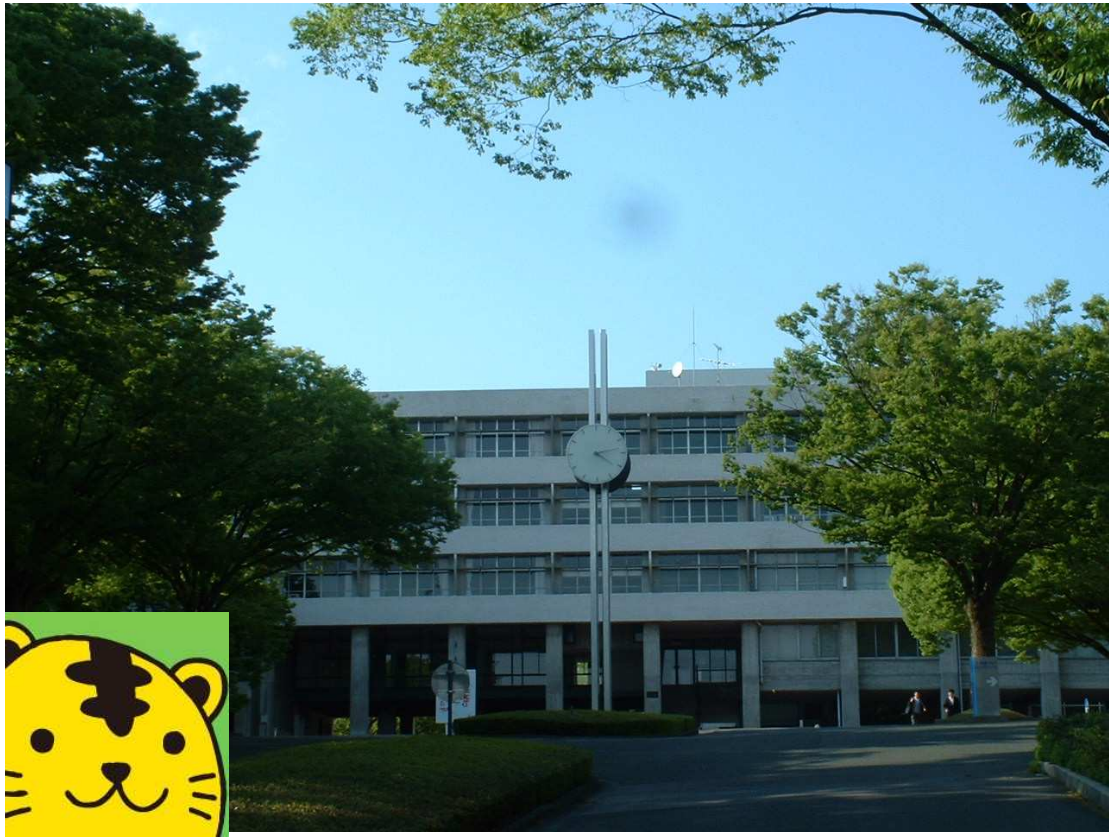
Stueckelberg model
and
Composite Z'



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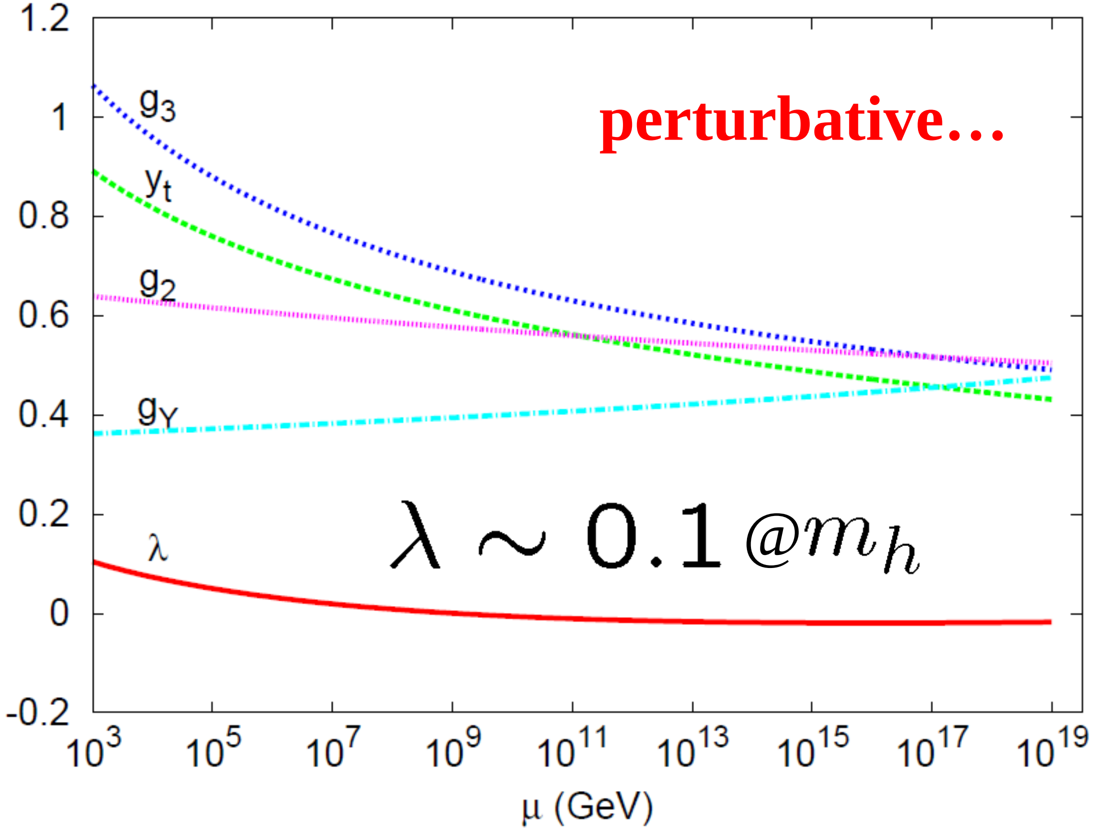
PRD90(2014)096004

(arXiv:1409.4954)

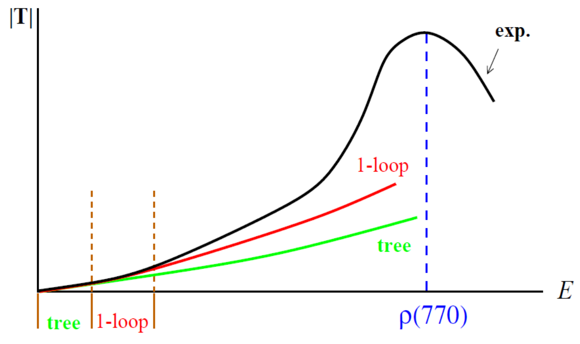
§1 Where is Strong Coupling?

- SM is almost perfectly consistent with experiments. ← M_ν DM, …
- SM is applicable up to the Planck scale. ↔ **Naturalness**
- SM has no strong coupling regime.

Classical conformality ???



QCD was lucky…



Schematic view of P-wave $\pi\pi$ scattering amplitude

But, the Landau pole can be less than the Planck scale,
if the coupling is slightly larger than the hypercharge!

$U(1)_{B-L}$ model

	$SU(3)_c$	$SU(2)_W$	$U(1)_Y$	$U(1)_{B-L}$
q_L	**3**	**2**	$\frac{1}{6}$	$\frac{1}{3}$
u_R	**3**	**1**	$\frac{2}{3}$	$\frac{1}{3}$
d_R	**3**	**1**	$-\frac{1}{3}$	$\frac{1}{3}$
ℓ_L	**1**	**2**	$-\frac{1}{2}$	-1
ν_R	**1**	**1**	0	-1
e_R	**1**	**1**	-1	-1
H	**1**	**2**	$+\frac{1}{2}$	0
χ	**1**	**1**	0	$+2$

$$g^2/(4\pi) \gtrsim 0.015$$

gives

$$\Lambda \lesssim \Lambda_{Pl}$$

cf.) For $U(1)_Y$

$$g_Y^2/(4\pi) \simeq 0.01$$

$$\Rightarrow \Lambda \sim 10^{42}\ \mathrm{GeV}$$

A strong dynamics in high energy may generate

$$\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \psi + G_S (\bar{\psi}\psi)^2 + G_V (\bar{\psi}\gamma^\mu \psi)^2 + G_\Sigma (\bar{\psi}\sigma^{\mu\nu}\psi)^2 + \cdots$$

What is the remnant?

→ **Composite Z'**

Composite scalar χ

M.H, PRD90(2014)096004

§2 Strong Stueckelberg Model is equiv. to NJL Model

Stueckelberg Model for the massive photon

$$\mathcal{L} = \mathcal{L}_\psi + \mathcal{L}_g + \mathcal{L}_{\rm gf}$$

$$\mathcal{L}_\psi = \bar{\psi} i \partial\!\!\!/ \psi + g \bar{\psi} A\!\!\!/ \psi \qquad (-M\bar{\psi}\psi)$$

$$\mathcal{L}_g = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} (m A_\mu - \partial_\mu B)^2$$

$$\mathcal{L}_{\rm gf} = -\frac{1}{2\xi} (\partial_\mu A^\mu + \xi m B)^2$$

$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$

B Stückelberg scalar field

Normalizing $gA_\mu \to A_\mu \; gB \to B$ with $m = gf$, $\xi = 1$

$$\mathcal{L}_\psi = \bar\psi i\partial\!\!\!/\psi + \bar\psi A\!\!\!/\psi$$

$$\mathcal{L}_g \to -\frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}f^2\left(A_\mu - \frac{1}{gf}\partial_\mu B\right)^2$$

$$\mathcal{L}_{\rm gf} \to -\frac{1}{2g^2}(\partial_\mu A^\mu + gfB)^2$$

In the limit $g \to \infty$

$$\mathcal{L} = \bar\psi i\partial\!\!\!/\psi + \bar\psi A\!\!\!/\psi + \frac{1}{2}f^2A_\mu^2 - \frac{1}{2}f^2B^2$$

Gauss integral yields the NJL model:

$$\mathcal{L} \to \bar\psi i\partial\!\!\!/\psi - G_V(\bar\psi\gamma^\mu\psi)^2$$

$$A^\mu \sim \bar\psi\gamma^\mu\psi$$

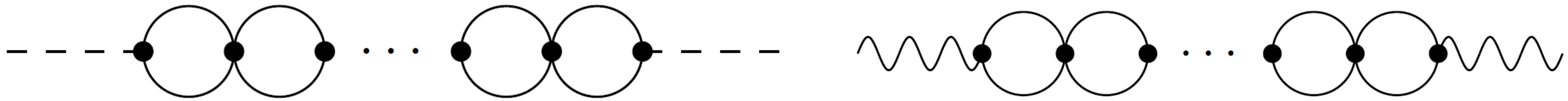
$$G_V = 1/(2f^2)$$

Stueckelberg model as low energy effective theory

$$\mathcal{L} = \bar{\eta} i \partial\!\!\!/ \eta + G_S(\overline{\eta^c}\eta)(\overline{\eta}\eta^c) - G_V(\overline{\eta}\gamma^\mu\eta)^2$$

with Majorana-type scalar four-fermion coupling

The kinetic terms for composite scalar and vector appear in low energy



Low energy EFT

We use the auxiliary fields, $\phi \sim \overline{\eta}\eta^c,\ \phi^\dagger \sim \overline{\eta^c}\eta,\ A_\mu \sim \overline{\eta}\gamma^\mu\eta$

$$\mathcal{L}_{\text{eff}} = \bar{\eta} i D\!\!\!\!/ \eta + Z_\phi |D_\mu\phi|^2 - M_\phi^2\phi^\dagger\phi - \lambda_\phi(\phi^\dagger\phi)^2 - \overline{\eta^c}\eta\phi - \overline{\eta}\eta^c\phi^\dagger - \frac{Z_A}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}f^2A_\mu^2,$$

$$Z_\phi = \frac{1}{16\pi^2}\log\Lambda^2/\mu^2$$

$$Z_A = \frac{1}{24\pi^2}\log\Lambda^2/\mu^2$$

→ **Compositeness conditions**

$$\begin{cases} Z_\phi(\Lambda) = 0 \\ Z_A(\Lambda) = 0 \end{cases}$$

Weinberg rotation

$$\varphi \equiv e^{i\frac{B(x)}{gf}}\eta, \quad \overline{\varphi} \equiv e^{-i\frac{B(x)}{gf}}\overline{\eta}, \quad \chi \equiv e^{-2i\frac{B(x)}{gf}}\phi, \quad \chi^\dagger \equiv e^{2i\frac{B(x)}{gf}}\phi^\dagger,$$

Redefinition of A_μ

$$\tilde{A}_\mu \equiv A_\mu + \frac{1}{gf}\partial_\mu B$$

$$\xi_B \equiv e^{i\frac{B(x)}{gf}}$$

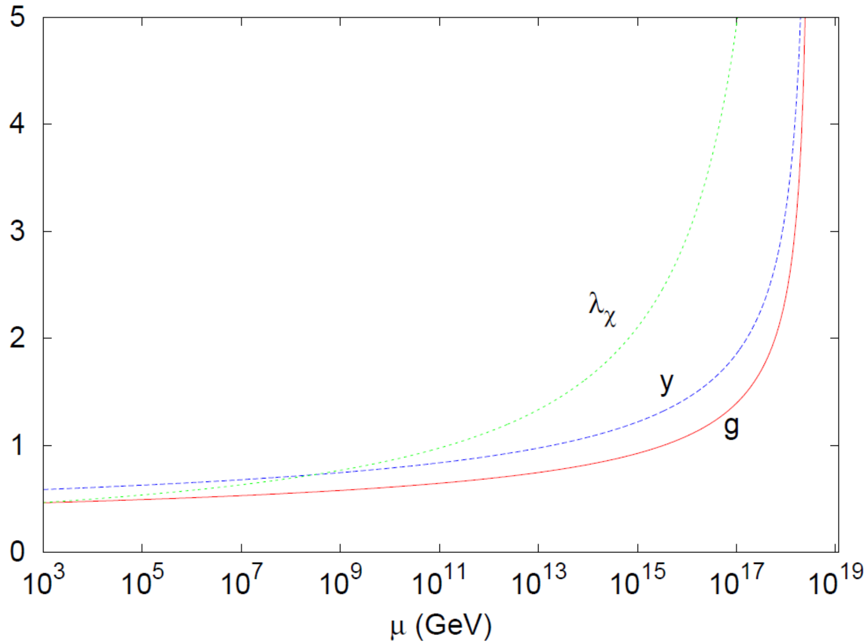
$B(x)$ is a redundant field and we add $\delta(\xi_B - 1)$

This delta function in the partition function corresponds to G.F. term.

We then obtain Stueckelberg model in low energy:

$$\begin{aligned}\mathcal{L}_{\rm eff} = \; & \bar{\varphi}(i\partial\!\!\!/ + g\tilde{A}\!\!\!/)\varphi + |(\partial_\mu + 2ig\tilde{A}_\mu)\chi|^2 - \tilde{M}_\chi^2\chi^\dagger\chi \\ & -\lambda(\chi^\dagger\chi)^2 - y\overline{\varphi^c}\varphi\chi - y\overline{\varphi}\varphi^c\chi^\dagger \\ & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \boxed{\frac{1}{2}g^2 f^2\left(\tilde{A}_\mu - \frac{1}{gf}\partial_\mu B\right)^2}\end{aligned}$$

RGE analysis (BHL, 1990)



$$\Lambda = 1/\sqrt{8\pi G} = 2.435 \times 10^{18}\ \mathrm{GeV}$$

★ NJL at the compositeness scale

$$\boxed{\frac{1}{g^2(\Lambda)}} = \frac{1}{y^2(\Lambda)} = 0, \quad \frac{\lambda(\Lambda)}{y^4(\Lambda)} = 0$$

Compositeness conditions

$$(16\pi^2)\mu\frac{\partial}{\partial\mu}g_i = a_i g_i^3 \qquad a_Y = \frac{41}{6}, \quad a_2 = -\frac{19}{6}, \quad a_3 = -7$$

$$(16\pi^2)\mu\frac{\partial}{\partial\mu}g_{B-L} = g_{B-L}\left[a\, g_{B-L}^2 + 2a'\, g_{B-L}\, g_{\rm mix} + a_Y\, g_{\rm mix}^2\right],$$

$$(16\pi^2)\mu\frac{\partial}{\partial\mu}g_{\rm mix} = g_{\rm mix}\left[a_Y\,(g_{\rm mix}^2 + 2g_Y^2) + a\, g_{B-L}^2\right] + 2a'\, g_{B-L}(g_{\rm mix}^2 + g_Y^2),$$

$$a = \frac{32}{9}N_g + \frac{4}{3}N_\chi, \qquad a' = \frac{16}{9}N_g$$

$$(16\pi^2)\mu\frac{\partial}{\partial\mu}y_t = y_t\left[\frac{9}{2}y_t^2 - \left(8g_3^2 + \frac{9}{4}g_2^2 + \frac{17}{12}(g_Y^2 + g_{\rm mix}^2) + \frac{2}{3}g_{B-L}^2 + \frac{5}{3}g_{B-L}\, g_{\rm mix}\right)\right]$$

MH, Iso, Orikasa,
PRD89 (2014) 016019;
PRD89 (2014) 056010.

Full set of RGE's

$$(16\pi^2)\mu\frac{\partial}{\partial\mu}y_M = y_M\left[(4+2N_\nu)y_M^2 - 6g_{B-L}^2\right]$$

$$Y_M^{ij} = \mathrm{diag}(y_M, \cdots, 0, \cdots)$$
$$\mathrm{tr}[(Y_M^{ij})^2] = N_\nu y_M^2$$

$$(16\pi^2)\mu\frac{\partial\lambda_H}{\partial\mu} = 24\lambda_H^2 + \lambda_{\rm mix}^2 - 6y_t^4 + 12\lambda_H y_t^2 + \frac{3}{8}\left(2g_2^4 + (g_2^2 + g_Y^2 + g_{\rm mix}^2)^2\right) - 3\lambda_H(3g_2^2 + g_Y^2 + g_{\rm mix}^2),$$

$$N_\nu = 3$$

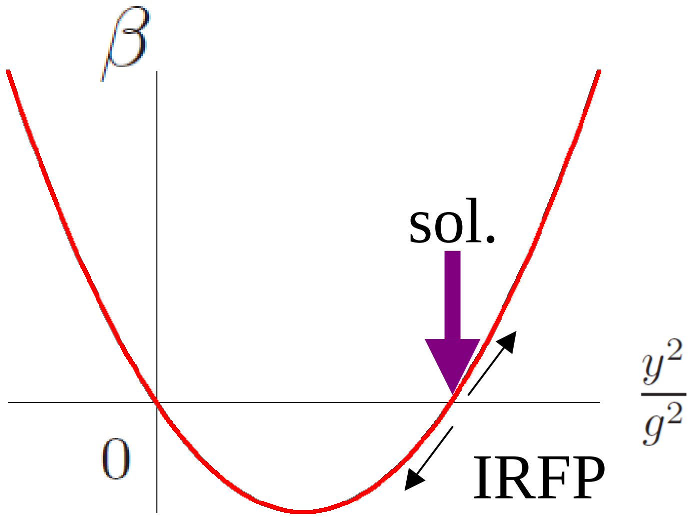
$$(16\pi^2)\mu\frac{\partial\lambda_\chi}{\partial\mu} = 20\lambda_\chi^2 + 2\lambda_{\rm mix}^2 - 16N_\nu y_M^4 + 8N_\nu\lambda_\chi y_M^2 + 96g_{B-L}^4 - 48\lambda_\chi g_{B-L}^2$$

$$(16\pi^2)\mu\frac{\partial\lambda_{\rm mix}}{\partial\mu} = 2\lambda_{\rm mix}\left[6\lambda_H + 4\lambda_\chi + 2\lambda_{\rm mix} + 3y_t^2 + 2N_\nu y_M^2 - \frac{3}{4}(3g_2^2 + g_Y^2 + g_{\rm mix}^2) - 12g_{B-L}^2\right] + 12g_{B-L}^2\, g_{\rm mix}^2$$

Infrared fixed point

(Majorana) Yukawa coupling

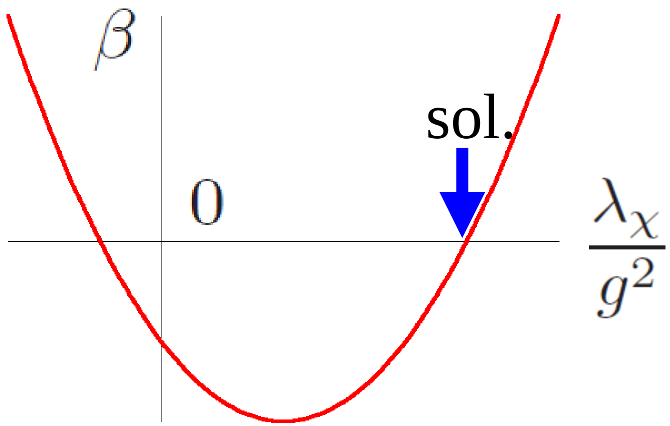
$$(8\pi^2)\,\mu\frac{\partial}{\partial\mu}\left(\frac{y^2}{g^2}\right) = b\,g^2\cdot\frac{y^2}{g^2}\left(\frac{y^2}{g^2} - \frac{a+c}{b}\right)$$



Quartic coupling

$$(16\pi^2)\mu\frac{\partial}{\partial\mu}\left(\frac{\lambda_\chi}{g^2}\right) = 20g^2\left(\frac{\lambda_\chi}{g^2} - k_+\right)\left(\frac{\lambda_\chi}{g^2} - k_-\right)$$

$$k_+ \equiv \tfrac{2}{25}(9+\sqrt{546}) \simeq 2.589$$
$$k_- \equiv \tfrac{2}{25}(9-\sqrt{546}) \simeq -1.149$$



Essence of RGE's

$$\beta_g \equiv \mu\frac{\partial}{\partial\mu}g = \frac{a}{16\pi^2}g^3,$$
$$\beta_y \equiv \mu\frac{\partial}{\partial\mu}y = \frac{y}{16\pi^2}\left[by^2 - cg^2\right]$$
$$\beta_{\lambda_\chi} \equiv \mu\frac{\partial}{\partial\mu}\lambda_\chi = \frac{1}{16\pi^2}\left[20\lambda_\chi^2 + \lambda_\chi(24y^2 - 48g^2) - 48y^4 + 96g^4\right]$$

$$a = 12$$
$$b = 10$$
$$c = 6$$

§.3 Summary

★ The possibility of composite Z' was discussed.
In particular, the nature is controlled by the IRFP.
The strength of the gauge coupling and the existence of
the Stueckelberg mass term suggest the compositeness of Z'.

$< \chi >= v_\chi/\sqrt{2}$

Mass $M_{\nu_R}^2 \simeq 2y^2 v_\chi^2, \quad M_\chi^2 \simeq 2\lambda_\chi v_\chi^2, \quad M_{Z'}^2 \simeq 4g^2 v_\chi^2 + \boxed{g^2 f^2}$

Evidence of Composite Z' $g^2/(4\pi) \gtrsim 0.015 \quad \Delta \equiv \frac{M_{Z'}^2}{g^2} - 4v_\chi^2 = f^2 > 0$

Prediction from IRFP

$$\frac{M_\chi}{M_{\nu_R}} = \frac{\sqrt{\lambda_\chi}}{y} \approx 1.2$$

Future work Phenomenology of Z', Masses of quarks and lepton…