>
<tr>
<td>$\kappa_\ell$</td>
<td>$\pm 8%$</td>
<td>2 – 5%</td>
<td>1.9%</td>
<td>0.98%</td>
<td>1.3%</td>
<td>0.72%</td>
<td>3.5/1.4/\leq1.3%</td>
<td>0.51%</td>
</tr>
<tr>
<td>$\kappa_d = \kappa_b$</td>
<td>10 – 13%</td>
<td>4 – 7%</td>
<td>0.93%</td>
<td>0.60%</td>
<td>0.51%</td>
<td>0.4%</td>
<td>1.7/0.32/0.19%</td>
<td>0.39%</td>
</tr>
<tr>
<td>$\kappa_u = \kappa_\ell$</td>
<td>14 – 15%</td>
<td>7 – 10%</td>
<td>2.5%</td>
<td>1.3%</td>
<td>1.3%</td>
<td>0.9%</td>
<td>3.1/1.0/0.7%</td>
<td>0.69%</td>
</tr>
</tbody>
</table>

Higgs working group report: 1310.8361
Coupling Measurement in Future Colliders

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hhh Coupling Measurement in Future Colliders
Conclusions

$hhh$ coupling deviation in MSSM: $< 3\%$

$hhh$ coupling deviation in NMSSM: $> 50\%$

$\text{ILC(LumUp) measurement constraint to hhh coupling deviation in MSSM: } < 0.3\%$

$\text{ILC(LumUp) measurement constraint to hhh coupling deviation in NMSSM: } < 1.0\%$