Introduction to Supersymmetry
(Supersymmetry Breaking, Mediation, etc...)

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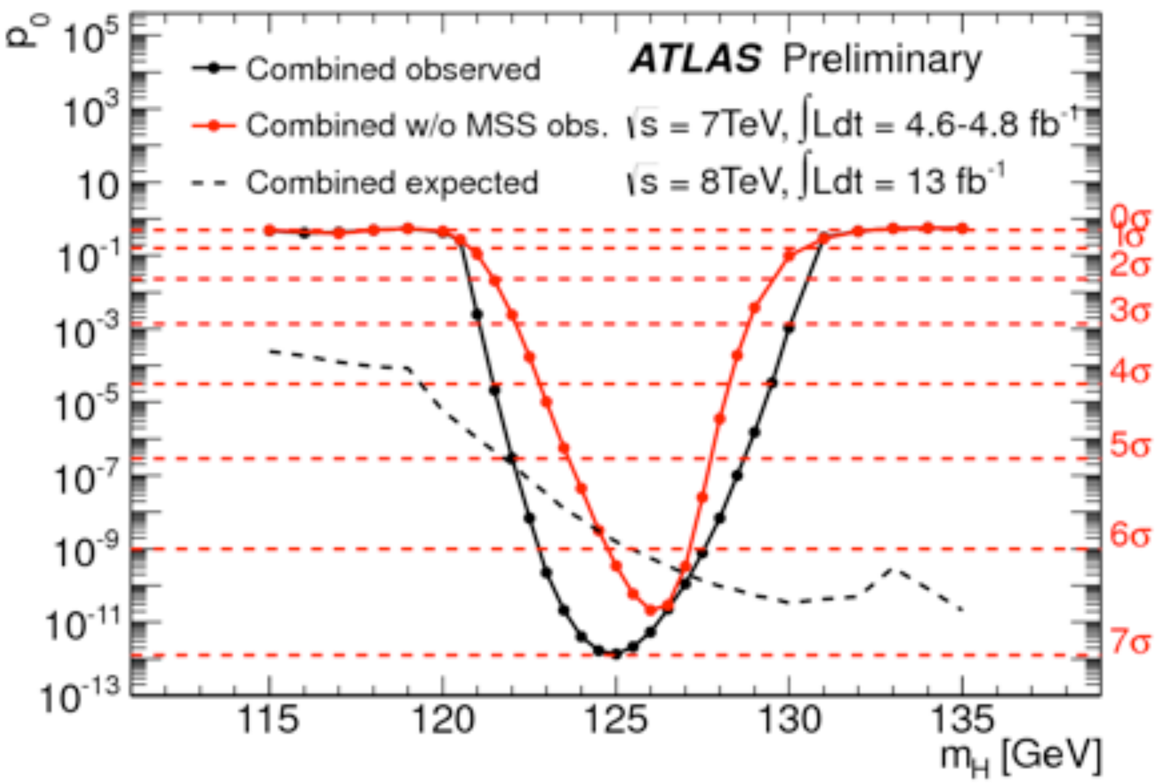
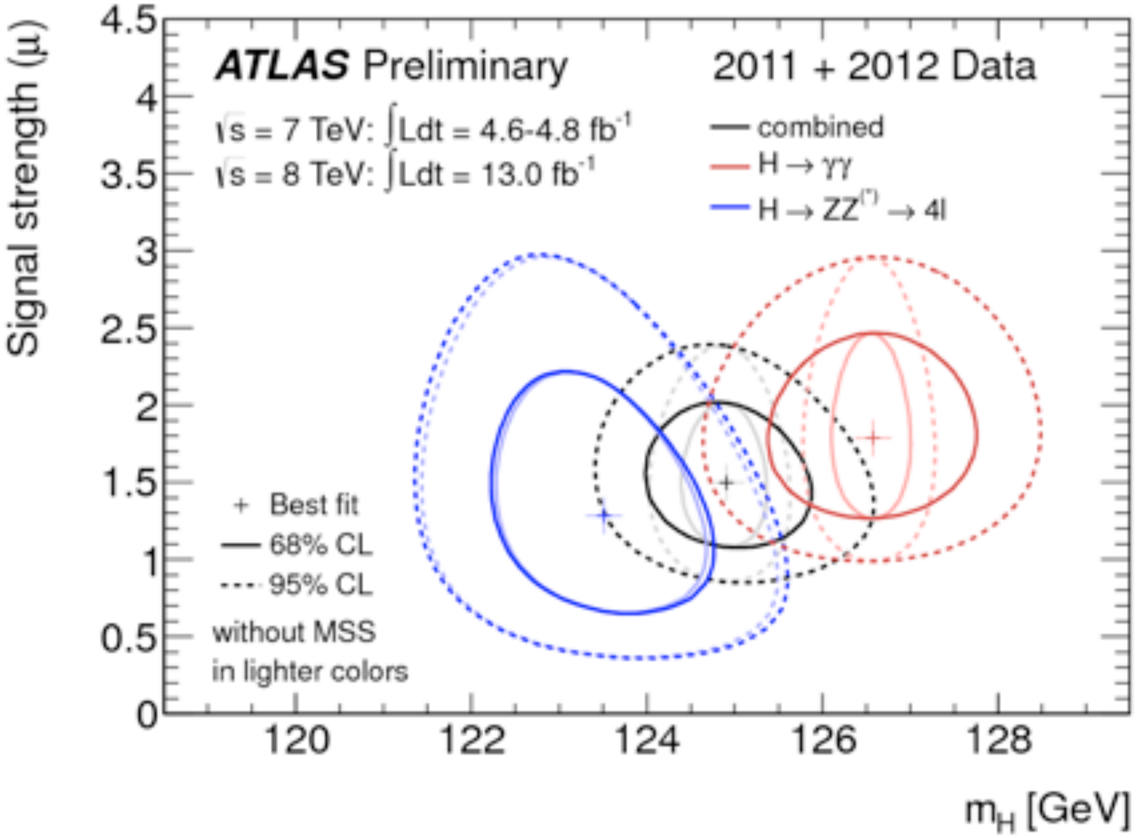
Introduction

Higgs(-like particle) has been discovered!





$H \to \gamma\gamma$ peak at 125(127?)GeV (6σ)

$H \to ZZ$(4lepton) peak at 125(123?)GeV (4σ)

$H \to WW$ 2.8σ excess

$H \to b\bar{b}$ has not been confirmed. (too many background)

$H \to \tau\bar{\tau}$ has been found ($\sigma/\sigma_{SM}=1.1\pm 0.4$ @ CMS)

Basic properties look consistent with the SM Higgs boson!

Introduction

What do we learn from the discovery?

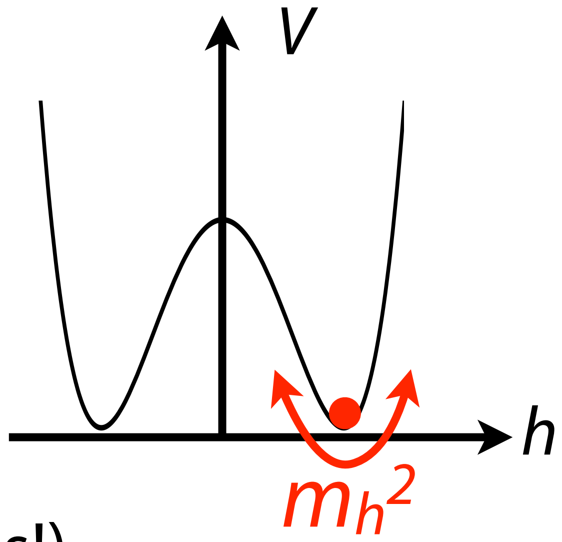
1. Higgsless models are almost excluded!
2. Higgs is more like an elementary scalar!

The simplest implementation

$$V = -\, m_{higgs}^2/2\; h^\dagger h + \lambda/4\; (h^\dagger h)^2$$

$$m_{higgs} = \lambda^{1/2}\, v \;\; [\, v = 174.1\mathrm{GeV}]$$

$$m_{higgs} \sim 125\mathrm{GeV} \quad \longrightarrow \quad \lambda \sim 0.5$$

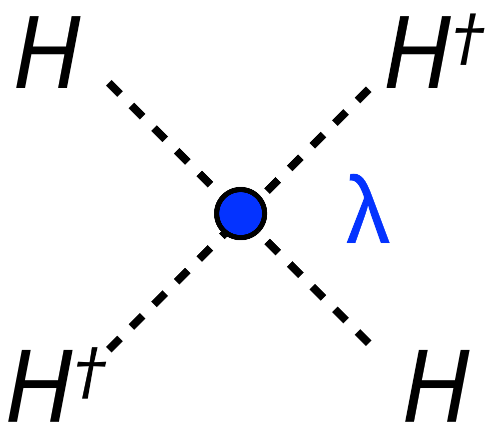


(We knew *v=174.1GeV* before the discovery of Higgs!)

The quartic coupling is small and this simple elementary scalar Higgs description works consistently!

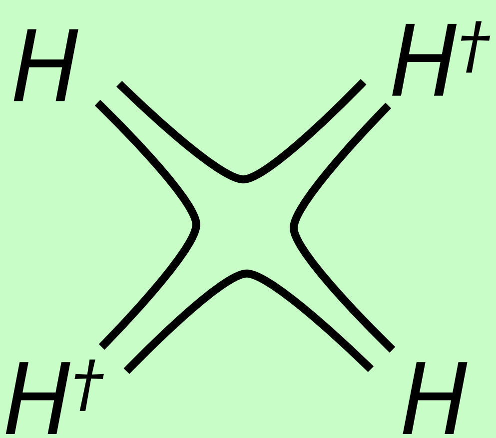
Introduction

The size of λ provides us hints on the Physics behind the SM!



λ is the coefficient of the quartic coupling...

If the Higgs is a composite state bounded by dynamics at around *O(100)GeV* scale,



λ is expected to be very large ($\simeq 4\pi$)

(Exceptional models : NGB Higgs

→ Top Yukawa coupling is difficult...)

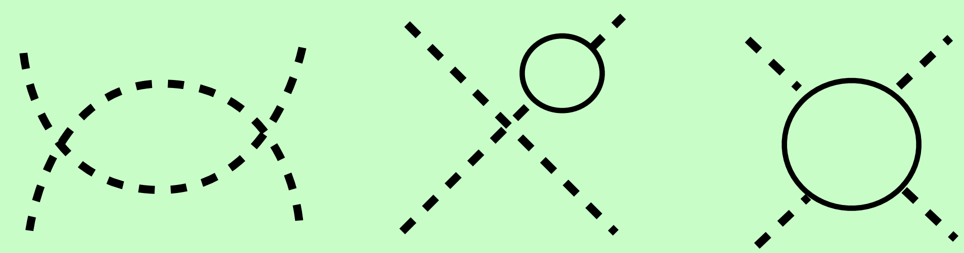
$m_h \simeq 126GeV$ ($\lambda \simeq 0.5$) suggests that the observed Higgs is more like an elementary scalar!

Introduction

Furthermore, the elementary scalar Higgs description can be consistent even up to the Planck scale for $m_h \simeq 126GeV$!

RGE of the quartic coupling...

$$\frac{d\lambda}{d\ln E/E_0} = \frac{1}{16\pi^2}\left(\boxed{12\lambda^2 + 12\lambda y_t^2} - \boxed{12y_t^4} + \ldots\right)$$



($y_t \simeq 0.95$ Top Yukawa coupling)

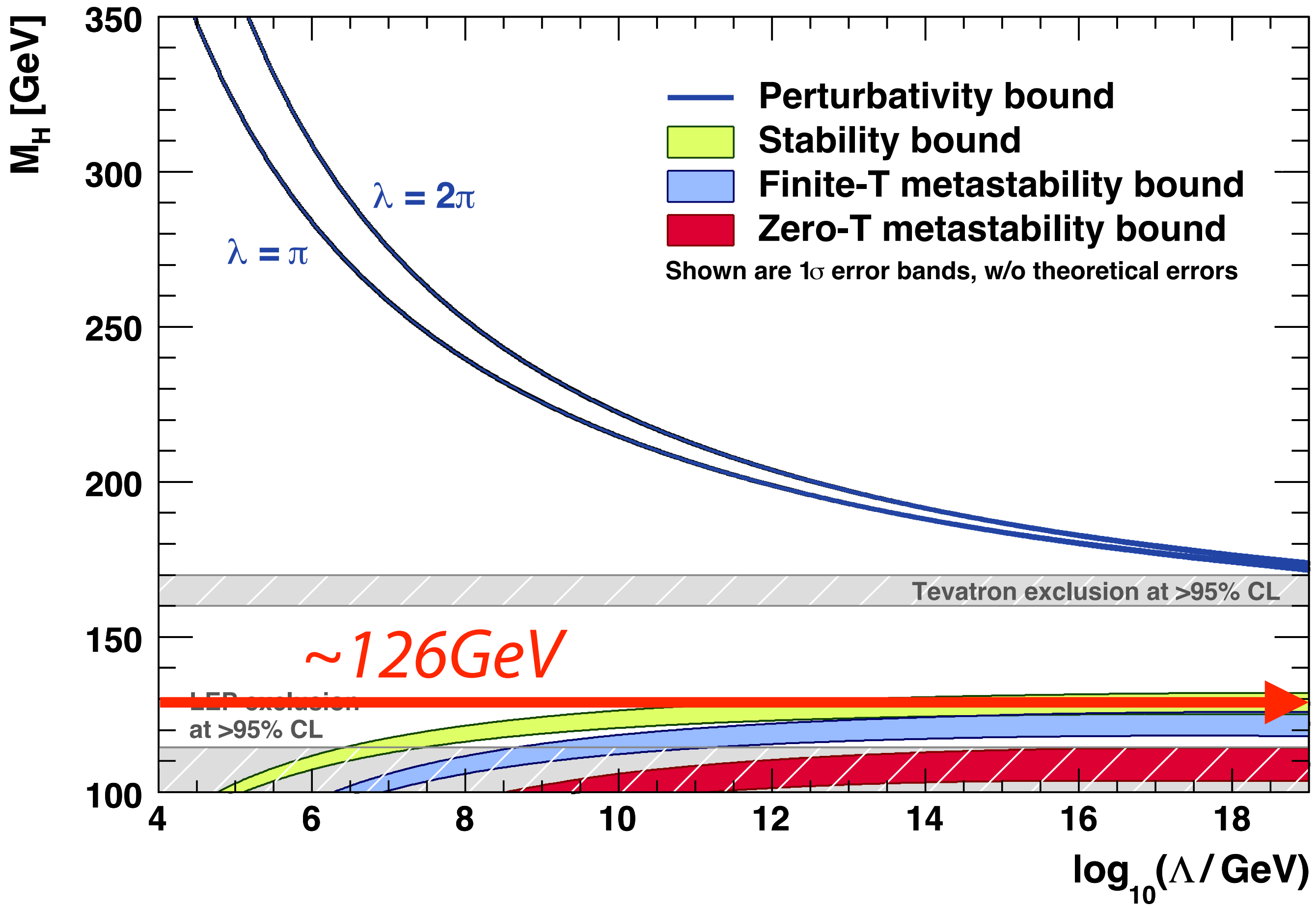
makes λ large at the high energy → Landau pole

draws λ at the high energy scale → Vacuum instability

Introduction

Furthermore, the elementary scalar Higgs description can be consistent even up to the Planck scale for $m_h \simeq 126GeV$!

['09, Ellis,Espinoza,Giudice,Hoecker, Riotto]



No positive Hints on New Physics?

Introduction

We have a lot of motivation for new physics beyond!

Quantum Gravity? 10^{18}GeV

Grand Unification? $10^{14\text{-}17}$GeV

Inflation? $10^{\sim 16}$GeV

Neutrino? $10^{2\sim 15}$GeV

log E

Standard Model $\sim 10^{2}$GeV

Dark Matter (WIMP)? $10^{2\text{-}5}$GeV

Dark Matter (axion)? $10^{9\text{-}12}$GeV

There should be a lot of unknown possibilities!

Among them, the grand unification is the most attractive!

The matter content of the SM looks complicated...

	$SU(3)$	$SU(2)$	$U(1)$
$q_L^{1,2,3} = \begin{pmatrix} u_L^{1,2,3} \\ d_L^{1,2,3} \end{pmatrix}$	3	2	$1/6$
$\bar{U}_R^{1,2,3}$ $\bar{D}_R^{1,2,3}$	3^* 3^*	- -	$-2/3$ $1/3$
$l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	-	2	$-1/2$
$\bar{E}_R$	-	-	1

Introduction

Once we embed $SU(3) \times SU(2) \times U(1)$ into $SU(5)$...

Matter multiplets are embedded into only two multiplets!

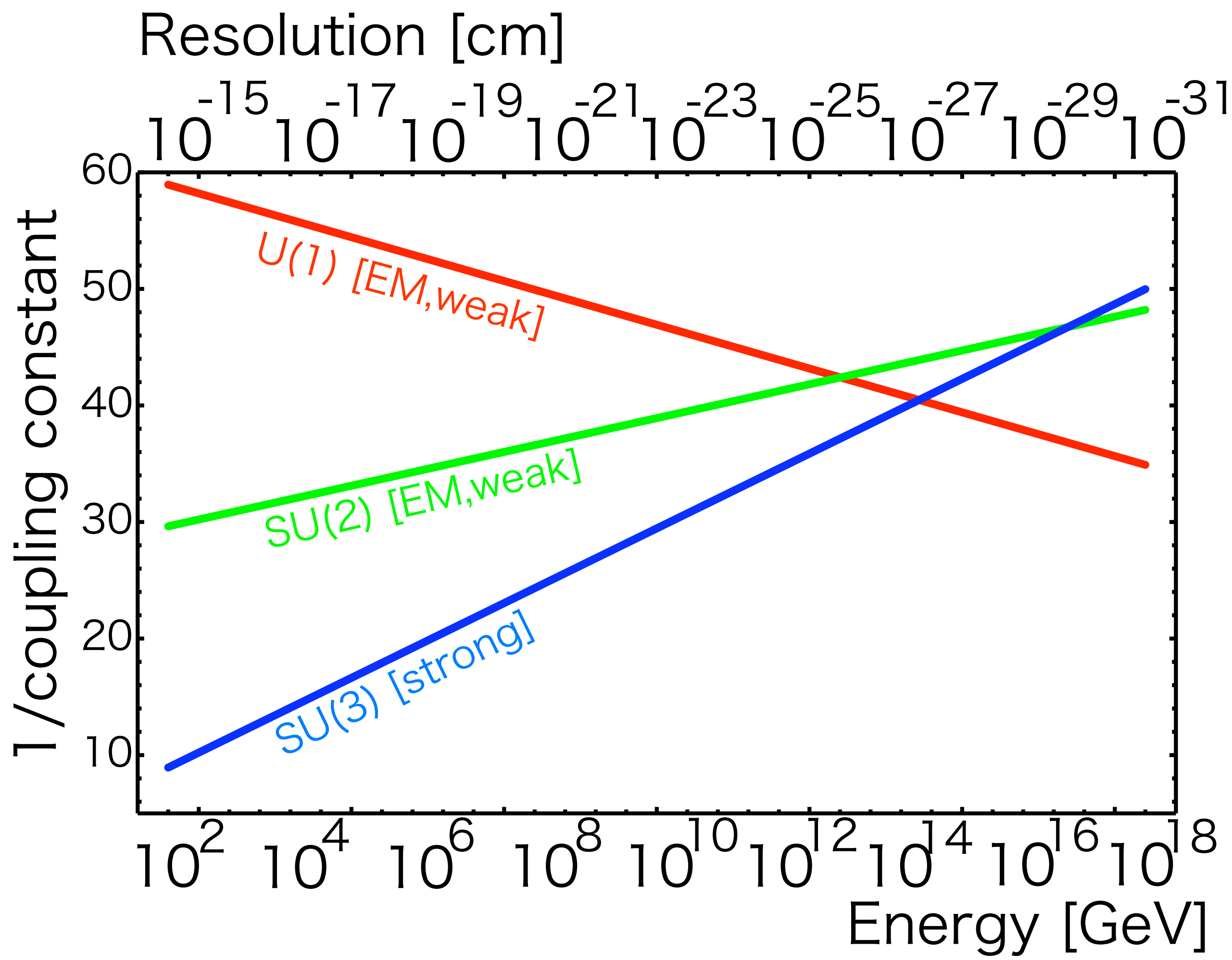
$$\psi(5^*) = \begin{pmatrix} \bar{D}_R^1 \\ \bar{D}_R^2 \\ \bar{D}_R^3 \\ L_L^1 \\ L_L^2 \end{pmatrix} \qquad \psi(10) = \begin{pmatrix} 0 & \bar{U}_R^3 & -\bar{U}_R^2 & U_L^1 & D_L^1 \\ -\bar{U}_R^3 & 0 & \bar{U}_R^1 & U_L^2 & D_L^2 \\ \bar{U}_R^2 & -\bar{U}_R^1 & 0 & U_L^3 & D_L^3 \\ -U_L^1 & -U_L^2 & -U_L^3 & 0 & \bar{E}_R \\ -D_L^1 & -D_L^2 & -D_L^3 & -\bar{E}_R & 0 \end{pmatrix}$$

It seems more than a coincidence!

[In addition, in the Grand Unified Theory, the neutrality of the atom (i.e. the charge quantization of U(1) can be easily understood]

Introduction

The Grand Unification is also suggested by the fact that the three gauge coupling constants tend to unify at the very high energy scale at *$10^{\sim 14\text{-}17}$GeV*



It seems more than a coincidence too!

Introduction

Once we consider at $10^{\sim 14-17}GeV$, we encounters the so-called Hierarchy problem :

Why (weak scale) << (GUT scale) ?

In the simplest model,

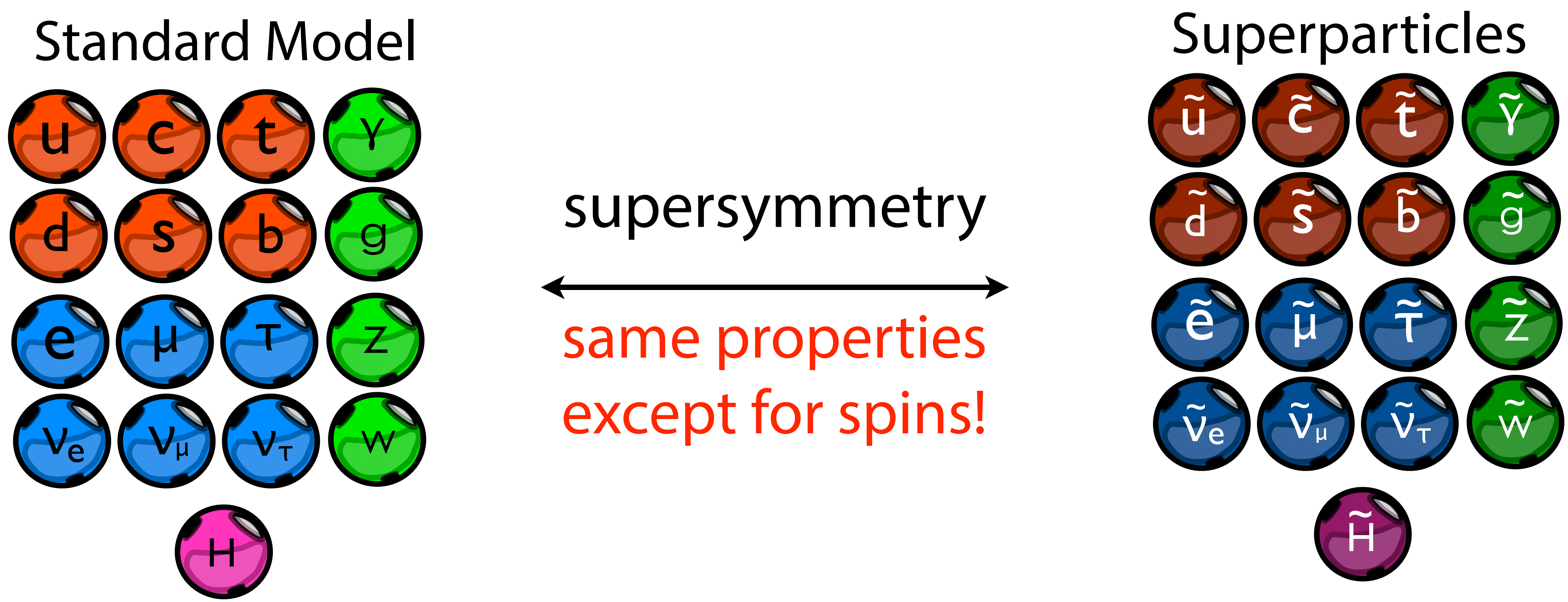
$$V = -m_{higgs}^2/2\, h^\dagger h + \lambda/4\, (h^\dagger h)^2$$

m_{higgs}^2 is not protected by any symmetries

(i.e. no symmetry is enhanced in the limit of $m_{higgs}^2 \to 0$)

The hierarchy problem must give us a hint on new physics which is not so above the $O(100)GeV$ scale!

Supersymmetric Standard Model



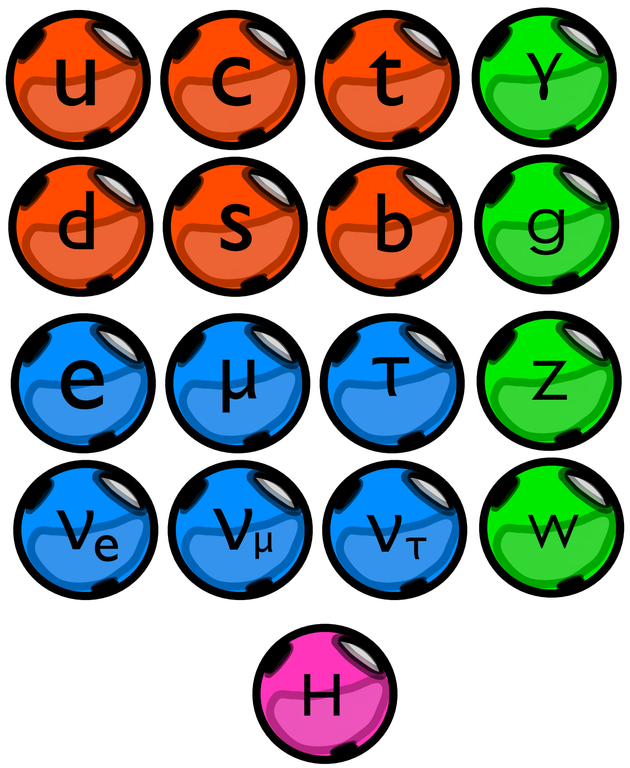
In the supersymmetric extension of the SM, we simply introduce superpartners of the SM particles.

We also extend the interactions so that the theory respects supersymmetry.

Introduction

Supersymmetric Standard Model

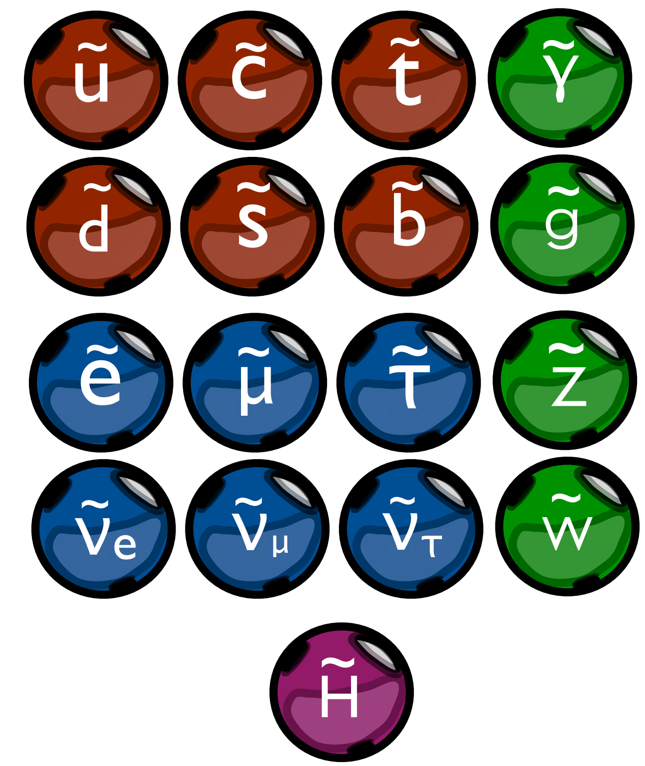
Standard Model



supersymmetry

same properties except for spins!

Superparticles



Higgs mass term is protected!

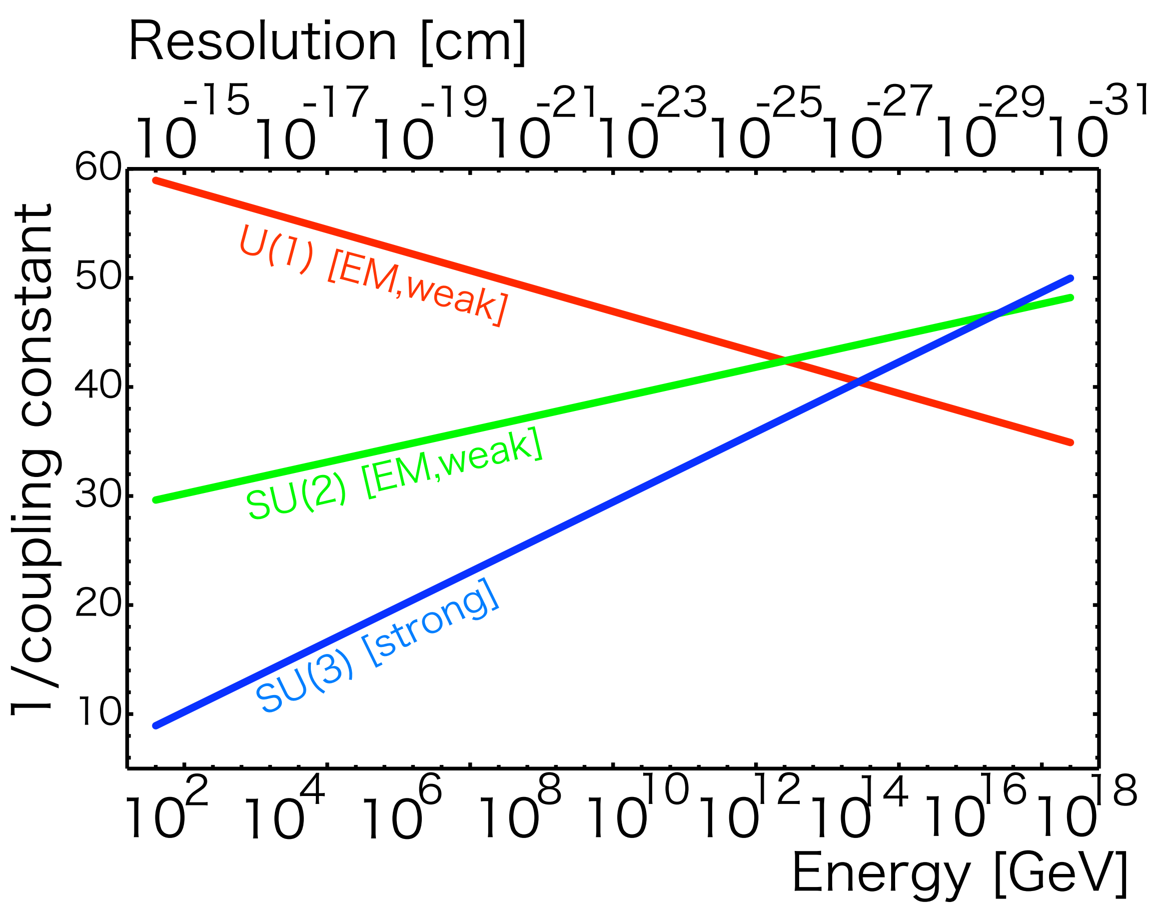
Higgs mass term = Higgsino mass term

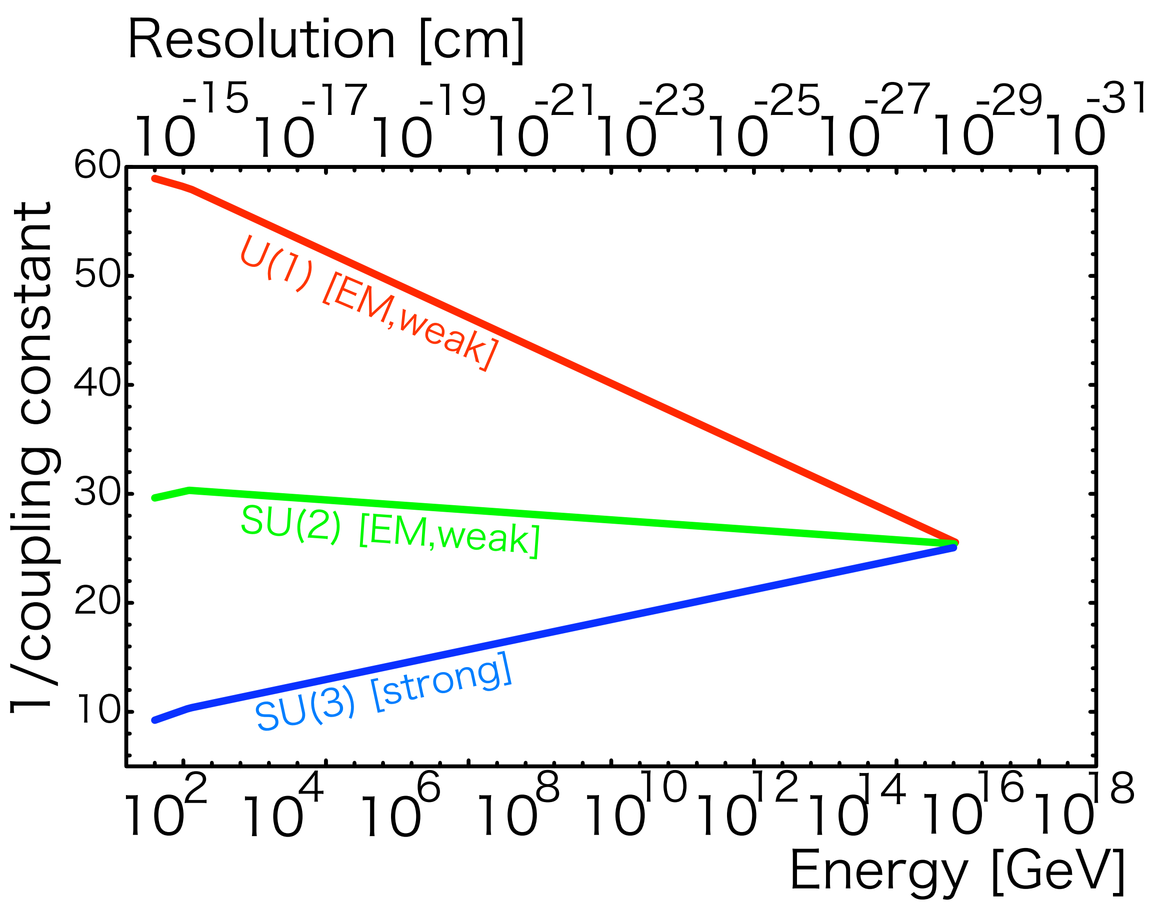
Higgs mass term can be protected by the chiral symmetry!

Hierarchy problem is solved if SUSY breaking is around TeV.

Bonus!

Just by introducing the superpartners at around TeV, the three gauge coupling constants become more precise!





It seems much more than a coincidence!

Why Supersymmetry?

No observation of superparticles so far....

gluino mass > *1-1.5 TeV*

Is SUSY still motivated?

✔ SUSY is an extension of the spacetime symmetry.

It is exciting if there is SUSY in nature!

(although it's not convincing at all...)

✔ SUSY models are consistent with the elementary Higgs.

It is now supported by the discovery of the Higgs.

✔ In the MSSM, the Higgs boson mass is interrelated to the mass scale of not yet observed sparticle masses!

It is interesting to ask which SUSY breaking scale the observed Higgs boson mass implies.

[In the SM, the Higgs boson mass is a free parameter]

Coleman-Mandula Theorem

Symmetry

Unitary operator *U* on Hilbert space is a symmetry transformation if :

1) *U* maps one-particle states → one-particle state

2) Many particle states = tensor products

A : infinitesimal generator of U

$$A(|p1>|p2>) = (A|p1>)|p2> + |p1>(A|p2>)$$

3) *U* (or *A*) commutes with the *S*-matrix

ex.) Spacetime symmetry : Lorentz symmetry + Translation (Poincare symmetry)

Internal symmetries : *SU(3)xSU(2)xU(1)* gauge symmetry, Baryon, Lepton symmetries, etc...

Coleman-Mandula Theorem

Can we extend spacetime symmetry larger than the Poincare symmetry?

Coleman-Mandula Theorem (No-Go theorem in d>2)

1) For any *M*, there are only a finite number of particle types with mass less than *M*.

2) Scattering occurs at almost all energies

3) The amplitudes for elastic two-body scattering are analytic functions of the scattering angle at almost all energies and angles.

Symmetry of S-matrix consists of the direct product of the Poincare symmetry and the internal symmetry!

Only exception = Supersymmetry!

Coleman-Mandula Theorem

1) $[\, B, P_\mu \,] = 0$, $\quad [\, B_a, B_b] = i\, C_{ab}{}^c\, B_c$

consider two particle state

$$B|p_1 m, p_2 n\rangle = \sum b(p_1, p_2)^{mn}_{m'n'} |p_1 m', p_2 n'\rangle$$

[m,n: indices of spins and internal symmetries]

Let us consider a scattering : $(p_1, p_2) \to (q_1, q_2)$

$$b(q_1, q_2)^{m'n'}_{lk} S(q_1, q_2; p_1, p_2)^{lk}_{mn} = S(q_1, q_2; p_1, p_2)^{m'n'}_{lk} b(p_1, p_2)^{lk}_{mn}$$

$$\mathrm{Tr}\, b(q_1, q_2) = \mathrm{Tr}\, b(p_1, p_2)$$

$$\mathrm{tr}\, b(q_1) + \mathrm{tr}\, b(q_2) = \mathrm{tr}\, b(p_1) + \mathrm{tr}\, b(p_2)$$

for any p's and q's with $p_1 + p_2 = q_1 + q_2$

[cf. $B|p\rangle|p'\rangle = (B|p\rangle)|p'\rangle + |p\rangle(B|p'\rangle)$]

Coleman-Mandula Theorem

1) $[\,B, P_\mu\,] = 0$, $\quad [\,B_a, B_b] = i\,C_{ab}{}^{c}\,B_c$

$\mathrm{tr}\,B = a^\mu P_\mu$ (a^μ : p and spin independent)

$\rightarrow\ B = a^\mu P_\mu \ \oplus\ B^\#$ $\quad$ ($B^\#$ traceless)

$[\,B^\#{}_a, B^\#{}_b] = i\,C_{ab}{}^{c}\,B^\#{}_c$ $\qquad$ ($\leftarrow$ Not true in SUSY!)

Let me skip the proof...

$B^\#$: commutes with $J_{\mu\nu}$ and momentum independent!

$\rightarrow$ $B^\#$ are internal symmetries!

[cf. semi-simple $B^\#$ case : $[\,J_{\mu\nu}, B^\#{}_a\,] \neq 0$, $B^\#{}_a$ goes to $D(\Lambda)_b{}^a\,B^\#{}_a$ under the Lorentz transformation Λ. We can show that $D(\Lambda)_b{}^a$ consists finite dimensional unitary representation of Λ which should not exist!
Thus, $= 1$, $[\,J_{\mu\nu}, B^\#{}_a\,] = 0$.]

Coleman-Mandula Theorem

2) $[\,A, P_\mu\,] \neq 0$: A changes the momentum of the state:

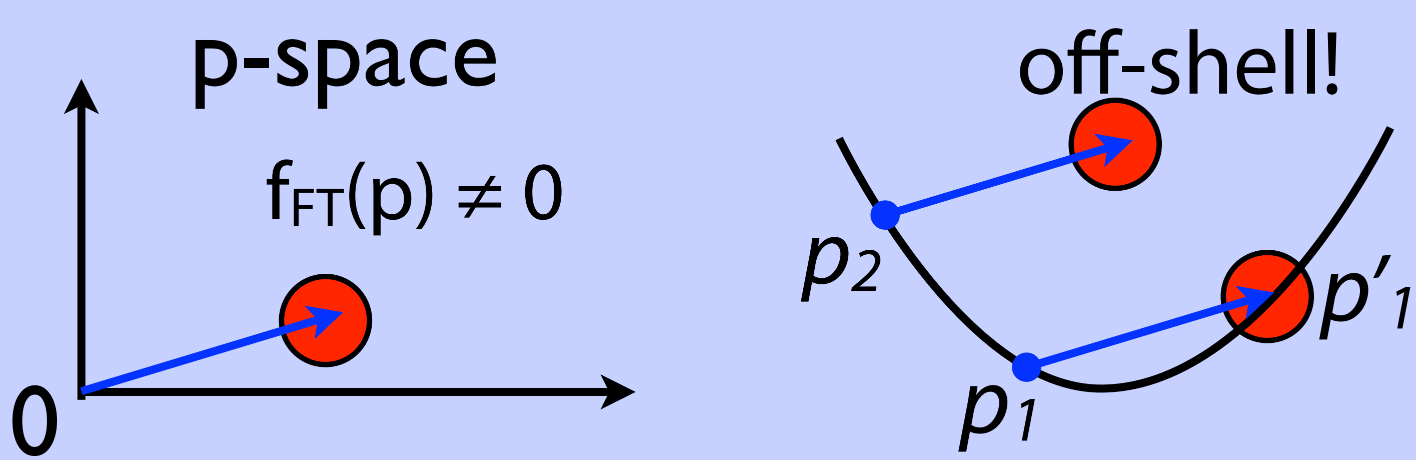
$$A|p\rangle = \int d^4p'\, A(p', p)|p'\rangle \qquad \text{(p, p': on-shell)}$$

Let us consider

$$A^f = \int d^4x\, e^{ixP} A e^{-ixP} f(x)$$

Then $\quad \langle p'|A^f|p\rangle = f_{FT}(p'-p) \times A(p', p)$

We may choose f(x) so that $f_{FT}(p)$ is non-zero in a tiny region.

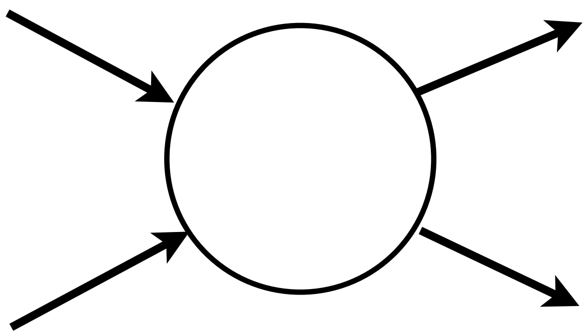


$$A^f|p_1\rangle \neq 0$$

$$A^f|p_2\rangle = 0$$

Coleman-Mandula Theorem

Let us consider a scattering : $(p_1, p_2) \rightarrow (q_1, q_2)$



In particular, we choose

$$A^f|p_1\rangle \neq 0 \qquad A^f|q_2\rangle = 0$$

$$A^f|p_2\rangle = 0 \qquad A^f|q_1\rangle = 0$$

so that

$$A^f|p_1, p_2\rangle = f_{FT}A(p_1', p_1)|p_1', p_2\rangle$$

$$A^f|q_1, q_2\rangle = 0$$

Then, $[S, A^f] = 0$ leads to

$$\langle q_1, q_2|S|p_1', p_2\rangle = 0$$

A^f forbids scattering process where (p'_1, p_2) goes into "any" (q_1, q_2) states! → contradicts with the 3rd condition!

Coleman-Mandula Theorem

$[\,A, P_\mu\,] \neq 0$ generators are at most,

$$A \;=\; \sum_{n=0}^{N} A^{(n)}(p)_{\mu_1,\cdots,\mu_n} \frac{\partial}{\partial p_{\mu_1}} \cdots \frac{\partial}{\partial p_{\mu_n}},$$

with finite N.

Note : $[p, [p, ... ,A]]_N$ commutes with P!

$$[p_{\mu_1}, [p_{\mu_2} \cdots, A] \cdots] = A^{(N)}_{\mu_1 \cdots \mu_N}(p)$$

$$\rightarrow \quad A^{(N)}_{\mu_1 \cdots \mu_N}(p) = a_{\lambda \mu_1 \cdots \mu_N} p^\lambda + b_{\mu_1 \cdots \mu_N}.$$

[Lemma : for $[\,B, P_\mu\,] = 0$, $B = a_\mu P^\mu + B^\#$

(a_μ :constant 4 vector x 1, b: traceless Hermitian matrix)]

Coleman-Mandula Theorem

$[A, P_\mu] \neq 0$ generators are at most,

$$A = \sum_{n=0}^{N} A^{(n)}(p)_{\mu_1,\cdots,\mu_n} \frac{\partial}{\partial p_{\mu_1}} \cdots \frac{\partial}{\partial p_{\mu_n}},$$

with finite N.

Note : A commutes with $P^\mu P_\mu$!

$$p^{\mu_1} A^{(N)}_{\mu_1 \cdots \mu_N} = a_{\lambda\mu_1\mu_2\cdots\mu_N} p^\lambda p^{\mu_1} + b_{\mu_1\mu_2\cdots\mu_N} p^{\mu_1} = 0$$

$N > 0 \ \rightarrow b = 0$

$a_{\lambda\mu\nu\ldots} = - a_{\mu\lambda\nu\ldots} \rightarrow a = 0$ for $N > 1$.

$(a_{\lambda\mu\nu\ldots} = - a_{\mu\lambda\nu\ldots} = - a_{\mu\nu\lambda\ldots} = a_{\nu\mu\lambda\ldots} = a_{\nu\lambda\mu\ldots} = - a_{\lambda\nu\mu\ldots} = - a_{\lambda\mu\nu\ldots})$

$$A = a_{\mu\nu} p^\mu \frac{\partial}{\partial p_\nu} + b \qquad a_{\mu\nu} = -a_{\nu\mu}$$

absorbed by spacetime Lorentz transf.

Coleman-Mandula Theorem

$[A, P_\mu] \neq 0$ generators are at most,

$$A = \sum_{n=0}^{N} A^{(n)}(p)_{\mu_1,\cdots,\mu_n} \frac{\partial}{\partial p_{\mu_1}} \cdots \frac{\partial}{\partial p_{\mu_n}},$$

with finite N.

Note : A commutes with $P^\mu P_\mu$!

$$p^{\mu_1} A^{(N)}_{\mu_1\cdots\mu_N} = a_{\lambda\mu_1\mu_2\cdots\mu_N} p^\lambda p^{\mu_1} + b_{\mu_1\mu_2\cdots\mu_N} p^{\mu_1} = 0$$

$N > 0 \rightarrow b = 0$

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$A =$ Lorentz transformation $\oplus B$, $([B, P_\mu] = 0)$

Coleman-Mandula Theorem

Coleman-Mandula Theorem (No-Go theorem in d>2)

1) For any *M*, there are only a finite number of particle types with mass less than *M*.

2) Scattering occurs at almost all energies

3) The amplitudes for elastic two-body scattering are analytic functions of the scattering angle at almost all energies and angles.

Symmetry of S-matrix consists of the direct product of the Poincare symmetry and the internal symmetry!

$$A = J_{\mu\nu} \oplus P_{\mu} \oplus B^{\#}$$

Only exception = Supersymmetry!

Supersymmetry

Supersymmetry :	Boson $\rightarrow$ Fermion
	Fermion $\rightarrow$ Boson

Symmetry = Bosonic symmetry B
+ Fermionic symmetry F

Bosonic symmetry : changes spins of states by integers.
Fermionic symmetry : changes spins of states by half integers.
[Here, spin-statistic relation is assumed.]

Poincare, internal symmetries = Bosonic symmetry

Supersymmetry = Fermionic symmetry

Supersymmetry

The generators of B and F can be given by:

$$B = b^\dagger K_{bb}\, b + f^\dagger K_{ff}\, f$$

$$F = f^\dagger K_{fb}\, b + b^\dagger K_{bf}\, f$$

b and f are annihilating operators of bosons and fermions.

$$[\, b_i{}^\dagger, b_j \,] = \delta_{ij}, \quad \{\, f_i{}^\dagger, f_j \,\} = \delta_{ij},$$

(anti)-commutators of B, F are bi-linear!

$$[\, B, B \,] = b^\dagger K_{bb}'\, b + f^\dagger K_{bb}'\, f$$

$$[\, F^{(\dagger)}, B \,] = f^\dagger K_{fb}'\, b + b^\dagger K_{bf}'\, f$$

$$\{\, F^{(\dagger)}, F \,\} = b^\dagger K_{bb}''\, b + f^\dagger K_{ff}''\, f$$

They are also generators of symmetry!

[[F,F], {B,B}, {B,F} are not bi-linear! So don't care!   ]

Supersymmetry

The generators of B and F can be given by:

$$B = b^{\dagger} K_{bb}\, b + f^{\dagger} K_{ff}\, f$$

$$F = f^{\dagger} K_{fb}\, b + b^{\dagger} K_{bf}\, f$$

b and f are annihilating operators of bosons and fermions.

$$[\, b_i^{\dagger}, b_j \,] = \delta_{ij}, \quad \{\, f_i^{\dagger}, f_j \,\} = \delta_{ij},$$

In the presence of Fermionic symmetry, generators of symmetry forms "graded" algebra!

$$[\, B, B \,] = B, \quad [\, F^{(\dagger)}, B \,] = F^{(\dagger)}, \quad \{\, F^{(\dagger)}, F \,\} = B \;\leftarrow \text{New!}$$

Coleman-Mandula theorem is not fully applicable!

Supersymmetry

Symmetry : Graded symmetry algebra

$$[\,B, B\,] = B, \quad [\,F^{(\dagger)}, B\,] = F^{(\dagger)}, \quad \{\,F^{(\dagger)}, F\,\} = B$$

B is closed by themselves and constrained by the CM theorem

$$B = J_{\mu\nu} \oplus P_{\mu} \oplus B^{\#}$$

F changes spin 1/2 by the CM theorem

$$F = Q_{\alpha}^{n} \ (\alpha\text{: spin}, n = 1,\ldots N)$$

If F changes spin n/2 ($n>1$), $\{\,F^{\dagger}, F\,\} = B$ has spin n.

The CM theorem does not allow B with spin $n>1$.

$\rightarrow \{\,F^{\dagger}, F\,\} = 0$ for spin n/2 ($n>1$)

On the positive definite Hilbert space : $\{\,F^{\dagger}, F\,\} = 0 \rightarrow F = 0$

$$\langle \text{state}|\,\{\,F^{\dagger}, F\,\}\,|\text{state}\rangle = |\,F|\text{state}\rangle|^{2} + |\,F^{\dagger}\,|\text{state}\rangle|^{2} > 0$$

Supersymmetry

Explicit $N=1$ Supersymmetry Algebra :

$$\{Q_\alpha, \overline{Q}_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu,$$
$$\{Q_\alpha, Q_\beta\} = 0,$$
$$[Q_\alpha, P_\mu] = 0.$$

Q_α has a spin 1/2, and hence not commutes with $J_{\mu\nu}$

($N>1$ does not allow chiral representation of the gauge interactions... Phenomenologically less motivated as is.)

※ Supersymmetry commutes with P_μ

SUSY predicts degenerated boson and fermion spectrum!

Supersymmetry

SUSY multiplet (N=1)

massive case : let us take $P = (M,0,0,0)$

$$a_a = Q_a/(2M)^{1/2} \text{ satisfies } \{ a_a, (a_b)^\dagger \} = \delta_a{}^b$$

Irreducible one-particle state of SUSY consists of

$\mid j >$	(spin j)
$(a_b)^\dagger \mid j >$	(spin j±1/2)
$\varepsilon^{ab} (a_a)^\dagger (a_b)^\dagger \mid j >$	(spin j)

spin\ j	0	1/2	1	3/2
0	2	1		
1/2	1	2	1	
1		1	2	1
3/2			1	2
2				1

quark
lepton
Higgs

massive
gauge
bosons

Supersymmetry

SUSY multiplet (N=1)

massless case : let us take P = (E,0,0,E)

$a_1 = Q_1 / 2(E)^{1/2}$ satisfies $\{ a_1 , (a_1)^\dagger \} = 1$

$Q_2, Q_2^\dagger = 0$ for this choice of momentum

Irreducible one-particle state of SUSY consists of

$\| \lambda >$	(helicity λ)
$(a_b)^\dagger \| \lambda>$	(helicity λ+1/2)

massless particles form shorter multiplets!

Supersymmetry

SUSY multiplet ($N=1$)

massless case : let us take $P = (E,0,0,E)$

$$a_1 = Q_1 / 2(E)^{1/2} \text{ satisfies } \{ a_1 , (a_1)^\dagger \} = 1$$

$$Q_2, Q_2^\dagger = 0 \text{ for this choice of momentum}$$

Irreducible one-particle state of SUSY consists of

helicity\ λ	-2	-3/2	-1	-1/2	0	1/2	1	3/2
2								1
3/2							1	1
1						1	1	
1/2					1	1		
0				1	1			
-1/2			1	1				
-1		1	1					
-3/2	1	1						
-2	1							

CPT invariance requires λ and $-\lambda$...

Supersymmetry

SUSY multiplet ($N=1$)

massless case : let us take $P = (E,0,0,E)$

$a_1 = Q_1/2(E)^{1/2}$ satisfies $\{a_1, (a_1)^\dagger\} = 1$

$Q_2, Q_2^\dagger = 0$ for this choice of momentum

In relativistic field theory, pairing of $\pm\lambda$ is automatic!

helicity\ λ	-2	-3/2	-1	-1/2	0	1/2	1	3/2
2	1							1
3/2	1	1					1	1
1		1	1			1	1	
1/2			1	1	1	1		
0				1+1	1+1			
-1/2			1	1	1	1		
-1		1	1			1	1	
-3/2	1	1					1	1
-2	1							1

equivalent

Supersymmetric Field Theory

Spin (or helicity) 0 multiplet : spin 0 x 2, spin 1/2 x 1

complex scalar φ : 2 boson
Weyl Fermion ψ : 2 fermion

On off-shell

complex scalar φ : 2 boson
Weyl Fermion ψ : 4 fermion

We want to have symmetries at off-shell!

Spin (or helicity) 0 multiplet : spin 0 x 2, spin 1/2 x 1

complex scalar φ : 2 boson
auxiliary scalar F : 2 boson
Weyl Fermion ψ : 4 fermion

Supersymmetric Field Theory

Free-Lagrangean

$$\mathcal{L}_{\text{free}} \;=\; -\partial^\mu \phi^{*i} \partial_\mu \phi_i - i\psi^{\dagger i} \overline{\sigma}^\mu \partial_\mu \psi_i + F^{*i} F_i,$$

Supersymmetry transformation

$$\delta\phi_i = \epsilon\psi_i,$$

$$\delta(\psi_i)_\alpha = i(\sigma^\mu \epsilon^\dagger)_\alpha \, \partial_\mu \phi_i + \epsilon_\alpha F_i,$$

$$\delta F_i = i\epsilon^\dagger \overline{\sigma}^\mu \partial_\mu \psi_i,$$

$$\delta\, \mathcal{L}_{\text{free}} \;=\; -\partial_\mu \left(\epsilon\sigma^\nu \overline{\sigma}^\mu \psi \, \partial_\nu \phi^* + \epsilon\psi \, \partial^\mu \phi^* + \epsilon^\dagger \psi^\dagger \, \partial^\mu \phi \right).$$

action is invariant!

$$X = \phi, \phi^*, \psi, \psi^\dagger, F, F^*$$

$$(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2})X \;=\; i(\epsilon_1 \sigma^\mu \epsilon_2^\dagger - \epsilon_2 \sigma^\mu \epsilon_1^\dagger)\, \partial_\mu X$$

Supersymmetric Field Theory

SUSY invariant interactions ?

$$\mathcal{L}_{\rm int} = \left(-\frac{1}{2}W^{ij}\psi_i\psi_j + W^i F_i + x^{ij}F_iF_j\right) + \text{c.c.} - U,$$

W's, x, and *U* are functions of φ and $\varphi^\dagger$.

SUSY requires

$$W^i = \frac{\delta W}{\delta \phi_i} \qquad \frac{\delta W}{\delta \phi_i^*} = 0 \qquad W^{ij} = \frac{\delta^2}{\delta\phi_i \delta\phi_j} W$$

and x = 0, U = 0.

Thus, the interactions are determined by a holomorphic function W (=superpotential)

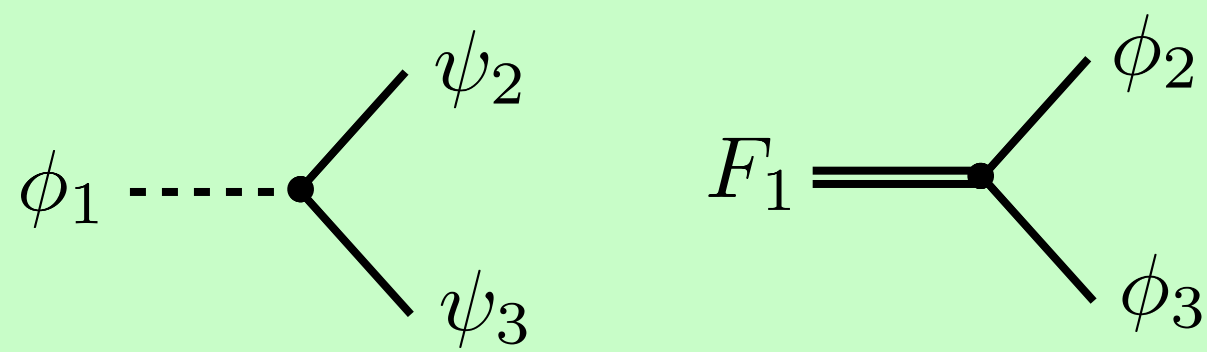
$$W = L^i\phi_i + \frac{1}{2}M^{ij}\phi_i\phi_j + \frac{1}{6}y^{ijk}\phi_i\phi_j\phi_k.$$

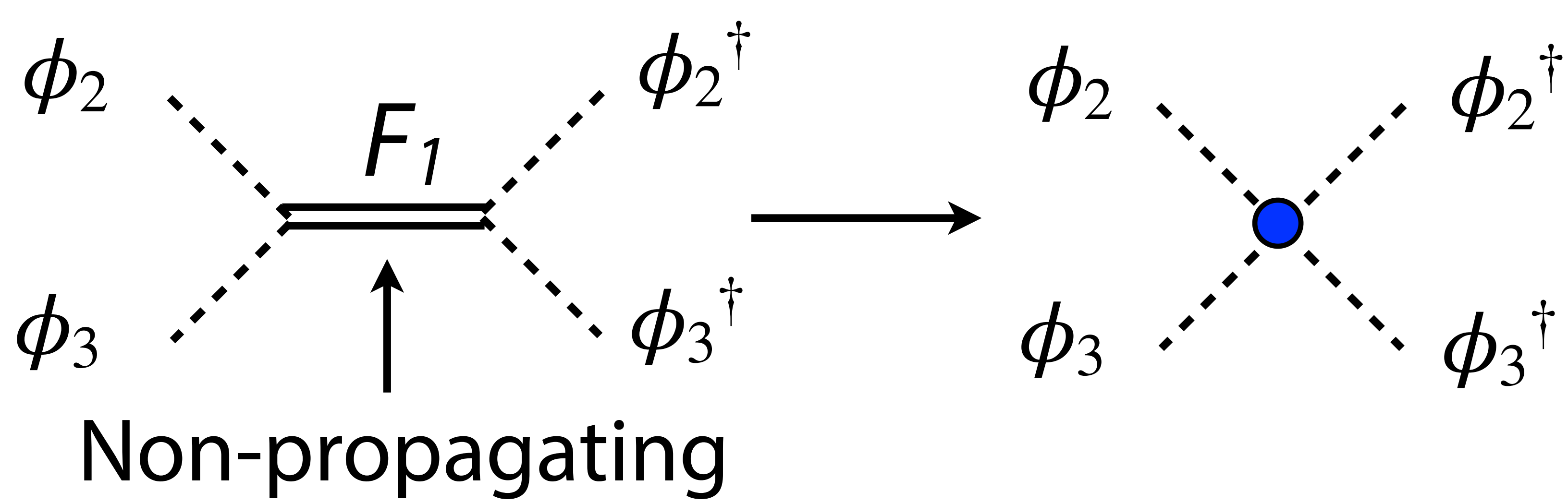
Supersymmetric Field Theory

ex) $W = y\phi_1\phi_2\phi_3$

$$\mathcal{L}_{int} = y\phi_1\psi_2\psi_3 + y\phi_2\psi_1\psi_3 + y\phi_3\psi_2\psi_1 \quad \text{[Yukawa-interaction]}$$

$$+yF_1\phi_2\phi_3 + yF_2\phi_1\phi_3 + yF_3\phi_1\phi_2 \quad \text{[scalar interactions]}$$





Supersymmetric Field Theory

Quark, Lepton, Higgs, Gauge boson are embedded into supermultiplets.

ex)

quark		squark
q	$\longleftrightarrow$	$\tilde{q}$
fermion		boson

squark, quark, F-term

$$Q = (\tilde{q},\ q,\ F)$$

gauge boson		gaugino
$F_{\mu\nu}$	$\longleftrightarrow$	λ_α
boson		fermion

gaugino, gauge boson, D-term

$$W_\alpha = (\lambda_\alpha,\ F_{\mu\nu},\ D)$$

$$\mathcal{L}_{\text{free}} = -\partial^\mu \phi^{*i} \partial_\mu \phi_i - i\psi^{\dagger i} \overline{\sigma}^\mu \partial_\mu \psi_i + F^{*i} F_i,$$

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} - i\lambda^{\dagger a} \overline{\sigma}^\mu D_\mu \lambda^a + \frac{1}{2} D^a D^a,$$

(F, D components are auxiliary field)

Quick review of superspace formalism

Spacetime = coset space of [Poincare group]/[Lorentz group]

Coordinate x_μ : parametrize the coset space

Poincare symmetry : $g = \exp[i\, a^\mu P_\mu + i\, \omega^{\mu\nu} J_{\mu\nu}] = \exp[i\, a^\mu P_\mu]h$

Quantum field : $\varphi(x) = L(x)\, \varphi(0)\, L^{-1}(x)$

$$L(x) = \exp[i\, x^\mu P_\mu]$$

Poincare transformation : $\varphi'(x') = g\, \varphi(x)\, g^{-1}$

$$= L(x')\, h\, \varphi(0)\, h^{-1}\, L^{-1}(x')$$

$$h\, \varphi(0)\, h^{-1} = \exp[i\, \omega^{\mu\nu} \Sigma_{\mu\nu}]\varphi(0)$$

$$x' = x + a + 2\omega x$$

Quick review of superspace formalism

Superpacetime
= coset space of [Super Poincare group]/[Lorentz group]

Coordinate x_μ, θ, $\theta^\dagger$: parametrize the coset space

Super Poincare : symmetry:

$$g = \exp[i\, a^\mu P_\mu + \xi Q + \xi^\dagger Q^\dagger + i\, \omega^{\mu\nu} J_{\mu\nu}]$$
$$= \exp[i\, a^\mu P_\mu + \xi Q + \xi^\dagger Q^\dagger] h$$

Quantum superfield : $\varphi(x,\theta,\theta^\dagger) = L(x,\theta,\theta^\dagger)\, \varphi(0)\, L^{-1}(x, \theta,\theta^\dagger)$

$$L(x,\theta,\theta^\dagger) = \exp[i\, x^\mu P_\mu + \theta Q + \theta^\dagger Q^\dagger]$$

Superpoincare transformation : $\varphi'(x',\theta',\theta'^\dagger) = g\, \varphi(x,\theta,\theta^\dagger)\, g^{-1}$

$$= L(x',\theta',\theta'^\dagger)\, h\, \varphi(0)\, h^{-1}\, L^{-1}(x',\theta',\theta'^\dagger)$$

For h =1,

$$x' = x + a + i\xi\sigma^\mu\theta^\dagger - i\theta\sigma^\mu\xi^\dagger \qquad \theta' = \theta + \xi \qquad \theta^{\dagger\prime} = \theta^\dagger + \xi^\dagger$$

Quick review of superspace formalism

Superpoincare transformation : $\varphi'(x',\theta',\theta'^{\dagger}) = g\, \varphi(x,\theta,\theta^{\dagger})\, g^{-1}$

$= L(x',\theta',\theta'^{\dagger})\, h\, \varphi(0)\, h^{-1}\, L^{-1}(x',\theta',\theta'^{\dagger})$

For h =1,

$x' = x + a + i\xi\sigma^{\mu}\theta^{\dagger} - i\theta\sigma^{\mu}\xi^{\dagger}$ $\quad \theta' = \theta + \xi$ $\quad \theta^{\dagger\prime} = \theta^{\dagger} + \xi^{\dagger}$

SUSY transformation can be expressed as derivative operators!

$$\begin{aligned}
\hat{Q}_{\alpha} &= i\frac{\partial}{\partial\theta^{\alpha}} - (\sigma^{\mu}\theta^{\dagger})_{\alpha}\partial_{\mu}, & \hat{Q}^{\alpha} &= -i\frac{\partial}{\partial\theta_{\alpha}} + (\theta^{\dagger}\overline{\sigma}^{\mu})^{\alpha}\partial_{\mu}, \\
\hat{Q}^{\dagger\dot{\alpha}} &= i\frac{\partial}{\partial\theta^{\dagger}_{\dot{\alpha}}} - (\overline{\sigma}^{\mu}\theta)^{\dot{\alpha}}\partial_{\mu}, & \hat{Q}^{\dagger}_{\dot{\alpha}} &= -i\frac{\partial}{\partial\theta^{\dagger\dot{\alpha}}} + (\theta\sigma^{\mu})_{\dot{\alpha}}\partial_{\mu}.
\end{aligned}$$

$$\begin{aligned}
\left\{\hat{Q}_{\alpha}, \hat{Q}^{\dagger}_{\dot{\beta}}\right\} &= 2i\sigma^{\mu}_{\alpha\dot{\beta}}\partial_{\mu} = -2\sigma^{\mu}_{\alpha\dot{\beta}}\hat{P}_{\mu}, \\
\left\{\hat{Q}_{\alpha}, \hat{Q}_{\beta}\right\} &= 0, \qquad \left\{\hat{Q}^{\dagger}_{\dot{\alpha}}, \hat{Q}^{\dagger}_{\dot{\beta}}\right\} = 0.
\end{aligned}$$

$$S'(x',\theta',\theta'^{\dagger}) - \varphi(x,\theta,\theta^{\dagger}) = (\,\xi\hat{Q}_{\alpha} + \xi^{\dagger}\hat{Q}^{\dagger}{}_{\alpha}\,)\, S(x,\theta,\theta^{\dagger})$$

Quick review of superspace formalism

Relation between superfield and component field (*φ,ψ,F*) ?

Taylor expansion:

$$S(x,\theta,\theta^\dagger) = a + \theta\xi + \theta^\dagger\chi^\dagger + \theta\theta b + \theta^\dagger\theta^\dagger c + \theta^\dagger\overline{\sigma}^\mu\theta v_\mu + \theta^\dagger\theta^\dagger\theta\eta + \theta\theta\theta^\dagger\zeta^\dagger + \theta\theta\theta^\dagger\theta^\dagger d.$$

a, b, c, d : complex scalar fields (8 real degrees)

ξ, χ, η, ζ : Wely fermions (16 real degrees)

v_μ : complex vector (8 real degrees)

too many components compared with (*φ,ψ,F*)

→ We need constraints to reduce the extra components.

Quick review of superspace formalism

SUSY covariant derivatives:

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} - i(\sigma^\mu\theta^\dagger)_\alpha\partial_\mu, \qquad D^\alpha = -\frac{\partial}{\partial\theta_\alpha} + i(\theta^\dagger\overline{\sigma}^\mu)^\alpha\partial_\mu,$$

$$D^{\dagger\dot{\alpha}} = \frac{\partial}{\partial\theta^\dagger_{\dot{\alpha}}} - i(\overline{\sigma}^\mu\theta)^{\dot{\alpha}}\partial_\mu, \qquad D^\dagger_{\dot{\alpha}} = -\frac{\partial}{\partial\theta^{\dagger\dot{\alpha}}} + i(\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu.$$

$$\left\{\hat{Q}_\alpha,\, D_\beta\right\} = \left\{\hat{Q}^\dagger_{\dot{\alpha}},\, D_\beta\right\} = \left\{\hat{Q}_\alpha,\, D^\dagger_{\dot{\beta}}\right\} = \left\{\hat{Q}^\dagger_{\dot{\alpha}},\, D^\dagger_{\dot{\beta}}\right\} = 0.$$

SUSY covariant derivatives commute with SUSY transformation!

Chiral Supermultiplet

$$D^\dagger_{\dot{\alpha}}\Phi = 0.$$

$$\Phi = \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y), \qquad y^\mu \equiv x^\mu + i\theta^\dagger\overline{\sigma}^\mu\theta,$$

This is what we want, i.e. (φ,ψ,F)!

SUSY Invariant action

The SUSY transformation of the highest components of the general supermultiplets (θ^4-term) and the chiral multiplet (θ^2-term) are given by total derivative!

$$\delta S|_{\theta^4} = \delta D = i\,\xi^\dagger \sigma^\mu \partial_\mu \eta + i\,\xi \sigma^\mu \partial_\mu \zeta^\dagger$$

$$\delta \Phi|_{\theta^2} = \delta F = i\,\xi^\dagger \sigma^\mu \partial_\mu \psi$$

(in Q's, increment of θ is accompanied by ∂_μ)

SUSY Invariant action

$$\int d^4x\,[\text{ general multiplet }]|_{\theta^4} + \int d^4x\,[\text{ chiral multiplet }]|_{\theta^2} + \text{h.c.}$$

Quick review of superspace formalism

✓ Holomorphic Function of chiral superfields are also chiral superfields!

$$W(\Phi) = m^2\, \Phi_i + m\, \Phi_i\, \Phi_j + y\, \Phi_i\, \Phi_j\, \Phi_k$$

(chiral)x(chiral)=(chiral)

✓ (chiral)†x(chiral)=(general)

$$\begin{aligned}
\Phi^{*i}\Phi_j \quad = \quad & \phi^{*i}\phi_j + \sqrt{2}\theta\psi_j\phi^{*i} + \sqrt{2}\theta^\dagger\psi^{\dagger i}\phi_j + \theta\theta\phi^{*i}F_j + \theta^\dagger\theta^\dagger\phi_j F^{*i} \\
& +\theta^\dagger\overline{\sigma}^\mu\theta\left[i\phi^{*i}\partial_\mu\phi_j - i\phi_j\partial_\mu\phi^{*i} - \psi^{\dagger i}\overline{\sigma}_\mu\psi_j\right] \\
& +\frac{i}{\sqrt{2}}\theta\theta\theta^\dagger\overline{\sigma}^\mu(\psi_j\partial_\mu\phi^{*i} - \partial_\mu\psi_j\phi^{*i}) + \sqrt{2}\theta\theta\theta^\dagger\psi^{\dagger i}F_j \\
& +\frac{i}{\sqrt{2}}\theta^\dagger\theta^\dagger\theta\sigma^\mu(\psi^{\dagger i}\partial_\mu\phi_j - \partial_\mu\psi^{\dagger i}\phi_j) + \sqrt{2}\theta^\dagger\theta^\dagger\theta\psi_j F^{*i} \\
& \boxed{\begin{aligned} & +\theta\theta\theta^\dagger\theta^\dagger\left[F^{*i}F_j - \frac{1}{2}\partial^\mu\phi^{*i}\partial_\mu\phi_j + \frac{1}{4}\phi^{*i}\partial^\mu\partial_\mu\phi_j + \frac{1}{4}\phi_j\partial^\mu\partial_\mu\phi^{*i}\right. \\
& \quad \left. +\frac{i}{2}\psi^{\dagger i}\overline{\sigma}^\mu\partial_\mu\psi_j + \frac{i}{2}\psi_j\sigma^\mu\partial_\mu\psi^{\dagger i}\right].\end{aligned}}
\end{aligned}$$

Quick review of superspace formalism

SUSY Invariant action

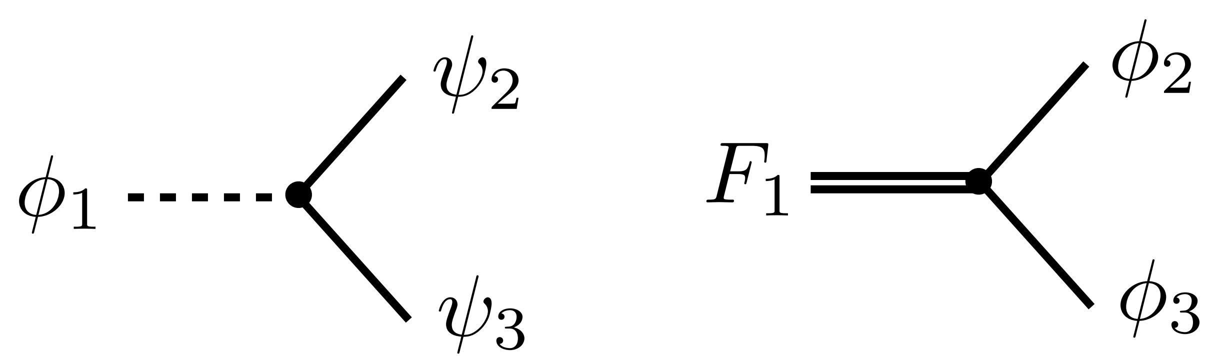
$$\int d^4x\, L = \int d^4x\, [\, \Phi_i^\dagger \Phi_i \,]\big|_{\theta^4} + \int d^4x\, W(\Phi)\, \big|_{\theta^2} + \text{h.c.}$$

$$= \int d^4x\, d^4\theta\;\; \Phi_i^\dagger \Phi_i + \int d^4x\, d^2\theta\, W(\Phi) + \text{h.c.}$$

$$\int d^2\theta d^2\theta^\dagger\, \Phi^*\Phi \;=\; -\partial^\mu\phi^*\partial_\mu\phi + i\psi^\dagger\overline{\sigma}^\mu\partial_\mu\psi + F^*F + \ldots.$$

$$\int d^2\theta W(\Phi) = -\frac{1}{2}W^{ij}\psi_i\psi_j + W^i F_i$$

ex) $W = y\Phi_1\Phi_2\Phi_3$



Quick review of superspace formalism

Scalar potential

$$\int d^2\theta d^2\theta^\dagger\, \Phi^*\Phi \;=\; -\partial^\mu\phi^*\partial_\mu\phi + i\psi^\dagger\overline{\sigma}^\mu\partial_\mu\psi + F^*F + \ldots .$$

$$\int d^2\theta W(\Phi) = -\frac{1}{2}W^{ij}\psi_i\psi_j + W^i F_i$$

$$\rightarrow\; V = -F^*F + W^i F_i + h.c.$$

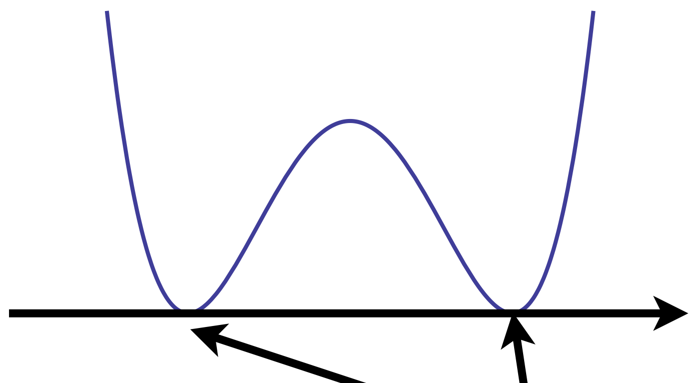
By solving the equation of motion of "F": $F_i = -W_i^*$

$$V = -F^*F + W^i F_i + h.c. = F_i^* F_i = W_i^* W_i \geqq 0$$

ex) $W = m/2\,\Phi^2 + y/3\,\Phi^3$

$$F_\Phi = -m\,\Phi + y\,\Phi^2$$

$$V = |m\,\Phi + y\,\Phi^2|^2$$



V=0 @ minima

Quick review of superspace formalism

Gauge theory

Theory is invariant under "local" symmetry :

$$\varphi'(x) = e^{i\alpha(x)T}\varphi(x)$$

How about in the superspace?

$$\Phi'(x,\theta,\theta^\dagger) = e^{i\alpha(x)T}\,\Phi(x,\theta,\theta^\dagger)\ ?$$

$\alpha(x)$ is not superfield → the left hand side is no more superfield...

"local" symmetry should be "local" in superspace!

$$\Phi'(x,\theta,\theta^\dagger) = e^{i\Lambda(x,\theta,\theta)T}\,\Phi(x,\theta,\theta^\dagger)\ !$$

$\Lambda(x,\theta,\theta^\dagger)$: chiral superfield (minimal construction)

Quick review of superspace formalism

In SUSY, the kinetic term is given by,

$$\int d^4x\, d^4\theta\ \ \Phi_i^\dagger \Phi_i$$

This is "not" invariant under the gauge transformation $\Phi' = e^{i\Lambda T}\Phi$

$$\int d^4x\, d^4\theta\ \ \Phi_i'^\dagger \Phi_i' = \int d^4x\, d^4\theta\ \ \Phi_i^\dagger e^{-i\Lambda^\dagger T} e^{i\Lambda T}\, \Phi_i'$$

→ We need connection (gauge) fields!

Real superfields ($V^\dagger = V$) provide connection fields if they shift :

$$e^{V'} = e^{i\Lambda^\dagger T}\, e^{V}\, e^{-i\Lambda\, T}$$

Then, $\int d^4x\, d^4\theta\ \ \Phi_i^\dagger\, e^V\, \Phi_i$ is invariant !

U(1) → V is one real superfield

Non-Abelian → V : real superfields in adjoint representation

Quick review of superspace formalism

Real superfields $V^\dagger = V$:

$$V(x,\theta,\theta^\dagger) = a + \theta\xi + \theta^\dagger\xi^\dagger + \theta\theta b + \theta^\dagger\theta^\dagger b^* + \theta^\dagger\overline{\sigma}^\mu \underline{\theta A_\mu} + \theta^\dagger\theta^\dagger\theta(\lambda - \frac{i}{2}\sigma^\mu\partial_\mu\xi^\dagger)$$
$$+\theta\theta\theta^\dagger(\lambda^\dagger - \frac{i}{2}\overline{\sigma}^\mu\partial_\mu\xi) + \theta\theta\theta^\dagger\theta^\dagger(\frac{1}{2}D + \frac{1}{4}\partial_\mu\partial^\mu a).$$

We have gauge boson and gaugino!

Fields other than A_μ, λ, D can be gauged away!

ex) U(1) gauge theory

$$V' = V - i\Lambda + i\Lambda^\dagger$$
$$\Lambda(y,\theta) = \phi(y) + \theta\psi(y) + \theta^2 F(y)$$

$$\begin{aligned} a &\to a + i(\phi^* - \phi), \\ \xi_\alpha &\to \xi_\alpha - i\sqrt{2}\psi_\alpha, \\ b &\to b - iF, \\ A_\mu &\to A_\mu + \partial_\mu(\phi + \phi^*), \\ \lambda_\alpha &\to \lambda_\alpha, \\ D &\to D. \end{aligned}$$

Quick review of superspace formalism

Real superfields $V^\dagger = V$:

$$V(x,\theta,\theta^\dagger) = a + \theta\xi + \theta^\dagger\xi^\dagger + \theta\theta b + \theta^\dagger\theta^\dagger b^* + \theta^\dagger\overline{\sigma}^\mu\underline{\theta A_\mu} + \theta^\dagger\theta^\dagger\theta(\lambda - \frac{i}{2}\sigma^\mu\partial_\mu\xi^\dagger)$$
$$+\theta\theta\theta^\dagger(\lambda^\dagger - \frac{i}{2}\overline{\sigma}^\mu\partial_\mu\xi) + \theta\theta\theta^\dagger\theta^\dagger(\frac{1}{2}D + \frac{1}{4}\partial_\mu\partial^\mu a).$$

We have gauge boson and gaugino!

Fields other than A_μ, λ, D can be gauged away!

→ Wess-Zumino gauge

$$V_{\text{WZ gauge}} = \theta^\dagger\overline{\sigma}^\mu\theta A_\mu + \theta^\dagger\theta^\dagger\theta\lambda + \theta\theta\theta^\dagger\lambda^\dagger + \frac{1}{2}\theta\theta\theta^\dagger\theta^\dagger D.$$

Quick review of superspace formalism

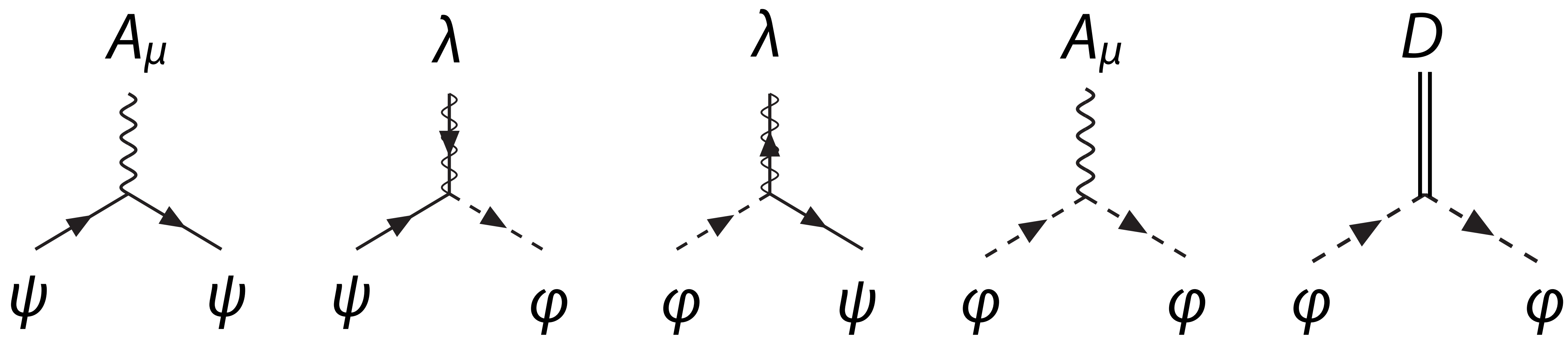
In the Wess-Zumino gauge

$$V_{\rm WZ\;gauge} \;=\; \theta^\dagger \overline{\sigma}^\mu \theta A_\mu + \theta^\dagger\theta^\dagger\theta\lambda + \theta\theta\theta^\dagger\lambda^\dagger + \frac{1}{2}\theta\theta\theta^\dagger\theta^\dagger D.$$

Matter kinetic functions are gauge symmetric!

$$\mathcal{L}_{\rm kin} = (\mathcal{D}_\mu\phi_i)^\dagger(\mathcal{D}^\mu\phi_i) + \psi^\dagger i\sigma\mathcal{D}_\mu\psi_i + F_i^\dagger F_i$$
$$-i\sqrt{2}\phi_i^\dagger\lambda\psi_i + i\sqrt{2}\psi_i^\dagger\lambda\phi_i - \phi_i^* D\phi_i$$

The kinetic term also leads to new interactions



Quick review of superspace formalism

Field Strength chiral superfield

$$\mathcal{W}_\alpha = -\frac{1}{4} D^\dagger D^\dagger \left(e^{-V} D_\alpha e^V \right),$$

$$D^\dagger_{\dot\alpha} \mathcal{W}_\alpha = 0$$

$$\mathcal{W}'_\alpha = e^{i\Lambda} \mathcal{W}_\alpha e^{-i\Lambda}$$

$$(\mathcal{W}^a_\alpha)_{\text{WZ gauge}} = \lambda^a_\alpha + \theta_\alpha D^a - \frac{i}{2}(\sigma^\mu \overline{\sigma}^\nu \theta)_\alpha F^a_{\mu\nu} + i\theta\theta(\sigma^\mu \nabla_\mu \lambda^{\dagger a})_\alpha,$$

Gauge Kinetic Function

$$\mathcal{L} = \mathrm{Re}[-\tau\, \mathrm{tr}[W_{\dot\alpha} W^{\dot\alpha}]]\Big|_{\theta\theta}$$

$$= -\frac{1}{4g^2} F^a_{\mu\nu} F^{a\mu\nu} + \frac{\theta_g}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} F^a_{\mu\nu} F^a_{\rho\sigma} + \frac{1}{g^2} \lambda^{\dagger a} i(\sigma^\mu_-) \mathcal{D}_\mu \lambda^a + \frac{1}{2g^2} D^a D^a$$

$$\tau = \frac{1}{g^2} + i\frac{\theta_g}{8\pi^2},$$

Auxiliary field!

Quick review of superspace formalism

Scalar potential

$$V = - F^{*}F \; + (W^{i}F_{i} + h.c.\,) - DD/2g^{2} + \varphi^{*}D\varphi$$

By solving the equation of motion of F and D

$$F_{i} = - W_{i}^{*}$$

$$D = g^{2}\Sigma\, \varphi^{*}\varphi$$

$$V = F^{*}F \; + DD/2 \; = W_{i}^{*}W_{i} + g^{2}(\Sigma\, \varphi^{*}\varphi)^{2}/2 \geqq 0$$

The positive definiteness of the energy is an important feature of the global supersymmetry!

Supersymmetric Standard Model

The minimal Supersymmetric Standard Model (The MSSM)

	$SU(3)$	$SU(2)$	$U(1)$	R_p
Q_L	3	2	1/6	-
$\overline{U}_R$	3	1	-2/3	-
$\overline{D}_R$	3	1	1/3	-
L_L	1	2	-1/2	-
$\overline{E}_R$	1	1	1	-
H_u	1	2	1/2	+
H_d	1	2	-1/2	+

Two Higgs doublets are required!

U(1)-SU(2) anomaly cancelation

- Interactions are given by an analytic function (superpotential)

$$W = y_u H_u Q_L \bar{U}_R + y_d H_d Q \bar{D}_R + y_e H_d L_L \bar{E}_R$$

All the SM interactions are easily extended!

In particular, the SM top Yukawa can appear as in the SM!

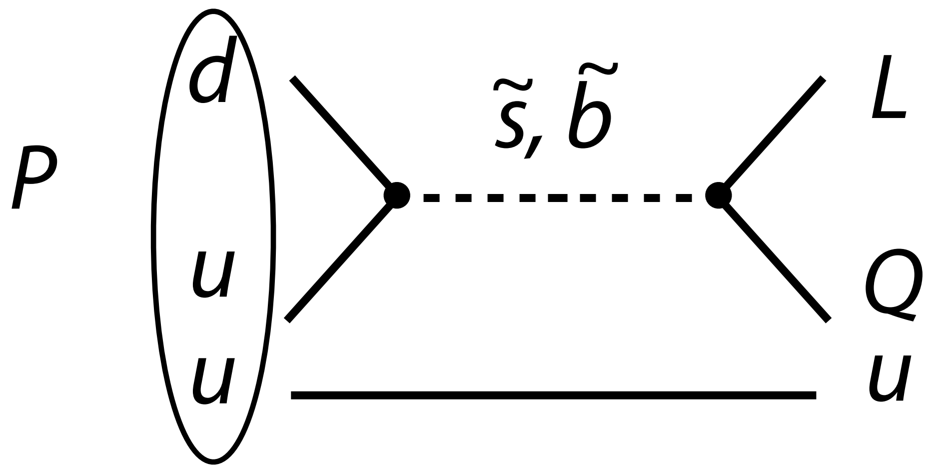
Supersymmetric Standard Model

Unacceptable B, L breaking interactions

$\Delta B = 1$

$$W_{RPV} = \alpha Q_L L_L \bar{D}_R + \beta L_L L_L \bar{E}_R + \delta \bar{D}_R \bar{D}_R \bar{U}_R + \mu' L_L H_u$$

$\Delta L = 1$



These lead to too rapid proton decay...

$p \rightarrow e\pi, \nu\pi, eK, \nu K, \ldots$

These operators are forbidden by introducing R-parity

(~ a discrete subgroup of L and B symmetry)

$$R_p = (-)^{3(B-L)+F}$$

R[SM particles] = +1

R[Superparticles] = -1

Supersymmetric Standard Model

Under the R-parity, the SM particles are even while the superpartners are odd.
(R-parity is not commute with SUSY)

LSP : the Lightest supersymmetric particle ($R_p = -1$)

The LSP is stable and a candidate of dark matter!

Who is the LSP?

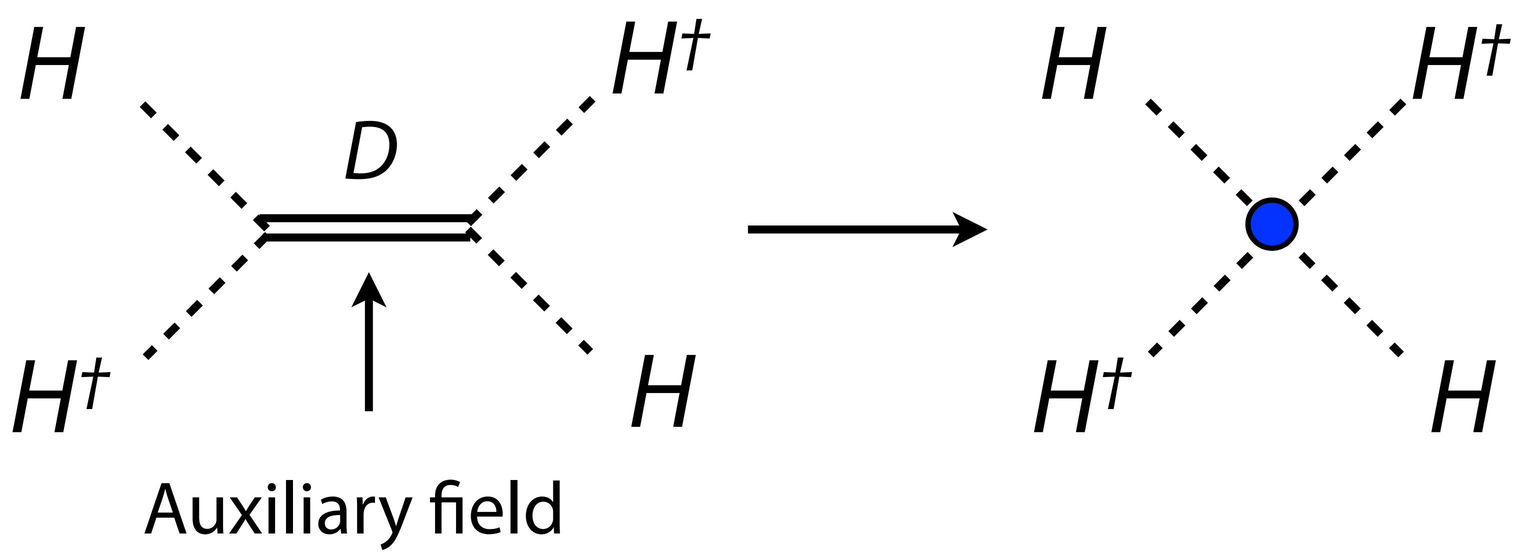
It depends on the SUSY breaking, mediations, etc.

- The lightest neutralino (Zino, Bino, 2 neutral Higgsino)
- Gravitino (superpartner of the gravition)

Higgs mass in Supersymmetric Standard Model

The most important prediction of the MSSM
= Higgs quartic coupling is given by the gauge couplings

[cf. in the SM, λ is a free parameter]



$\lambda = (g_1^2 + g_2^2)/2 \cos^2 2\beta$

($\tan\beta = v_u/v_d$)

In the MSSM, the Higgs mass (at the tree-level) is a prediction!

$$\boxed{m_{higgs} = \lambda^{1/2} v \sim m_Z \cos 2\beta}$$

→ Is it too light? SUSY breaking effects play important roles!

Supersymmetry Breaking

To be a realistic model we need SUSY breaking!

We have not seen any superparticles with mass spectrums degenerated with the SM counterparts....

We need to make the SUSY particles heavy.

→ Spontaneous Supersymmetry Breaking!

Supersymmetry Breaking

SUSY algebra : $\{Q_\alpha, Q^\dagger{}_\alpha\} = 2\,\sigma^\mu{}_{\alpha\alpha} P_\mu$

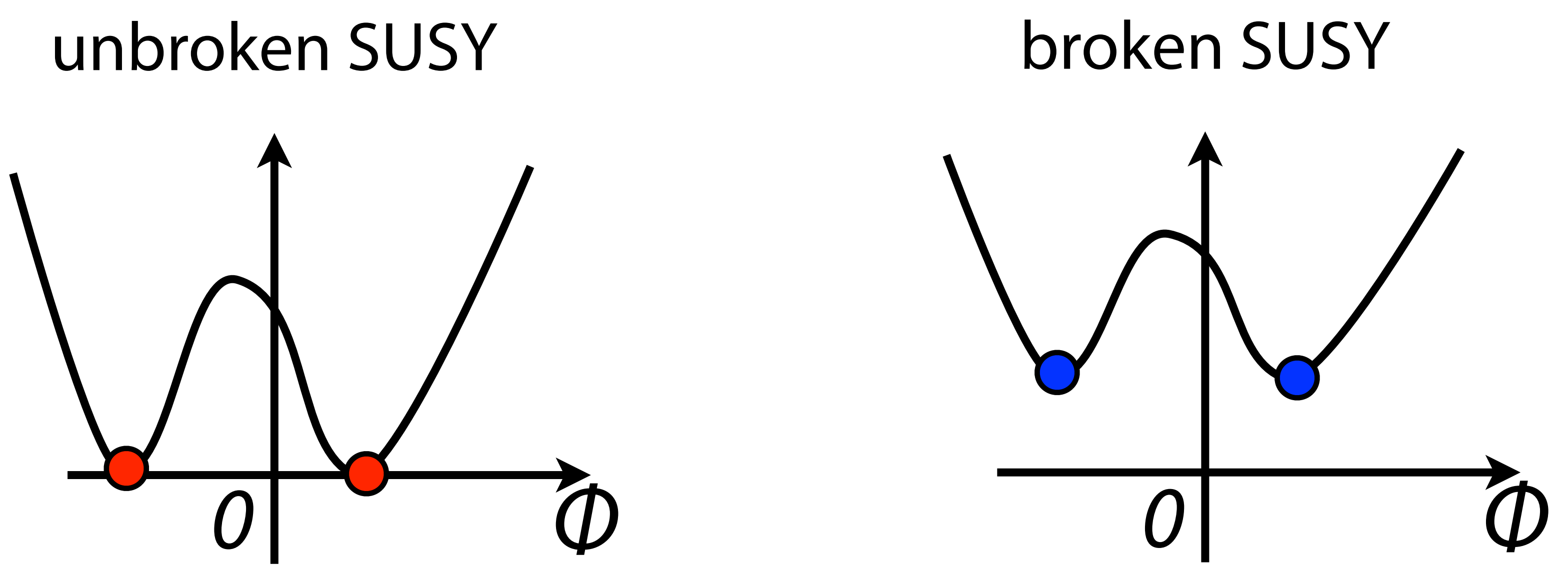
[supersymmetry is an extension of the spacetime symmetry!]

$$H = (Q_1 Q_1{}^\dagger + Q_1{}^\dagger Q_1 + Q_2 Q_2{}^\dagger + Q_2{}^\dagger Q_2)/4$$

$$< vac \,|\, H \,|\, vac > = (\,|Q_1|\, vac >|^2 + |Q_2|\, vac >|^2\,)/2$$

SUSY preserving vacuum : vacuum energy = 0 $\quad (\; Q_1 | 0 > = 0 \;)$

SUSY breaking vacuum : vacuum energy > 0 $\quad (\; Q_1 | 0 > \neq 0 \;)$



We need a model with non-vanishing vacuum energy !

Supersymmetry Breaking

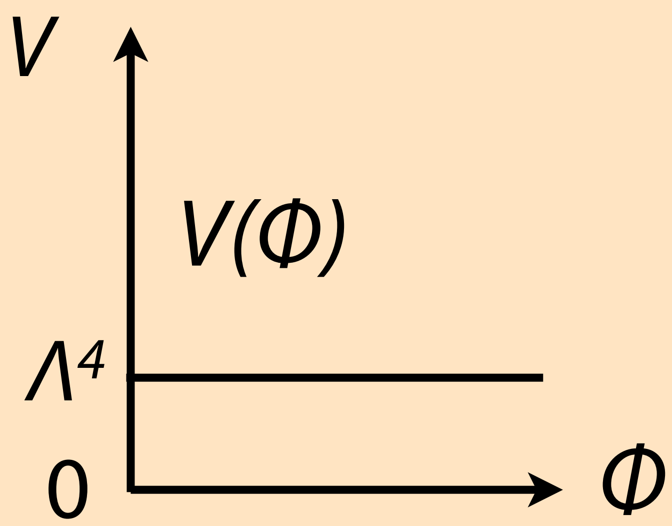
• Simplest example : single field perturbative model

The order parameter of ~~SUSY~~ = vacuum energy: $V = \Sigma\, | F_\Phi |^2$

$$F_\Phi = -W^\dagger{}_\Phi \neq 0 \qquad \longleftrightarrow \qquad \cancel{\text{SUSY}}$$

$$W = \Lambda^2\, \Phi$$

Energy is non-vanishing
for any field value.



SUSY is spontaneously broken!

SUSY is spontaneously broken!

$$F_\Phi = -W_\Phi{}^\dagger = -\,\Lambda^2$$

cf. $\delta_{SUSY}\,\psi = \xi \, x\, F \neq 0$

Supersymmetry Breaking

Flat universe?

SUSY breaking vacuum $V > 0$?

In supergravity

$$V = e^{K} \left(F^{*} F - 3\, M_{PL}^{2} \, |W|^{2} \right)$$

The flat universe is possible even if SUSY is broken for :

$$W = F/\sqrt{3} \times M_{PL}$$

cf. Gravitino Mass

$$m_{3/2} = W/M_{PL}^{2} = F/\sqrt{3}\, M_{PL}$$

Gravitino Mass ⇆ SUSY breaking scale

Supersymmetry Breaking

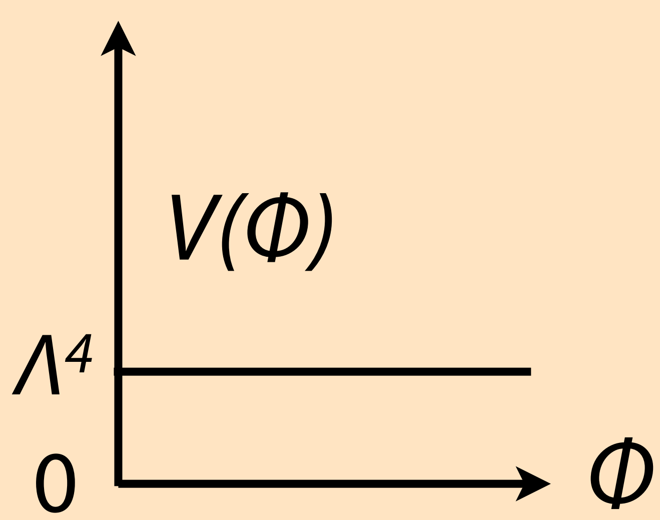
- Simplest example : single field perturbative model

The order parameter of ~~SUSY~~ = vacuum energy: $V = \Sigma \,|\, F_\Phi \,|^2$

$F_\Phi = -W^\dagger{}_\Phi \neq 0 \quad \longleftrightarrow \quad \cancel{\text{SUSY}}$

$W = \Lambda^2\,\Phi$

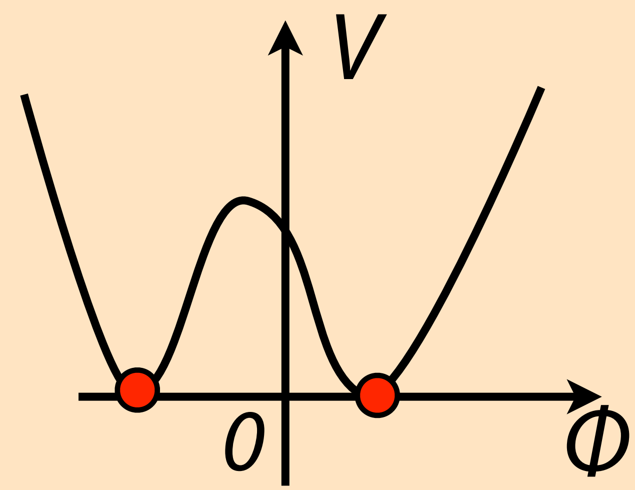
Energy is non-vanishing
for any field value.



SUSY is spontaneously broken!

$W = \Lambda^2\Phi + m\Phi^2 + \lambda\Phi^3$

V not only depends on Φ
but has zero energy state.

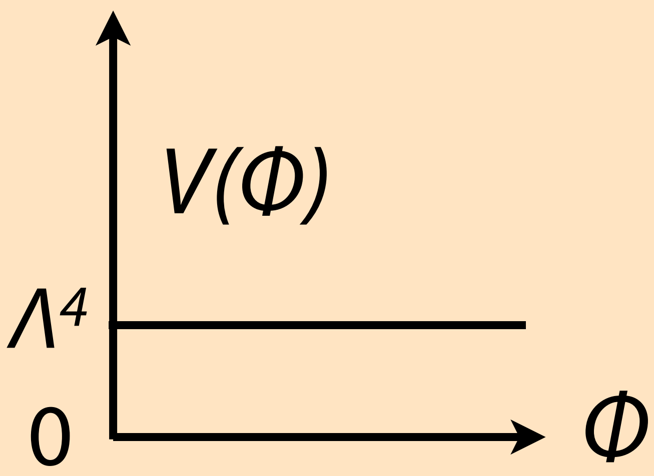


SUSY is not broken!

Supersymmetry Breaking

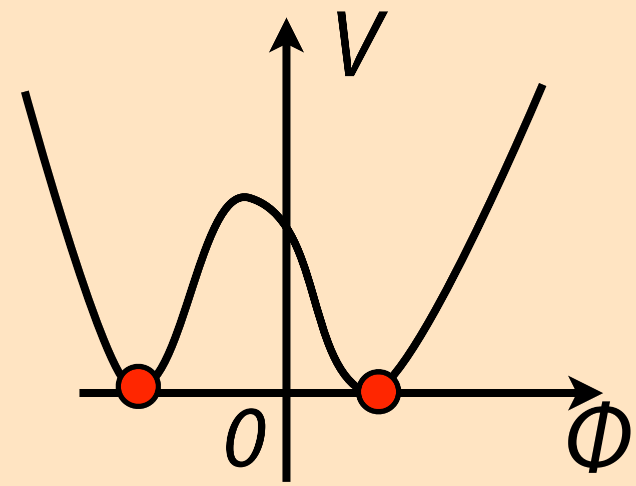
What is the difference in these models?

$W = \Lambda^2 \Phi$



SUSY is spontaneously broken!

$W = \Lambda^2\Phi + m\Phi^2 + \lambda\Phi^3$



SUSY is not broken!

R-symmetry (U(1) symmetry which is not commute with SUSY)!

$[\,R, Q\,] = -Q \qquad \theta \rightarrow e^{i\alpha}\theta, \qquad \theta^\dagger \rightarrow e^{-i\alpha}\theta^\dagger$

$\Phi = (\varphi, \psi, F)$

$Q_R \qquad Q_R\text{-}1 \qquad Q_R\text{-}2$

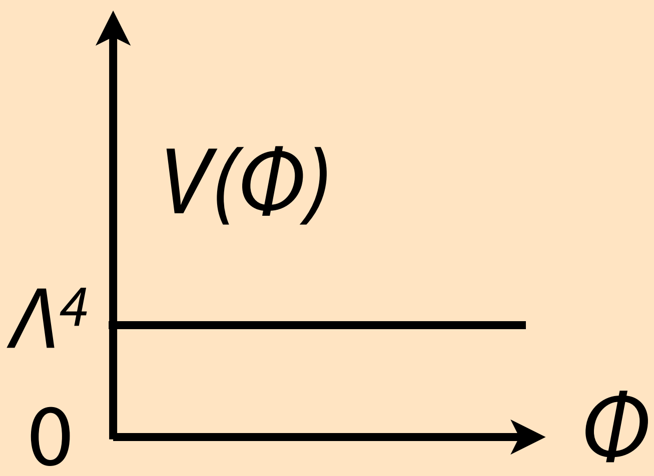
$W_a = (\lambda_a, F_{\mu\nu}, D)$

$1 \qquad 0 \qquad 0$

Superpotential W should have R-charge 2!

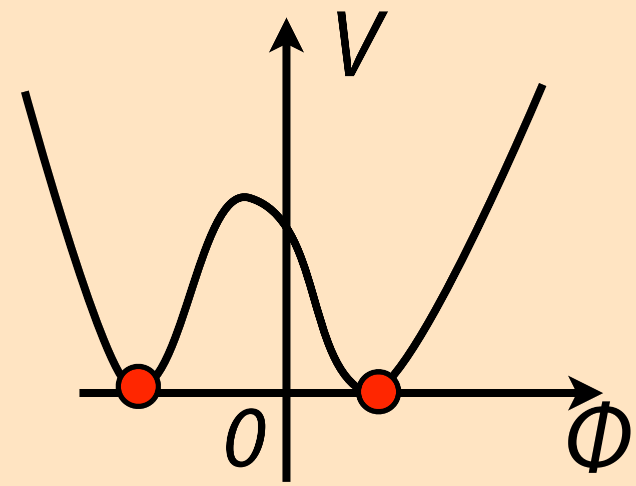
What is the difference in these models?

$$W = \Lambda^2 \Phi$$



SUSY is spontaneously broken!

$$W = \Lambda^2\Phi + m\Phi^2 + \lambda\Phi^3$$



SUSY is not broken!

This model has R-symmetry!

No R-symmetry!

R-charge of $\Phi = 2$.

R-symmetry is a necessary condition for spontaneous SUSY breaking when the model has generic superpotential under symmetries (Nelson&Seiberg `93)

Supersymmetry Breaking

Nelson&Seiberg `93

1) Assume that superpotential is generic under symmetries.

SUSY vacuum condition

$$-F_i^* = \partial W(\Phi_1, \ldots, \Phi_n)/\partial\Phi_i = 0$$

For generic superpotential, n-conditions for n-variables

In general, there is solutions!

= SUSY is not broken!

Supersymmetry Breaking

Nelson&Seiberg `93

1) Assume that superpotential is generic under symmetries.
2) Assume that the model possesses R-symmetry
3) Assume that R-symmetry is broken by the finite VEV of Φ_R

$$W(\Phi_1, ..., \Phi_n, \Phi_R) = \Phi_R^{2/qR} W(X_1, ..., X_n, 1)$$

SUSY vacuum condition n variables, n+1 conditions !

$$\partial W(X_1, X_2, ..., 1)/\partial X_i = 0$$

$$W(X_1, X_2, ..., 1) = 0$$

There is not always solutions! SUSY could be broken!

→ R-symmetry is a necessary condition!

cf. non-R U(1) symmetry : n variables, n conditions.

$$W(\Phi_1, ..., \Phi_n, \Phi_{n+1}) = W(X_1, ..., X_n, 1)$$

generically solvable!

Supersymmetry Breaking

O'Reifeartaigh model

$$W = \Lambda^2 \Phi - y \Phi X^2 + m X Y$$

This model has R-symmetry : $\Phi(2)$, $X(0)$, $Y(2)$

Z_2 symmetry : $\Phi(even)$, $X(odd)$, $Y(odd)$

Under these symmetries the model has a generic potential

SUSY vacuum conditions : $W_i = 0$

$$W_\Phi = \Lambda^2 - y X^2, \quad W_X = -2y\Phi X + mY, \quad W_Y = mX$$

SUSY breaking ($m^2 > y \Lambda^2$)

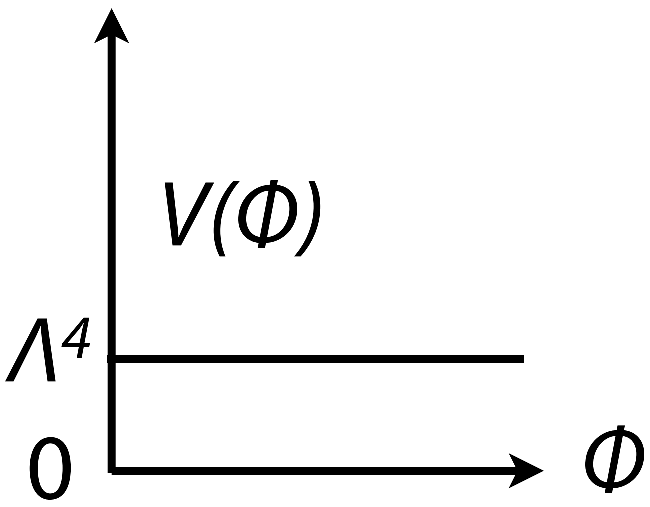
$\langle X \rangle = \langle Y \rangle = 0$ $\quad F_\Phi = \Lambda^2$ $\quad \Phi$ = flat potential

generic feature of F-term SUSY breaking!

Supersymmetry Breaking

O'Reifeartaigh model

Tree-level scalar potential = Flat!



※ Superpotential is not renormalized perturbatively!

$$W_{renormalized} = \Lambda^2\,\Phi - y\,\Phi\,X^2 + m\,X\,Y$$

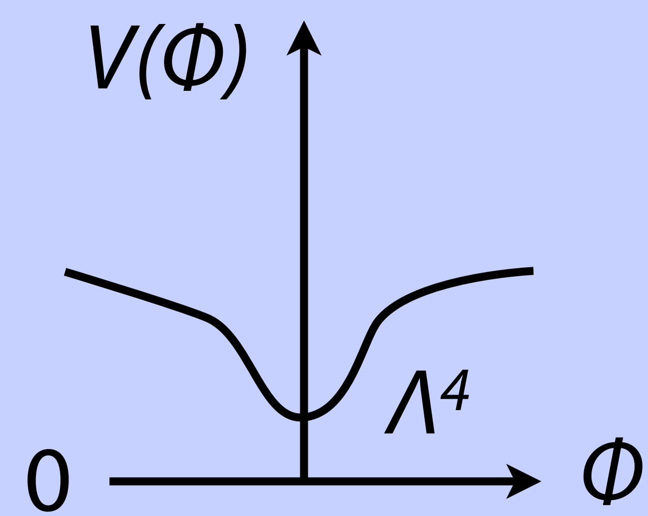
[SUSY wouldn't be restored radiatively]

※ Kahler potential (= kinetic term) is renormalized!

$$K \sim \Phi^\dagger\Phi - y^2/(16\pi^2 m^2)\,|\Phi^\dagger\Phi|^2 + \ldots$$

Φ gets a maass from the second term.

$$\langle\Phi\rangle = \theta^2 F_\Phi :\; m_\Phi{}^2 = y^2/16\pi^2\,x\,F_\Phi{}^2/m^2$$



Supersymmetry Breaking

Strong gauge dynamics

Supersymmetric QCD

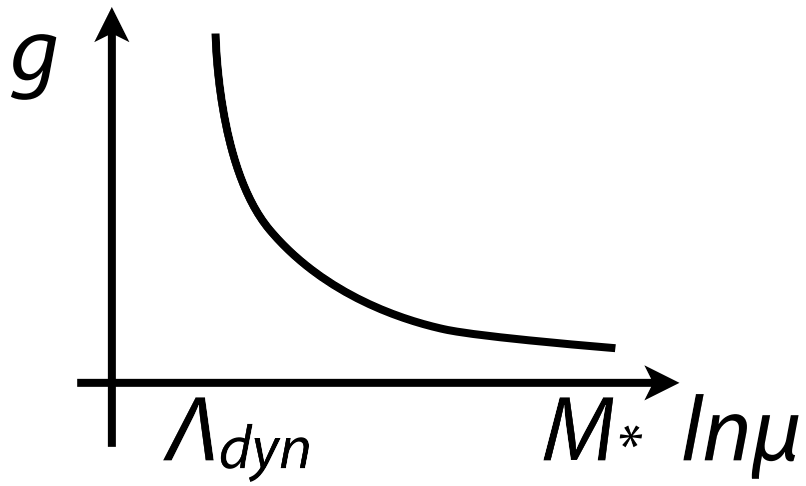
$SU(N_c)$ gauge theory with N_f flavors (q_i , $q^c{}_i$)

Beta function of the gauge coupling constant

$$dg/dt = -(3N_c - N_f)\, g^3/16\pi^2$$

Asymptotically free for $3N_c > N_f$

→ Non-trivial thing could happen at IR?



Dynamical scale

$$\Lambda_{dyn} \sim \exp(-8\pi^2/g_0^2(3N_c - N_f))\, M_*$$

Dimensional Transmutation!

Supersymmetry Breaking

Strong gauge dynamics

ex) Gaugino condensation for $N_f = 0$

(non-rigorous effective potential approach)

R-symmetry : $\lambda^{a\prime} = e^{i\alpha} \lambda^a$

R-symmetry is anomalous against $SU(N_c)$: $\theta_g \to \theta_g + 2N_c\alpha$

→ Still invariant under a fictitious R-symmetry :

$$\lambda^{a\prime} = e^{i\alpha}\lambda^a, \quad \tau' = \tau + i\,\alpha N_c/4\pi^2$$

Effective superpotential should have charge 2!

$$\mathcal{L} = \mathrm{Re}[-\tau\,\mathrm{tr}[W_{\dot\alpha}W^{\dot\alpha}]]\Big|_{\theta\theta} \to -\frac{1}{4g^2}F^a_{\mu\nu}F^{a\mu\nu} + \frac{\theta_g}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}$$

$$\tau = \frac{1}{g^2} + i\frac{\theta_g}{8\pi^2},$$

Supersymmetry Breaking

Strong gauge dynamics

ex) Gaugino condensation for $N_f = 0$

Holomorphic Dynamical Scale

$$\Lambda_{dyn} \sim exp(-8\pi^2/g_0^2\,(3N_c))\,M_*$$

$$\rightarrow \Lambda_{dyn} \sim exp(-8\pi^2\tau_0\,/(3N_c))\,M_*$$

Under the fictitious R-symmetry, $\lambda^{a\prime} = e^{i\alpha}\,\lambda^a$, $\tau' = \tau + i\,\alpha N_c/4\pi^2$
the dynamical scale rotates

$$\Lambda_{dyn}' = \Lambda_{dyn}\, e^{-i\,2\alpha/3}$$

Assuming no massless particle exists below Λ_{dyn},
only allowed effective potential is...

$$W_{eff} = a\,\Lambda_{dyn}^3 \quad \text{(fictitious R-charge 2)}$$

Supersymmetry Breaking

Strong gauge dynamics

ex) Gaugino condensation for $N_f = 0$

Gauge kinetic function : $W = -\tau\, W^{\alpha} W_{\alpha}$

$$W_{\alpha} = \lambda_{\alpha}{}^{a} + O(\theta)$$

$$\rightarrow \partial W/\partial\tau\,\big|_{\theta^0} = \lambda^a\lambda^a / 4\,i$$

$$< \lambda^a\lambda^a > = 4i\, \partial W_{eff} / \partial\tau\,\big|_{\theta^0}$$
$$= -\, 32\pi^2 / N_c\, a\, \Lambda_{dyn}{}^3$$

Gaugino condensation occurs!

Discrete $Z_{2N_c}\, R$ symmetry is spontanesously broken to Z_2 R symmetry! We have N_c distinct vacua!

Supersymmetry Breaking

Strong gauge dynamics

ex) Gaugino condensation for $N_f = 0$

Is SUSY broken?

We have N_c distinct vacua!

Witten index : $Tr(-)^F = N_c$

Witten index is non-zero only when there are E = 0 states!

($Q\,|\,\text{boson} \rangle = E^{1/2}\,|\,\text{fermion} \rangle$, $Q\,|\,\text{fermion} \rangle = E^{1/2}\,|\,\text{boson} \rangle$)

SQCD with $N_f = 0$ theory does not break SUSY even by non-perturbative effects!

(Model does not possess continuous R-symmetry... and hence, no surprise!)

Supersymmetry Breaking

Strong gauge dynamics

ex) Gaugino condensation for $N_f = 0$

$a \neq 0$? (more reliable path to show $a \neq 0$)

1) add N_c - 1 flavors (q_i , $q^c{}_i$),

2) At large vevs of q's, non-perturbative effective superpotential is generated by instanton effects (weak coupling!)

3) Add small mass "m" to N_c - 1 flavors

→ gaugino condensates via Konishi anomaly

($a \neq 0$ *at* weak coupling)

4) Using "exact" holomorphic equation, $<\lambda\lambda> = 2m\partial<\lambda\lambda>/\partial m$, we find $<\lambda\lambda> \neq 0$ for $m \to \infty$

Supersymmetry Breaking

Dynamical SUSY Breaking model
(Izawa-Yanagida-Intriligator-Thomas model)

SU(2) gauge theory :
4-fundamental representations: q_i (i = 1,2,3,4)
6-gauge singlets : $S_{ij} = -S_{ji}$ (i = 1,2,3,4)

$$W = S_{ij}\, q_i\, q_j$$

Model has anomaly free R-symmetry : S(2), q(0)

Let us consider $S_{ij} = S\,\varepsilon_{ij} >> \Lambda_{dyn}$

all the q's get heavy and model looks pure SU(2) theory!

Gaugino condensation should occur!

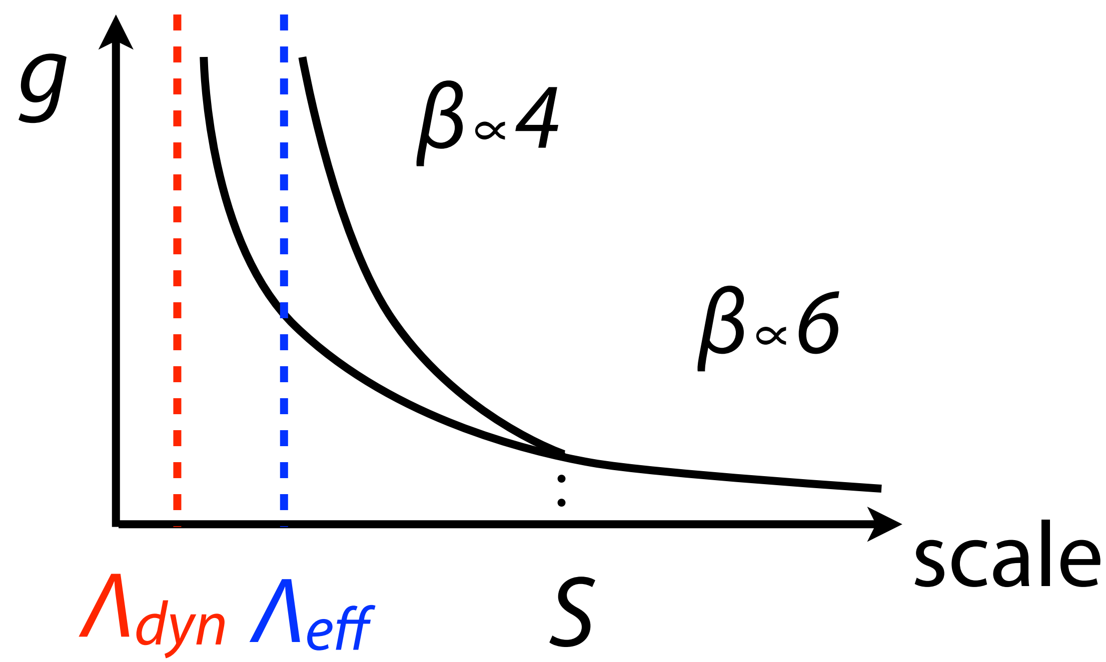
Supersymmetry Breaking

Dynamical SUSY Breaking model

(Izawa-Yanagida-Intriligator-Thomas model)

Gaugino condensation should occur!

The effective dynamical scale depends on "S"!



$$\Lambda_{eff}^3 = S\,\Lambda_{dyn}^2$$

$$W_{eff} = a\,\Lambda_{eff}^3 = a\,\Lambda_{dyn}^2\,S$$

Thus, SUSY is broken by the F-component of S!

$$F_S = \partial W_{eff}/\partial S = a\,\Lambda_{dyn}^2 \neq 0$$

At $S << \Lambda_{dyn}$, the Gaugino condensation picture is no more valid, but it is known that similar potential is generated!

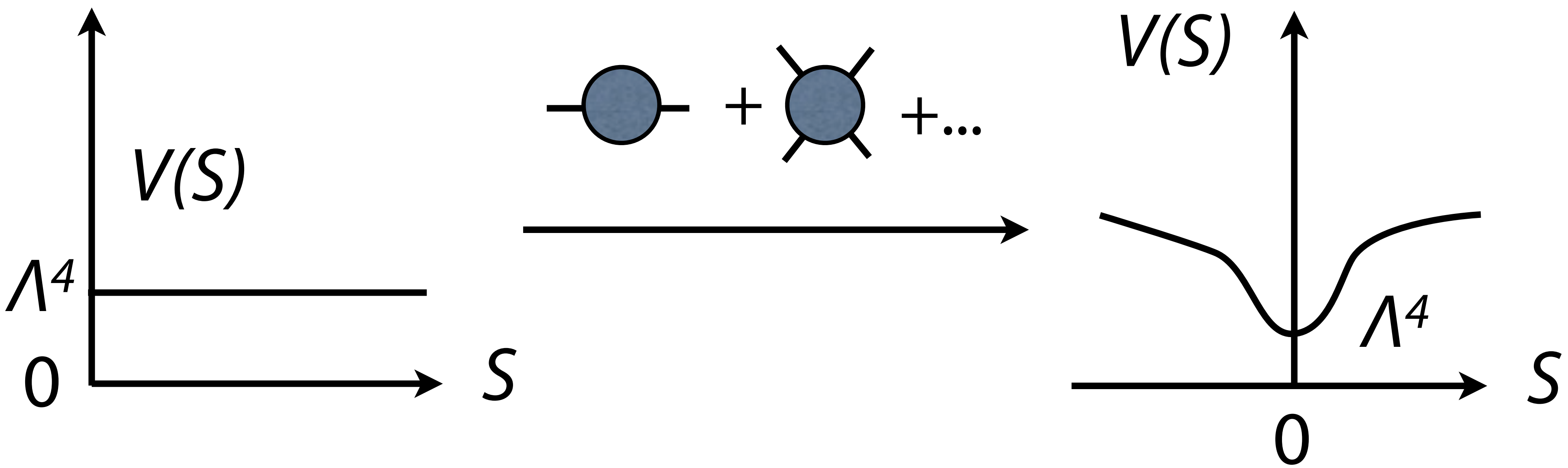
Supersymmetry Breaking

Dynamical SUSY Breaking model
(Izawa-Yanagida-Intriligator-Thomas model)

If the kinetic function of S is flat, i.e. minimal $[S^\dagger S]_D$, scalar potential of S is flat.

The kinetic function receives incalculable corrections from the SU(2) interactions...

$[S^\dagger S + (S^\dagger S)/\Lambda_{dyn}{}^2 + \ldots]_D$

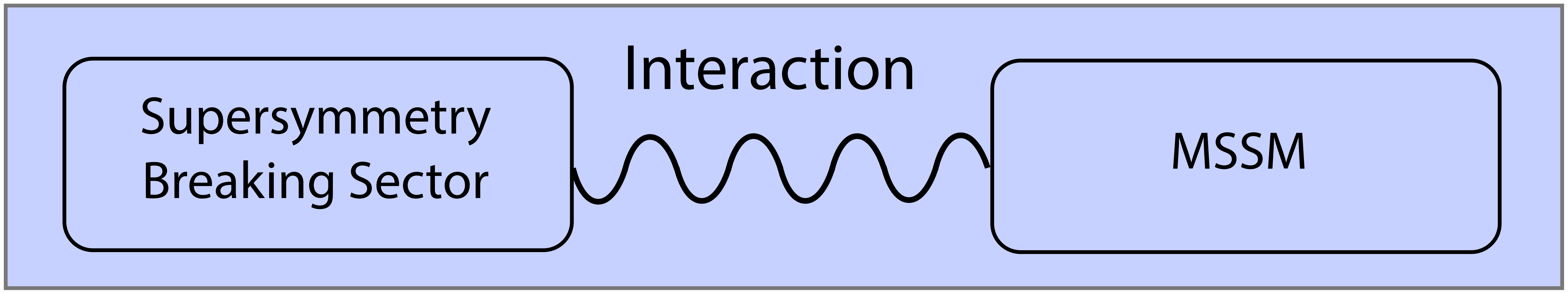


Such a lift of potential is important in cosmology!

Supersymmetry Breaking and SUSY spectrum

Now the time for model building....

We need SUSY breaking sector!



The superparticles in the MSSM obtain masses via the interactions to the SUSY breaking sector.

The MSSM spectrum depends more on how supersymmetry breaking is mediated than on how it is broken!

Supersymmetry Breaking and SUSY spectrum

Useful model independent parametrization = soft parameters

$$\begin{aligned}\mathcal{L}_{\rm soft} = &-\frac{1}{2}\left(M_3\tilde{g}\tilde{g} + M_2\tilde{W}\tilde{W} + M_1\tilde{B}\tilde{B}\right)\\ &-\left(a_u H_u \tilde{Q}_L \tilde{\bar{U}}_R + a_d H_d \tilde{Q}_L \tilde{\bar{D}}_R + a_e H_d \tilde{L}_L \tilde{\bar{E}}_R\right) + c.c.\\ &-m_Q^2|\tilde{Q}_L|^2 - m_{\bar{U}}^2|\tilde{\bar{U}}_R|^2 - m_{\bar{D}}^2|\tilde{\bar{D}}_R|^2 - m_L^2|\tilde{L}_L|^2 - m_{\bar{E}}^2|\tilde{\bar{E}}_R|^2\\ &-m_{H_u}^2|H_u|^2 - m_{H_d}^2|H_d|^2 - (B\mu_H H_u H_d + c.c.)\end{aligned}$$

$$M_{1,2,3}, a_{u,d,e}, m_{Q,U,D,E,L,H_u,H_d}, B = O(10^{2-3})\ \text{GeV}$$

Each mediation model gives these soft parameters in terms of more fundamental parameters...

Supersymmetry Breaking and SUSY spectrum

In terms of superspace formalism

Let us assume that SUSY breaking is provided by a F-term of the chiral field in a hidden sector :

$$Z(x,\theta) = F\,\theta^2$$

Gaugino mass term:

$$\int d^2\theta\, Z/M_*\, W_a W^a \rightarrow F/M_*\, \lambda\, \lambda,$$

$$\text{i.e. } M = F/M_*$$

Soft scalar squared mass :

$$\int d^4\theta\, Z^\dagger Z\, q^\dagger q/M_*^{\,2} \rightarrow F^\dagger F/M_*^{\,2}\, q^\dagger q,$$

$$\text{i.e. } m^2_{squark} = F^\dagger F/M_*^{\,2}$$

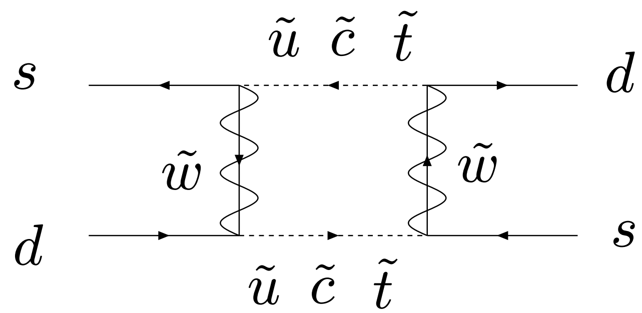
Explicit mediation models determine these interactions.

Supersymmetry Breaking and SUSY spectrum

Although we have no experimental evidence of supersymmetry, there are already good clues to restrict the model parameters.

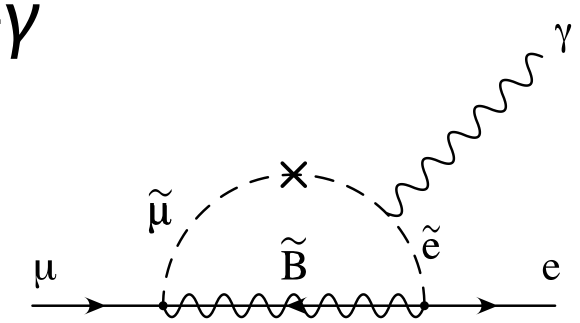
SUSY FCNC contributions
$\longrightarrow$ Flavor-violating soft masses must be suppressed!

K0-$\overline{K0}$ mixing



$$\frac{m^2_{\tilde{s}\tilde{d}}}{m^2_{\text{soft}}} \sim 10^{-(2-3)} \left(\frac{m_{\text{soft}}}{500\,\text{GeV}}\right)$$

$\mu \rightarrow e+\gamma$

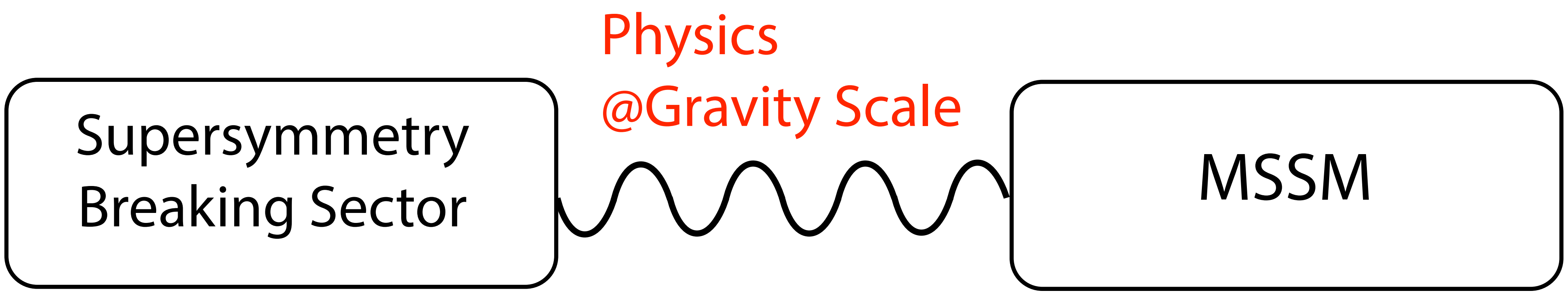


$$\frac{m^2_{\tilde{e}\tilde{\mu}}}{m^2_{\text{soft}}} \sim 10^{-(2-3)} \left(\frac{m_{\text{soft}}}{100\,\text{GeV}}\right)^2$$

Models with flavor-blind soft parameters are preferred!

Supersymmetry Breaking and SUSY spectrum

Exapmple 1 : mSUGRA



Universal scalar mass (almost by hand)

$$\int d^4\theta\, Z^\dagger Z\, \varphi^\dagger\varphi/3M_{PL}^{\,2} \rightarrow F^\dagger F/3M_{PL}^{\,2}\, \varphi^\dagger\varphi,$$

$$m^2_{sfermions} = m_0^2 = F^\dagger F/3M_{PL}^{\,2}$$

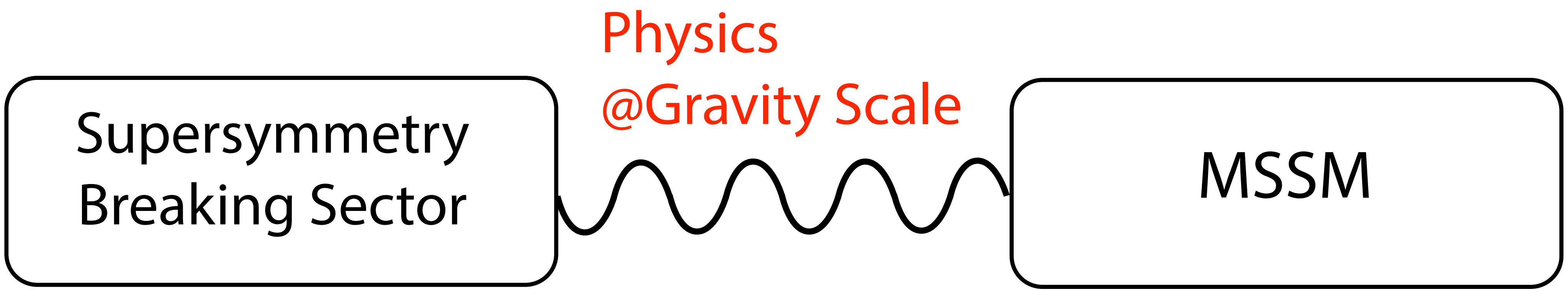
Universal gaugino mass (GUT)

$$\int d^2\theta\;\; cZ\,/M_{PL}\, W_a W^a \rightarrow cF/M_{PL}^{\,2}\, \lambda\lambda,$$

$$\text{i.e. } m_{gaugino} = m_{1/2} = cF/M_{PL}$$

Supersymmetry Breaking and SUSY spectrum

Exapmple 1 : mSUGRA



In the simplest case :

$$m^2_{\rm scalar} = m_0^2,\ \ m_{\rm gaugino} = m_{1/2},\ a_{u,d,e} = y_{y,d,e} \times A_0$$

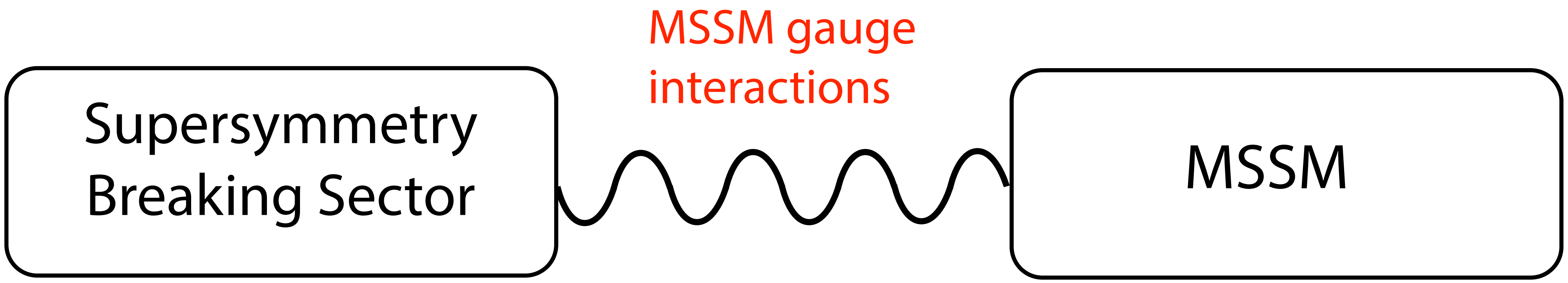
at the Planck scale.

All the soft masses are expected to be around the gravitino mass $m_{3/2} = O(1)$TeV.

The LSP is usually thought to be the lightest neutralino.

Supersymmetry Breaking and SUSY spectrum

Example 2 : Gauge Mediation



Messenger particles : usually $SU(5)_{GUT}$ multiplet

$$\Psi_D(3^*,1,1/3),\ \Psi_{D^c}(3,1,-1/3),\ \Psi_L(2,1,-1/2),\ \Psi_{L^c}(2,1,1/2),$$

$$W = (M_{mess} + Z)\Psi_D \Psi_{D^c} + (M_{mess} + Z)\,\Psi_L \Psi_{L^c}$$

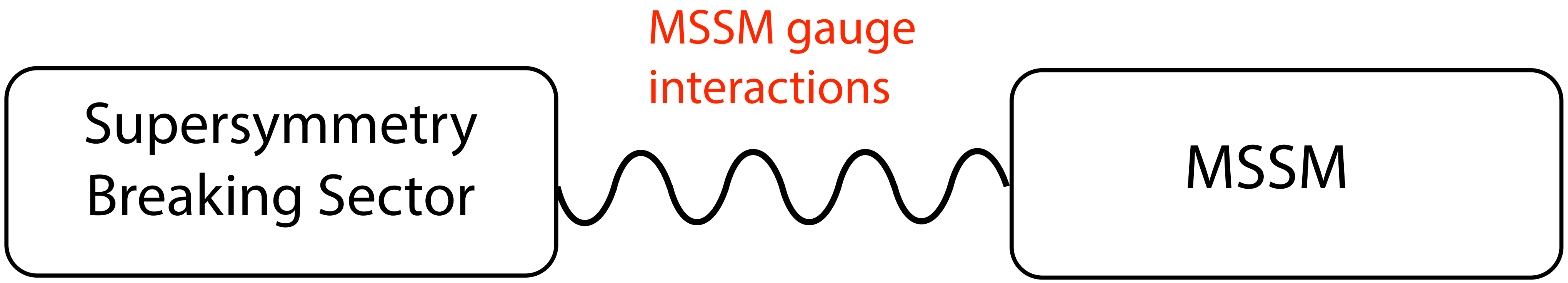
Messenger fermions : M_{mess}

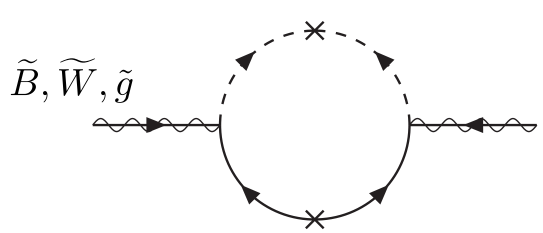
Messenger scalars : $M_{mess}^2 \pm F$

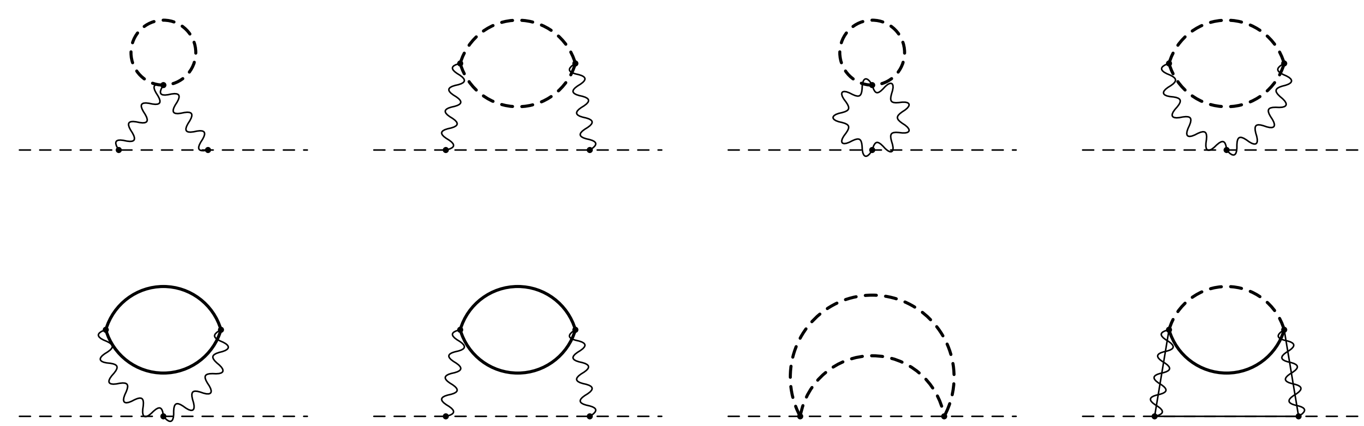
Messengers Masses are split due to the SUSY breaking effect!

Supersymmetry Breaking and SUSY spectrum

Example 2 : Gauge Mediation





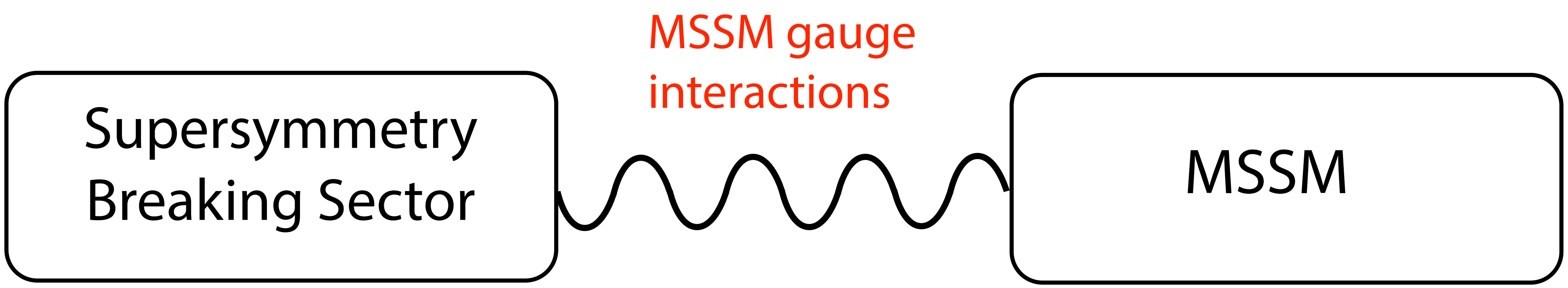


Gaugino mass @ 1-loop

scalar mass @ 1-loop

Supersymmetry Breaking and SUSY spectrum

Example 2 : Gauge Mediation



The SUSY breaking is mediated via the MSSM charged "messenger fields" which couples to the Hidden sector.

$$m_{\mathrm{gaugino}} = \frac{\alpha_a}{4\pi}\Lambda_{\mathrm{SUSY}} \qquad m^2_{\mathrm{scalar}} = 2\left(\frac{\alpha_a}{4\pi}\right)^2 C_a \Lambda^2_{\mathrm{SUSY}}$$

$$\Lambda_{\mathrm{SUSY}} = \frac{F}{M}$$ F : SUSY parameter M : Messenger scale

at the Messenger scale.

For a given SUSY breaking "F",

(Gauge Mediaiton) ≫ (Gravity Mediaiton)

For a fixed SUSY spectrum → gravitino is much lighter and the LSP!

Supersymmetry Breaking and SUSY spectrum

Example 3 : Anomaly Mediation

Supersymmetry Breaking Sector	SUGRA Effects	MSSM

In SUGRA, all the dimensionful supersymmetric parameters are accompanied by soft parameters even in the absence of direct couplings to the SUSY breaking sector!

Ex) Mass term in $W = \mu H_u H_d$

$\rightarrow$ SUSY breaking bi-linear term : $V = \mu\, m_{3/2} H_u H_d$

For a supersymmetric coupling with the mass dimension “n”, it is accompanied by a soft parameter n x $m_{3/2}$.

Supersymmetry Breaking and SUSY spectrum

Example 3 : Anomaly Mediation

Supersymmetry Breaking Sector

SUGRA Effects

MSSM

Gauge coupling : mass dimension 0 at the tree-level

→ gaugino mass is zero at the tree-level!

Gauge coupling has anomalous mass dimension at the loop-level!

→ gaugino mass is non-zero at the loop-level!

$M_a = \beta_a/g_a \times m_{3/2}$ (β_a : β function of gauge coupling)

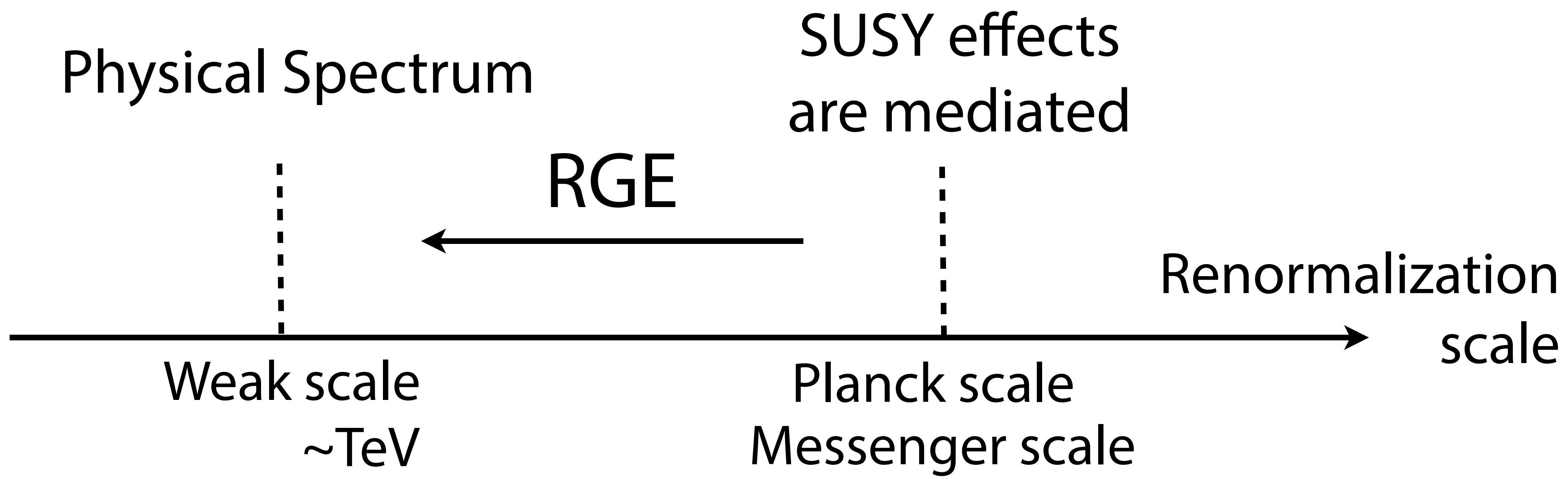
SU(2) gauge coupling is less scale dependent → the wino is the LSP!

Anomaly Mediation effects are subdominant if there are direct interactions to the SUSY breaking sector.

Supersymmetry Breaking and SUSY spectrum

The above soft parameters are given at the high energy scale.

$\longrightarrow$ We need to evolve the mass parameters down to around TeV scale to know the spectrum.



Supersymmetry Breaking and SUSY spectrum

Gaugino Masses Running

The RG equation of gaugino masses

$$\frac{d}{dt}M_a = \frac{1}{8\pi^2} b_a g_a^2 M_a \qquad (b_a = 33/5, 1, -3)$$

$$\left(\frac{d}{dt}\alpha_a^{-1} = -\frac{b_a}{2\pi}\right)$$

$$\frac{M_1}{g_1^2} = \frac{M_2}{g_2^2} = \frac{M_3}{g_3^2}$$ at any RG scale

↓

$$M_1 : M_2 : M_3 = 0.5 : 1 : 3.5$$ at the TeV range

This ratio is the prediction of the universal gaugino mass!

[Realized in both the mSUGRA and gauge mediation but not in the AMSB]

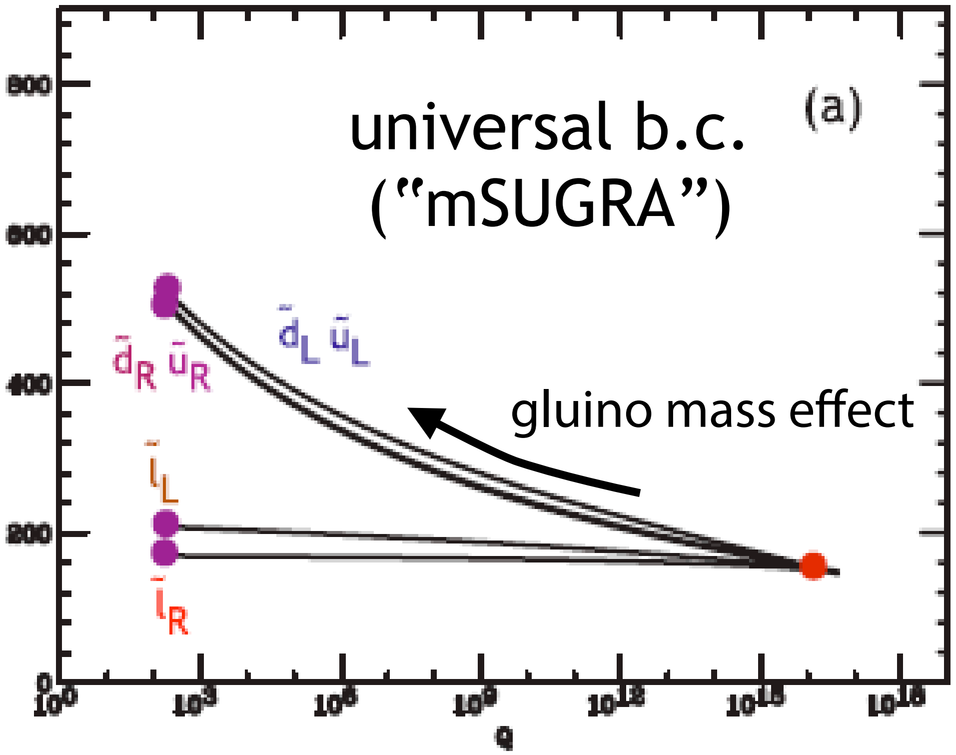
Checking the gaugino mass universality provides us very important hints on the origin of SUSY breaking.

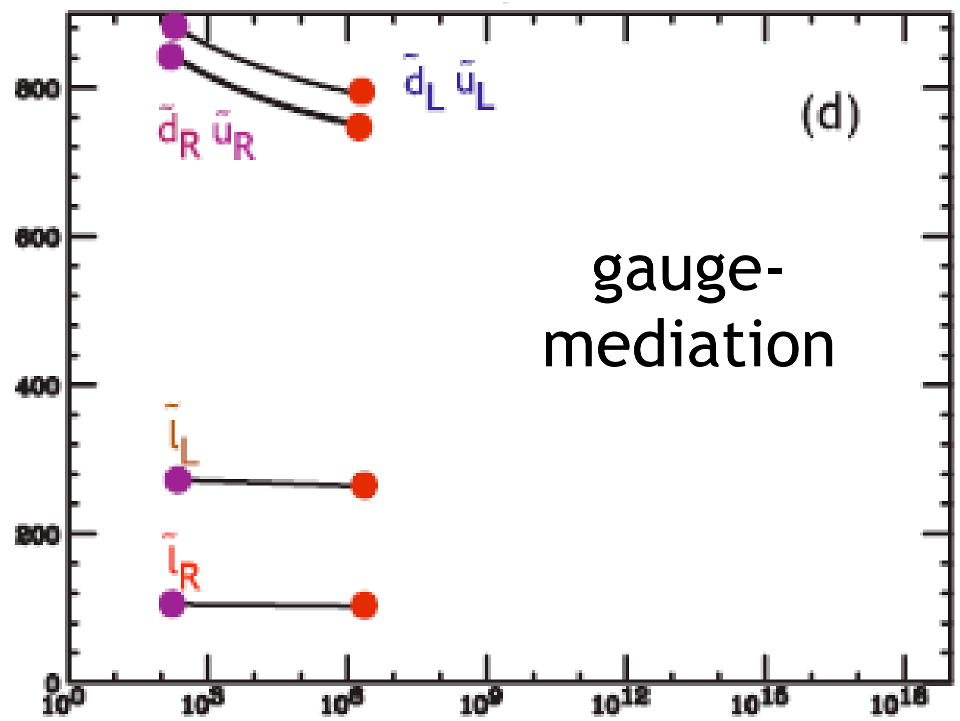
Supersymmetry Breaking and SUSY spectrum

squark/slepton Masses
(first 2 generations)

$$16\pi^2 \frac{d}{dt} m_\phi^2 = - \sum_{a=1,2,3} 8 g_a^2 C_a^\phi |M_a|^2$$

Gaugino mass effects raise the scalar masses at the low energy!





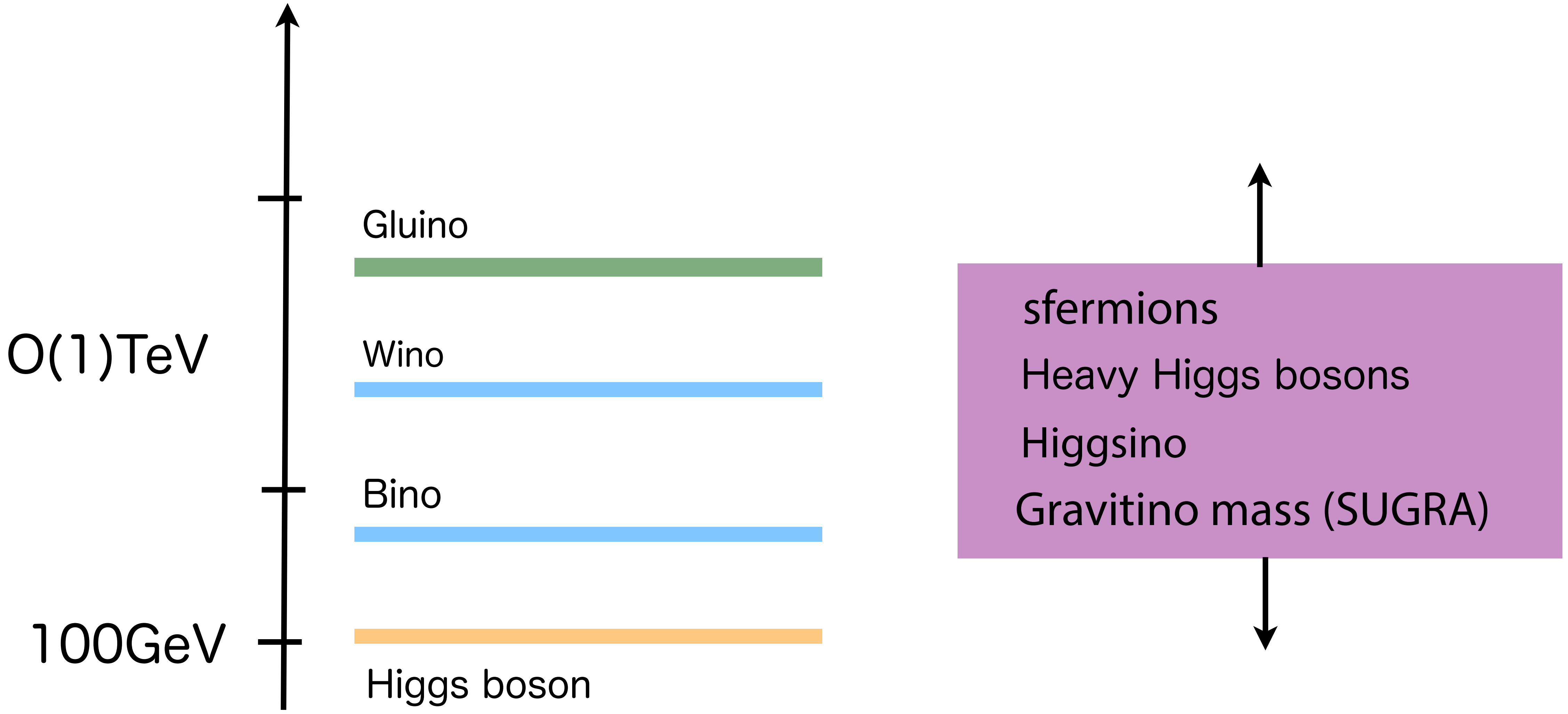
[borrowed from M.Peskin's lecture]

Typically, squarks are much heavier than sleptons.

Typically, squarks are degenerated compared with leptons due to large gluino contributions

Supersymmetry Breaking and SUSY spectrum

Typical Spectrum...

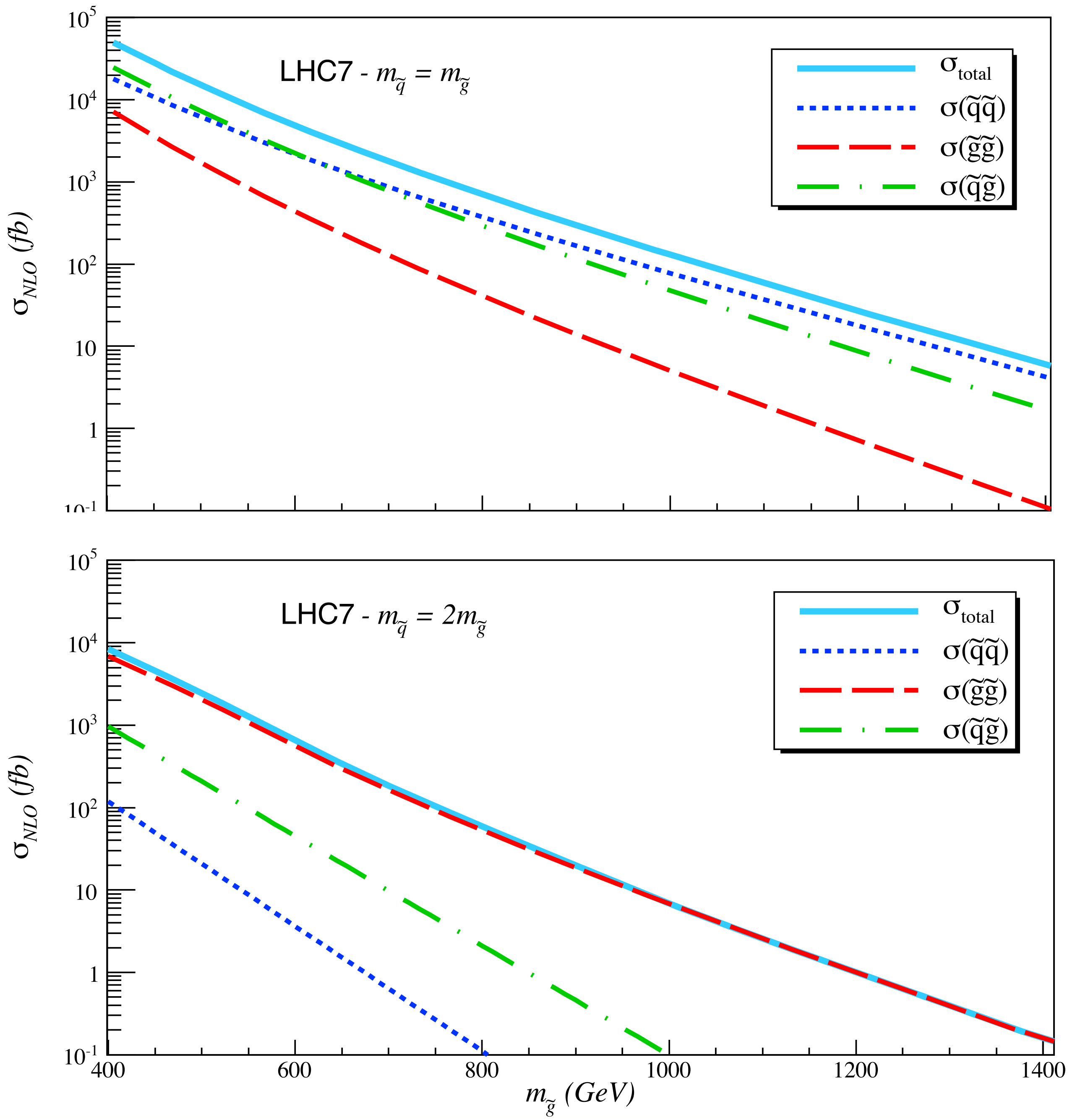


Gravitino mass (Gauge Mediation) : O(1)eV - O(1) GeV

Gravitino mass (Anomaly Mediation) : O(10-1000) TeV

SUSY at the LHC

Production cross section of the SUSY particles @ LHC



gluino and squark are mainly produced

$$
\begin{aligned}
gg &\longrightarrow \tilde{g}\tilde{g}, \quad \tilde{q}_i\tilde{q}_j^*, \\
gq &\longrightarrow \tilde{g}\tilde{q}_i, \\
q\bar{q} &\longrightarrow \tilde{g}\tilde{g}, \quad \tilde{q}_i\tilde{q}_j^*, \\
qq &\longrightarrow \tilde{q}_i\tilde{q}_j,
\end{aligned}
$$

If they are within TeV
→ they should have beed discovered...

SUSY at the LHC

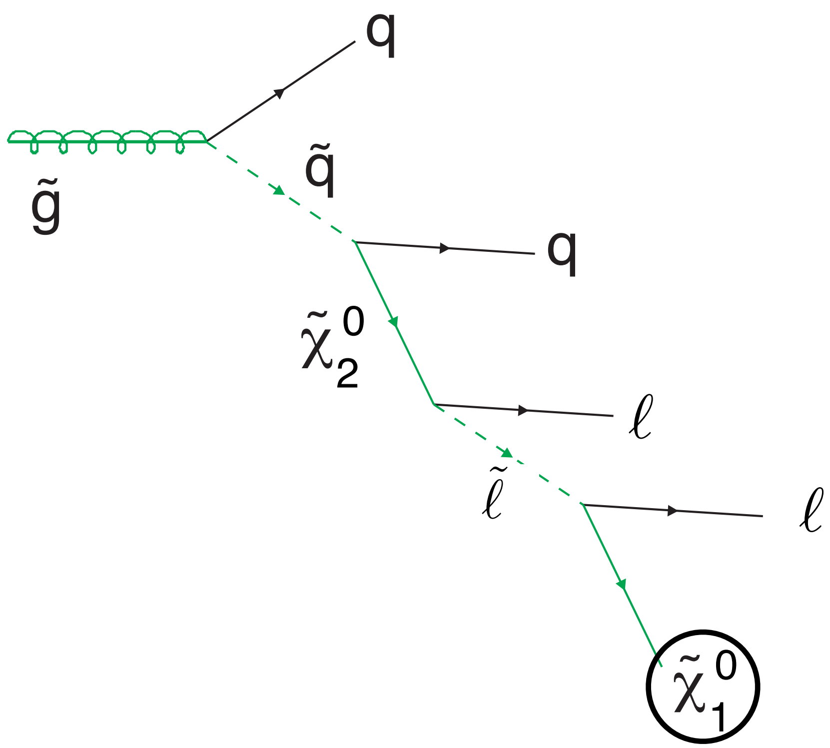
How do we look for the SUSY events ?

It depends on the LSP...

In the models with neutralino LSP (e.g. mSUGRA), the decays of the produced superparticles result in final state with two LSPs which escape the detector.

SUSY events : n jets + m leptons + missing E_T $(n \geq 0, m \geq 0)$

ex)



The LSP escapes the detector and results in the missing ET.

SUSY at the LHC

In the models with gravitino LSP (e.g. gauge mediation), the NLSP can have a long lifetime.

[NLSP : The lightest SUSY particle in the MSSM]

Decay length of the NLSP (decaying into gravitino)

$$d/\beta\gamma_{\rm NLSP} \sim 6\,{\rm m} \times \left(\frac{m_{\chi^0}}{100\,{\rm GeV}}\right)^{-5} \left(\frac{m_{3/2}}{1\,{\rm keV}}\right)^{2}$$

Prompt decaying NLSP

SUSY events : n jets + m leptons + missing E_T (n≥0,m≥0)

(+ photons)

Escaping neutralino NLSP

SUSY events : n jets + m leptons + missing E_T (n≥0,m≥0)

Escaping charged NLSP

SUSY events : n jets + m leptons + new charged tracks

SUSY at the LHC

SM backgrounds

SUSY events : n jets + m leptons + missing E_T

QCD multi-jets (ET>100GeV) ~1μb

Suppressed by large missing ET.

W/Z + jets ~ 10nb [$W \rightarrow \tau\nu$, $l\nu$, $Z \rightarrow \nu\nu$]

Top pair + jets ~ 800pb

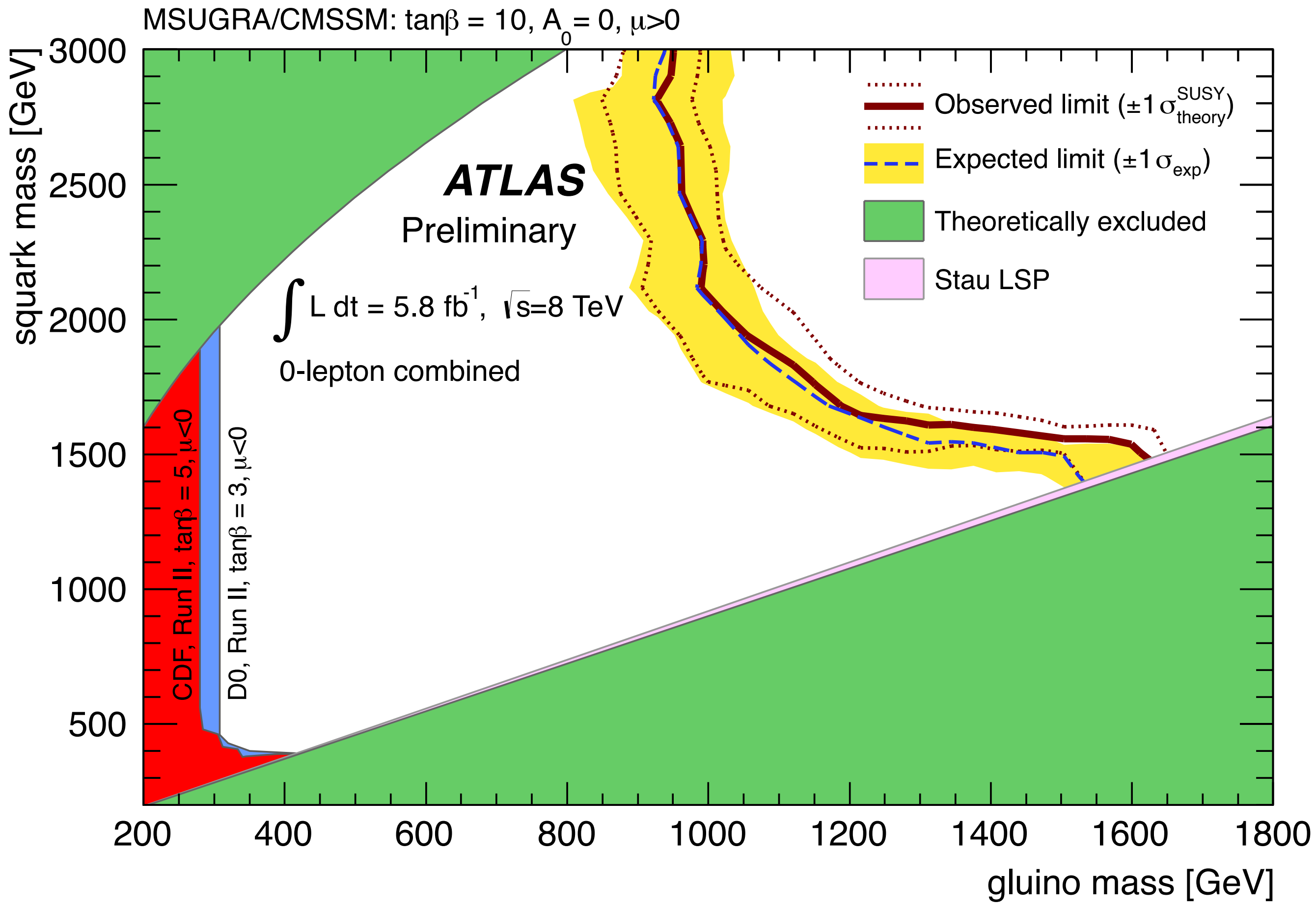
SUSY events can win with larger ET, more jets

SUSY events : n jets + m leptons + new charged tracks

Collect slow tracks to distinguish the charged tracks from the muon tracks.

SUSY at the LHC

ATLAS 2012



0-lepton + jets + missing E_T

95% exclusion limit

gluino mass > 950GeV [mgluino ≪ msquark]

gluino mass > 1.6TeV [mgluino = msquark]

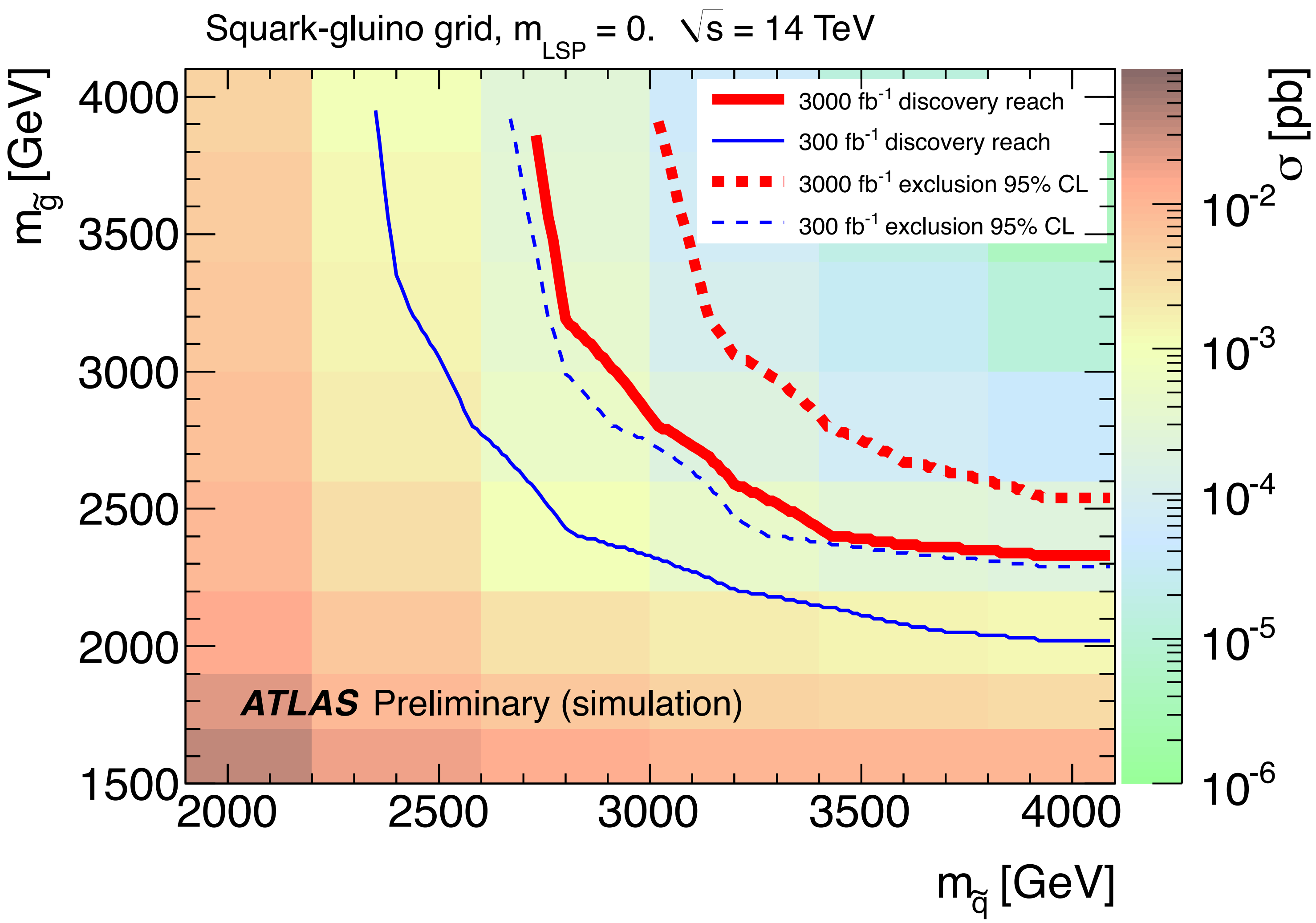
Large portion of the parameter space expected from the conventional naturalness has been excluded...

We were too serious about the naturalness?

The light SUSY but more intricate spectrum?

SUSY at the LHC

Prospects :



[borrowed from a talk by K.Terasi]

@14TeV run : gluino ~2TeV, squark ~2.3TeV with $300 fb^{-1}$

Higgs mass in the MSSM

What does 126GeV Higgs boson mean in SUSY models?

In the MSSM, the tree-level Higgs boson mass is given by the gauge coupling constants.

$$V = - m_{higgs}^2/2\, h^\dagger h + \lambda/4\, (h^\dagger h)^2$$

A combination of the SUSY breaking masses and the Higgsino mass

$\lambda = (g'^2+g^2)/2\, \cos^2 2\beta$
related to gauge couplings

$[\tan\beta = v_u/v_d]$

The predicted Higgs boson mass is around Z-boson mass,

$$m_{higgs} = \lambda^{1/2}\, v \sim m_Z \cos 2\beta$$

at the tree-level.

It looks inconsistent with the observed Higgs mass...

Higgs mass in the MSSM

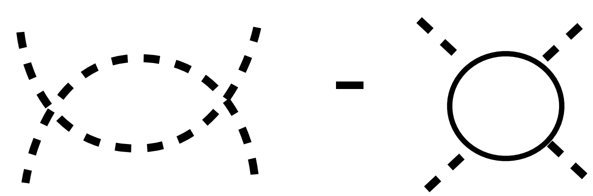
The radiative corrections to the Higgs boson mass logarithmically depends on the stop masses!

$$m_{h^0}^2 \lesssim m_Z^2 \cos^2 2\beta + \boxed{\frac{3}{4\pi^2} y_t^2 m_t^2 \sin^2\beta \left(\log \frac{m_{\tilde{t}}^2}{m_t^2} + \frac{A_t^2}{m_{\tilde{t}}^2} - \frac{A_t^4}{12 m_{\tilde{t}}^4} \right)} .$$

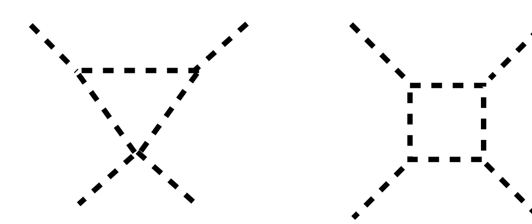
Tree-level quartic term:

$$\lambda = \frac{1}{2}(g_1^2 + g_2^2)\cos^2 2\beta$$

One-loop log enhanced:



One-loop finite:

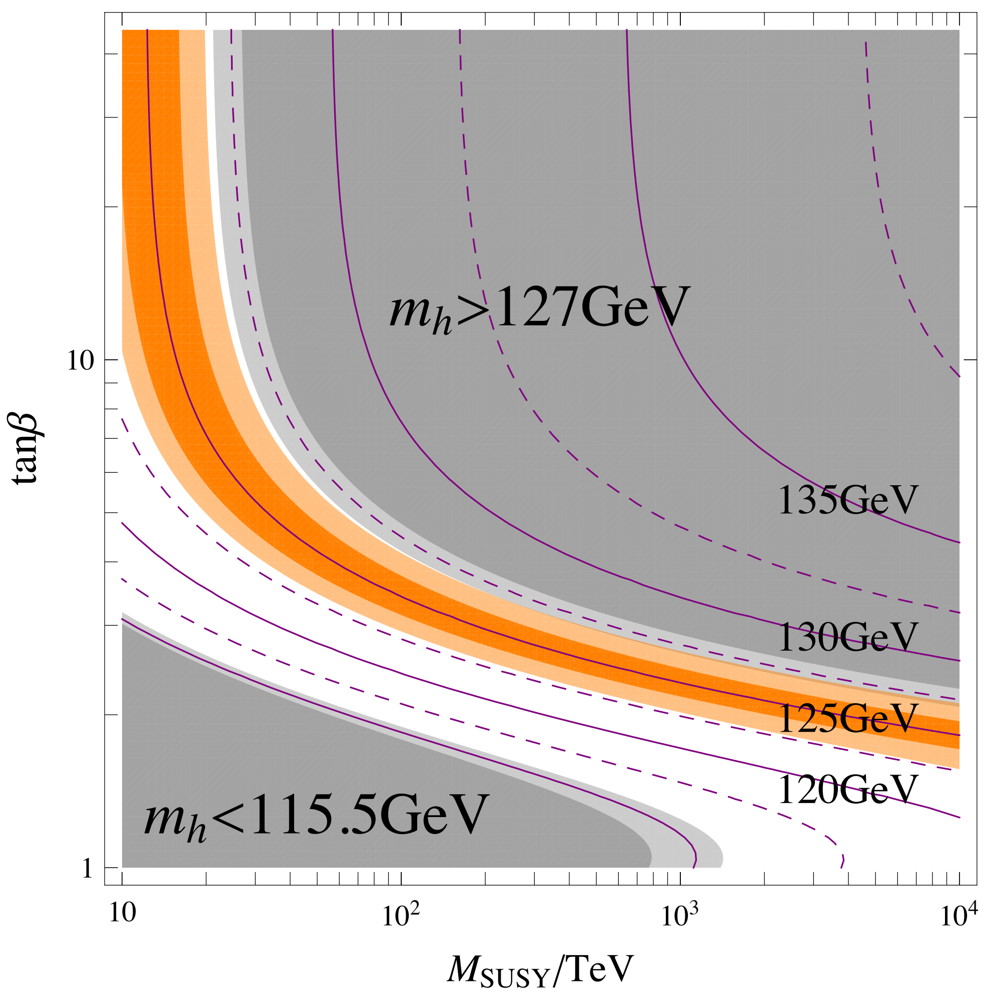


['91 Haber, Hempfling, '91 Ellis, Ridolfi, Zwirner, '91 Okada, Yamaguchi, Yanagida]

The heavier Higgs boson mass than m_Z can be obtained with large SUSY breaking effects!

Higgs mass in the MSSM

In the simplest case, m_{higgs} ~ *126 GeV* suggests the sfermion (stop) masses above *O(10-100)TeV* !



['12, MI, Matsumoto,Yanagida (μ_H=O(M_{susy}))]

✔ SUSY-FCNC/CP constraints are relaxed!

$$\sqrt{\tilde{m}_{LL}\tilde{m}_{RR}} \gtrsim 4000\,\mathrm{TeV} \times \sqrt{\left|\mathrm{Im}\left(\frac{m^{d\,2}_{12,LL}}{\tilde{m}^2_{LL}}\frac{m^{d\,2}_{12,RR}}{\tilde{m}^2_{RR}}\right)\right|}\,,$$

['96 Gabbiani, Gabrielli, Masiero, Silvestrini]

✔ Consistent with negative results at the LHC experiments.

gluino mass >*1 TeV* for M_{susy} >>*TeV*

Higgs mass in the MSSM

How about the naturalness arguments?

$\longrightarrow$ $m_{SUSY} = O(10\text{-}100) TeV$ requires fine-tuning of O(10^{-4}-10^{-6}).

✓ This is not satisfactory at all, but is much better than the SM which requires fine-tuning of O(10^{-28}-10^{-32}).

The naturalness arguments are still motivation for the “low scale” SUSY.

What fills the gap between O(10-100)TeV and O(100)GeV?

✓ At this point, I do not know the answer...

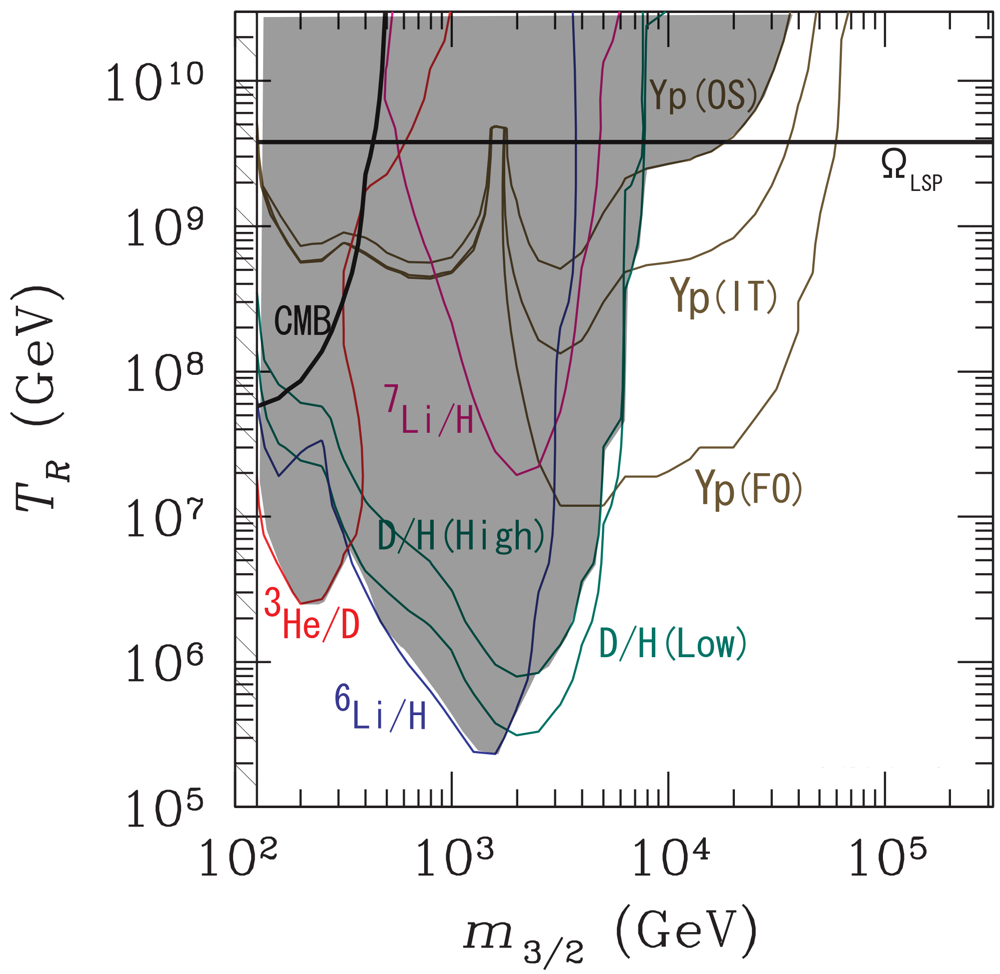
The measure of the naturalness should be defined on multidimensional parameter space with, for example, cosmological parameters...

Higgs mass in the MSSM

This is a good news in cosmology!

✔ The gravitino problem is solved for $m_{3/2}$ = O(10-100)TeV.

['05 Kohri, Moroi, Yotsuyanagi]



The gravitinos are produced by particle scattering in thermal bath in the early universe (abundance proportional to T_R). ['82 Weinberg]

$$Y_{3/2} = n_{3/2}/s \sim 10^{-12} \times (T_R / 10^9\ GeV)$$

[T_R : Reheating temperature after inflation]

✔ $m_{3/2}$=O(1)TeV → BBN constrains thermal history of cosmology...

✔ The model with $m_{sfermion}$ = $m_{3/2}$ = O(10-100)TeV can be consistent with simple baryogenesis such as leptogenesis!

[Leptogenesis requires $T_R > 10^9$GeV, '86 Fukugita,Yanagida]

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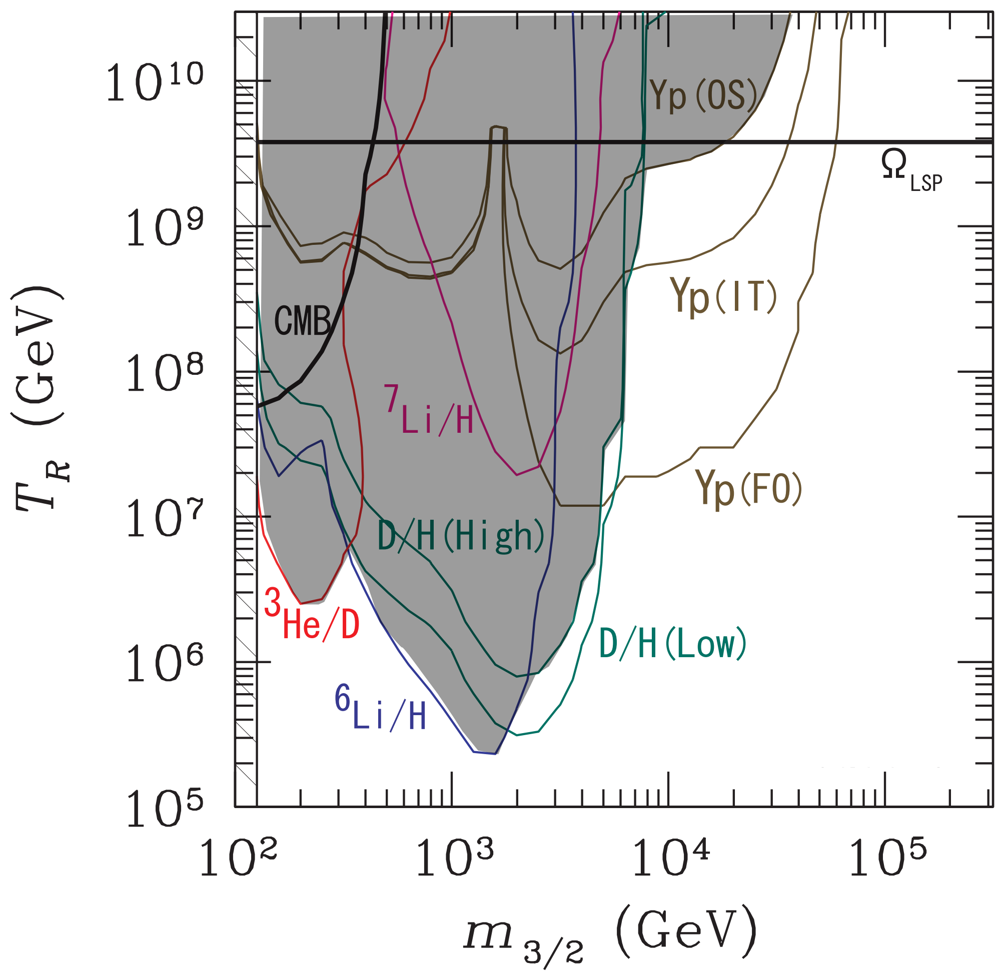


The gravitino decay rate is suppressed by the Planck scale ($\Gamma_{3/2} = m_{3/2}^3/M_{PL}^2$)

$$\tau_{3/2} \sim 0.01 sec \times (100 TeV / m_{3/2})^3$$
$$[\, \tau_{BBN} = O(1) sec \,]$$

✓ $m_{3/2}$=O(1)TeV → BBN constrains thermal history of cosmology...

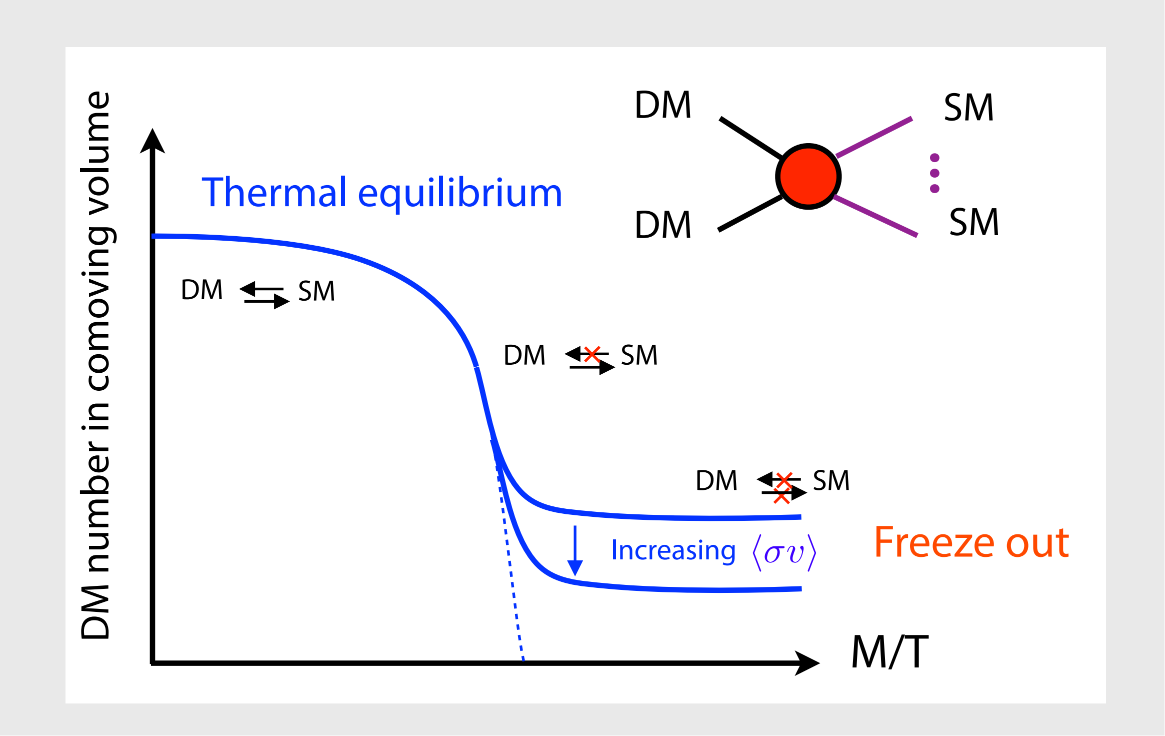
✓ The model with $m_{sfermion} = m_{3/2}$ = O(10-100)TeV can be consistent with simple baryogenesis such as leptogenesis!

[Leptogenesis requires $T_R > 10^9$GeV, '86 Fukugita,Yanagida]

Dark Matter

Who is Dark Matter?

The thermal relics of Weakly Interacting Massive Particles (WIMPs) are the most motivated candidate.

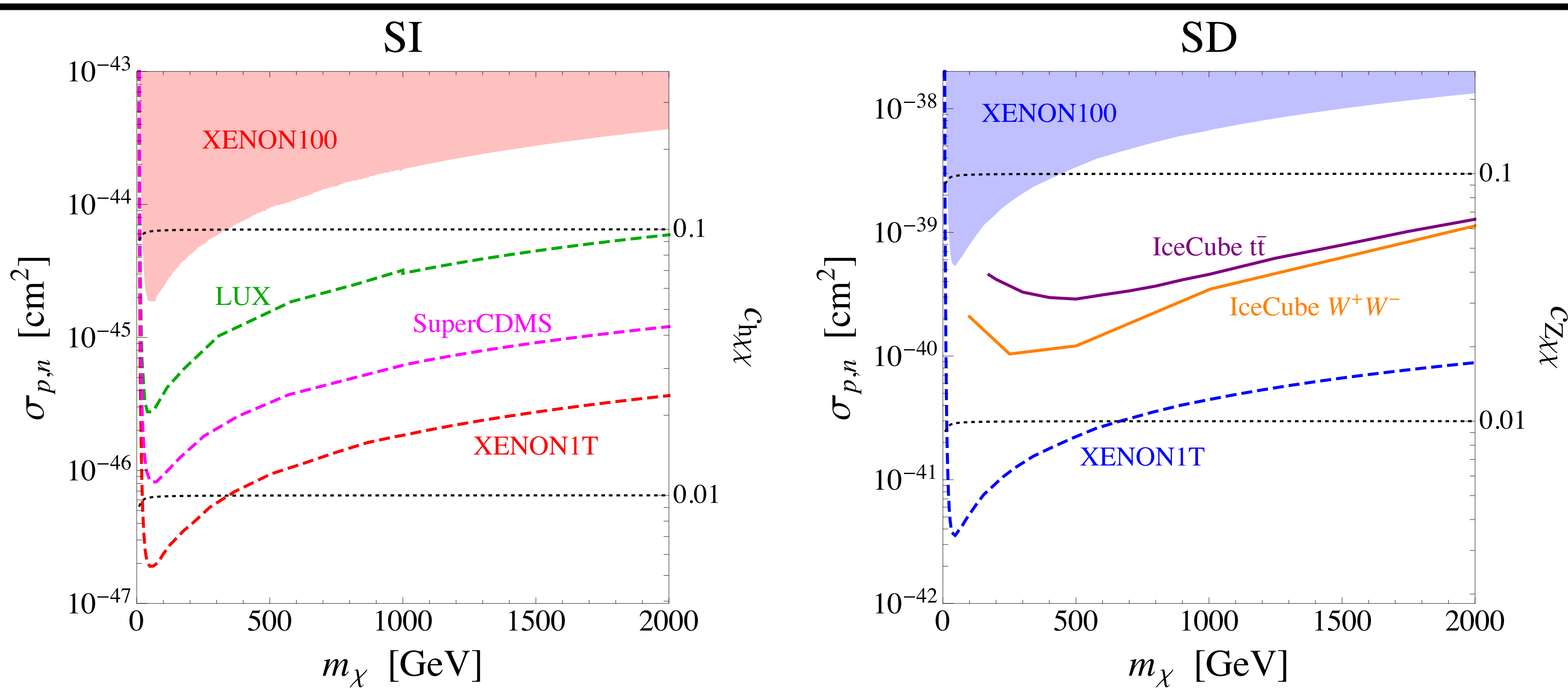


- DM is in thermal equilibrium for T > M.
- For M < T, DM is no more created
- DM is still annihilating for M < T for a while...
- DM is also diluted by the cosmic expansion
- DM cannot find each other and stop annihilating at some point
- DM number in comoving volume is frozen

$$\Omega_{DM} h^2 \simeq 0.1 \times \left(\frac{10^{-9}\,\mathrm{GeV}^{-2}}{\langle \sigma v \rangle} \right)$$

The WIMPs with the annihilation cross section $\langle \sigma v \rangle \sim 10^{-9} \mathrm{GeV}^{-2}$ at the early universe are very good candidates of Dark Matter.

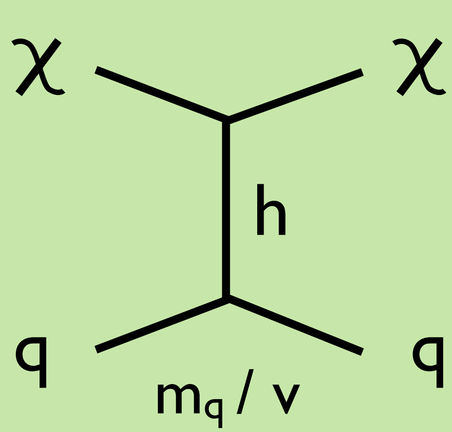
Dark Matter direct detection



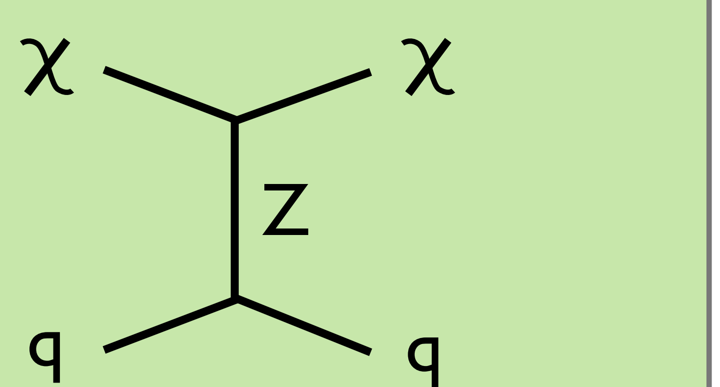
$$\mathcal{L} \supset \frac{c_{h\chi\chi}}{2}\, h(\chi\chi + \chi^\dagger\chi^\dagger) \;+\; c_{Z\chi\chi}\, \chi^\dagger \bar{\sigma}^\mu \chi Z_\mu,$$

c's depend on gauge coupling x neutralino mixing angles

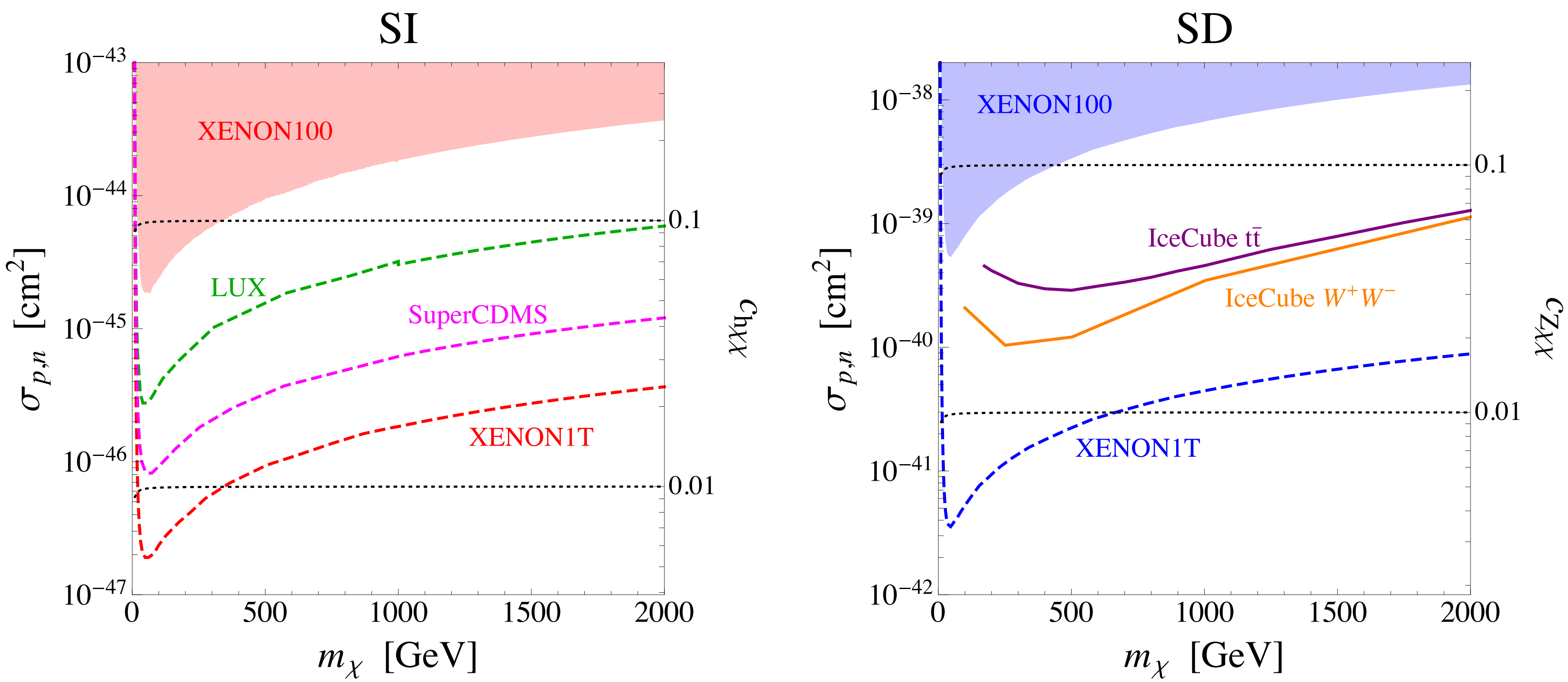
$$\sigma_{\rm SI} = 8 \times 10^{-45}\ {\rm cm}^2 \left(\frac{c_{h\chi\chi}}{0.1}\right)^2$$



$$\sigma_{\rm SD} = 3 \times 10^{-39}\ {\rm cm}^2 \left(\frac{c_{Z\chi\chi}}{0.1}\right)^2$$



Dark Matter direct detection



$$\mathcal{L} \supset \frac{c_{h\chi\chi}}{2}\, h(\chi\chi + \chi^\dagger\chi^\dagger) \;+\; c_{Z\chi\chi}\, \chi^\dagger\bar{\sigma}^\mu\chi Z_\mu,$$

$\sigma_{SI} \sim (m_q/v)^2 << \sigma_{SD} \sim g^2$ (per nucleon)

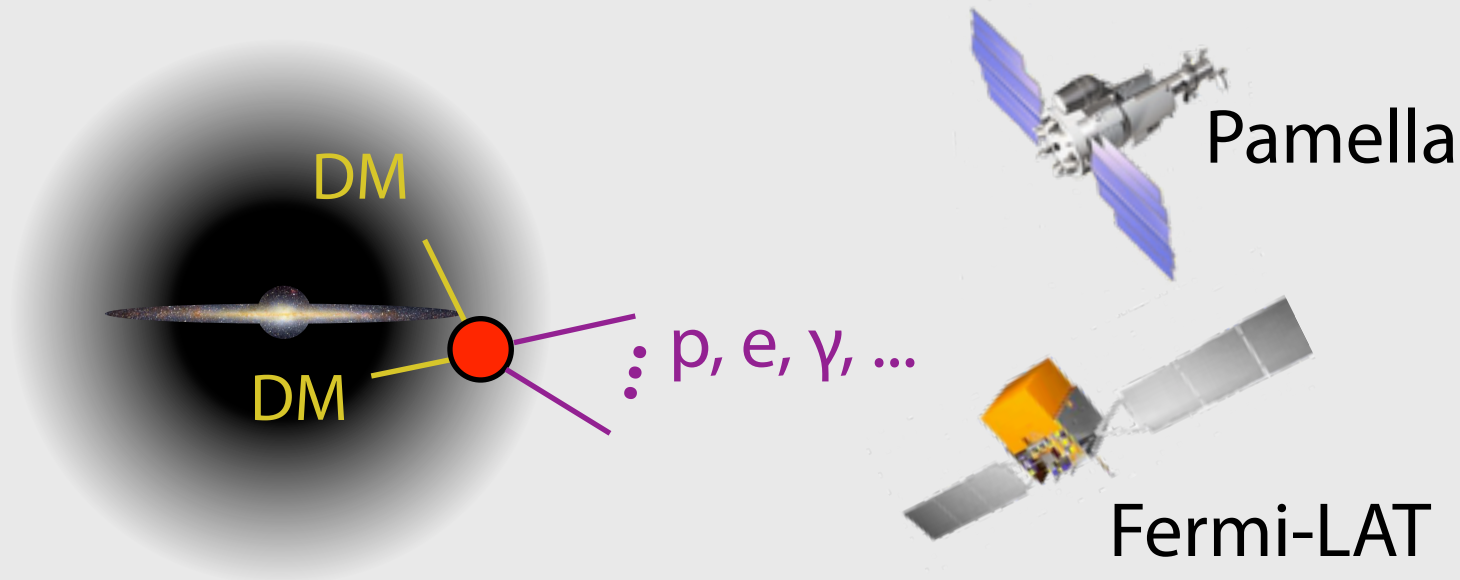
$\sigma_{SI} \sim A^2 >> \sigma_{SD} \sim J(J+1)$ (per nucleus)

Spin independent/dependent constraints are comparable in strength...

(LUX~2013, Xenon1T ~2015...)

Dark Matter indirect detection

The WIMPS are annihilating even now!



DM can be probed as a source of cosmic ray!

Cosmic Ray charged particle (proton, electron, etc...)

They change their direction during the propagation.

Gamma ray, neutrino fluxes : coming straight from the source.

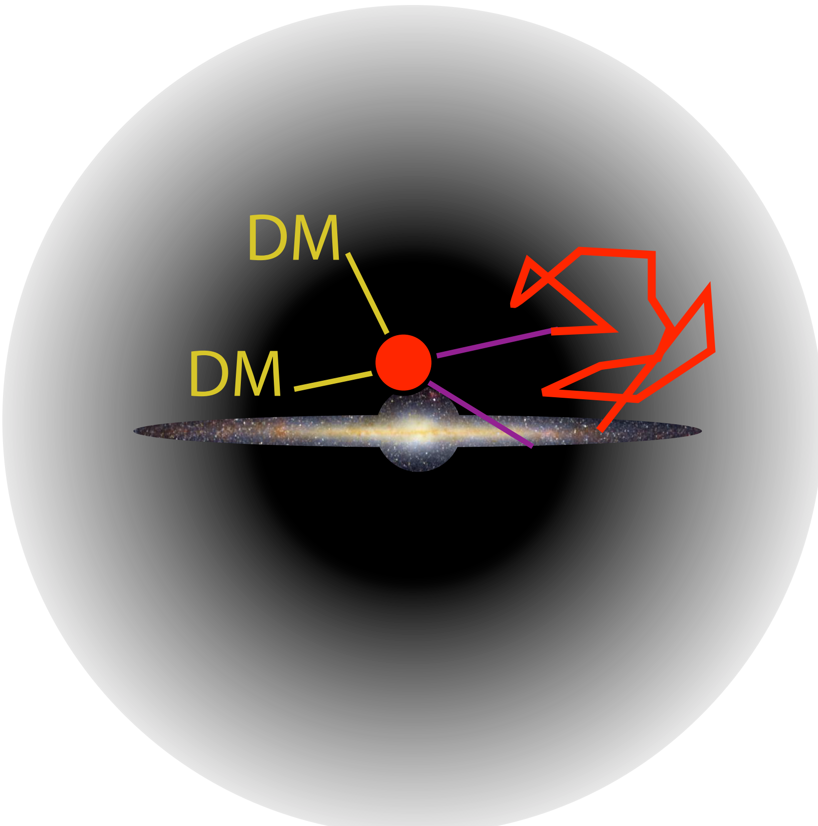
primary source : DM decay, annihilation

→ many independent targets (Galactic Center, Cluster, etc...)

secondary source : charged particles from DM decay, annihilation

Dark Matter indirect detection

Cosmic Ray charged particle (proton, electron, etc...)



Flux : $\psi(E) \sim Q(E) \times Min[\, t_{diff}, t_{loss}\,]$

t_{diff} = (time scale of diffusion)
$\sim 10^{17} sec \times (E/GeV)^{-\delta}$

t_{loss} = Energy loss rate $\sim E^{-1}$

Background (Super Nova) : $Q(E) \sim E^{-2}$

For primary proton, $t_{diff} \ll t_{loss}$

$$\psi_p(E) \sim Q(E)\, t_{diff} \sim E^{-2-\delta} \sim E^{-2.7}$$

For electron, $t_{diff} \ll t_{loss}$ for low energy, $t_{loss} \ll t_{diff}$ *for high energy*

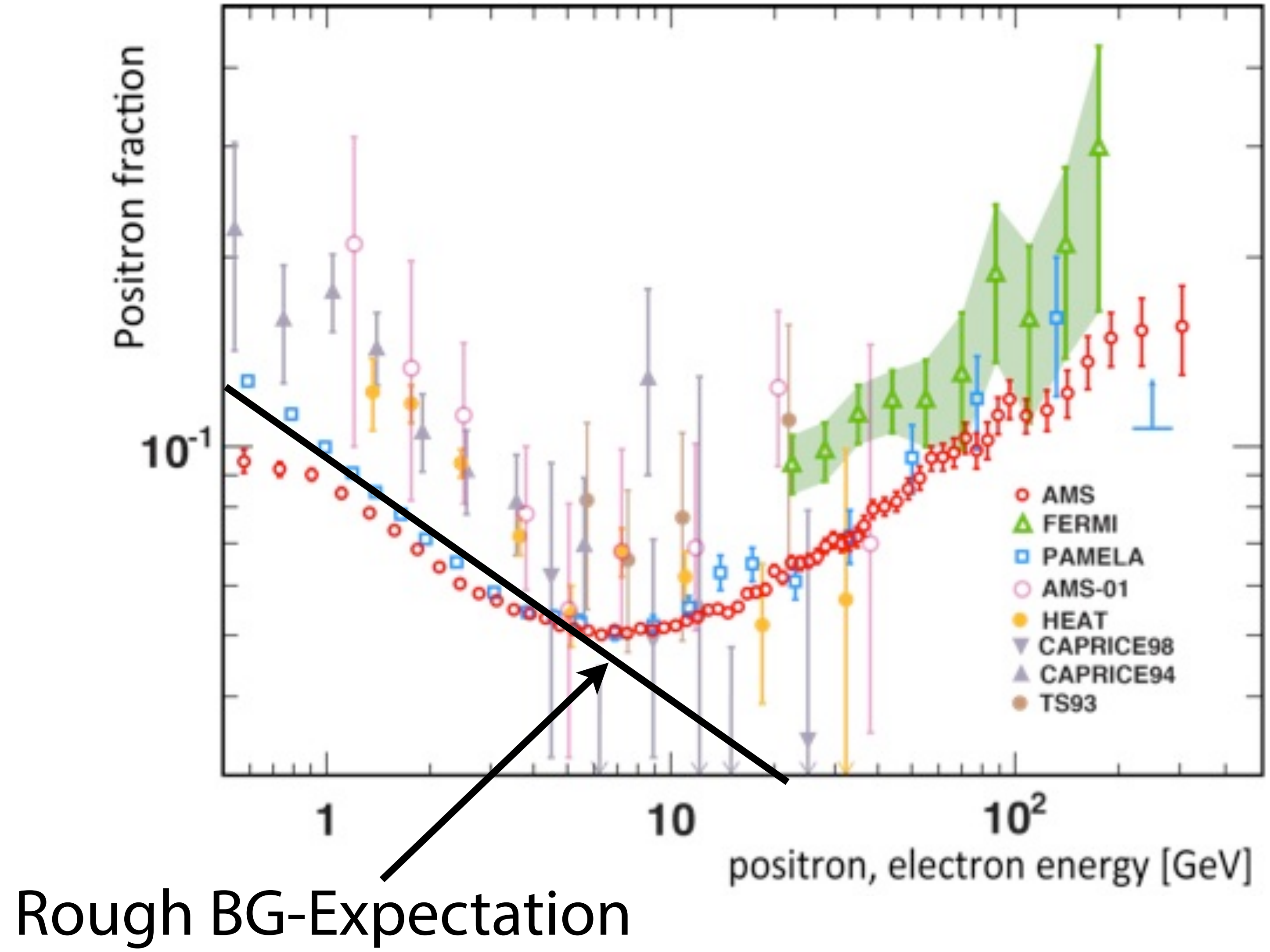
High energy Primary electron : $\psi_{prim\, e}(E) \sim Q(E)\, t_{loss} \sim E^{-3}$

High energy secondary electron, positron
from the proton flux: $\psi_{second\, e}(E) \sim Q_p(E)\, t_{loss} \sim E^{-3-\delta}$

Ratio of the Positron/Electron flux $\sim \psi_{second\, e}(E) / \psi_{prim\, e}(E) \sim E^{-\delta}$

→ Good probe for the DM contribution!

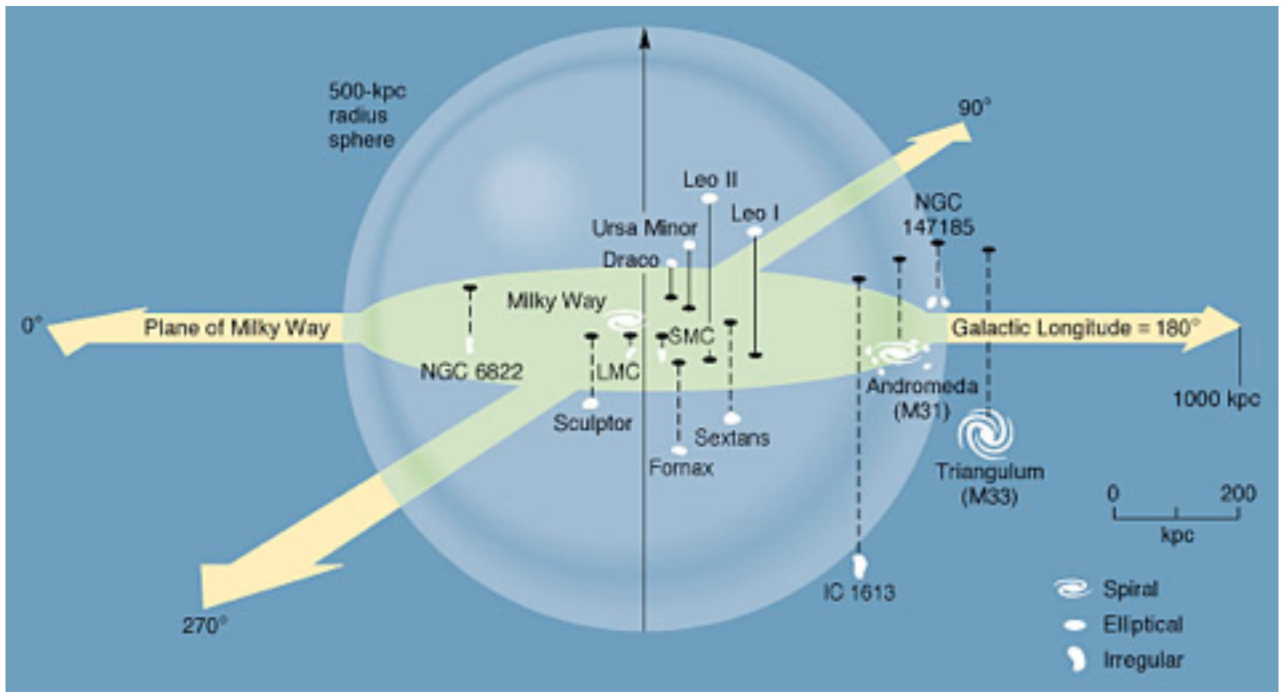
The deviation from $E^{-\delta}$ in the positron fraction has been observed by the Pamela, Fermi and AMS-02!



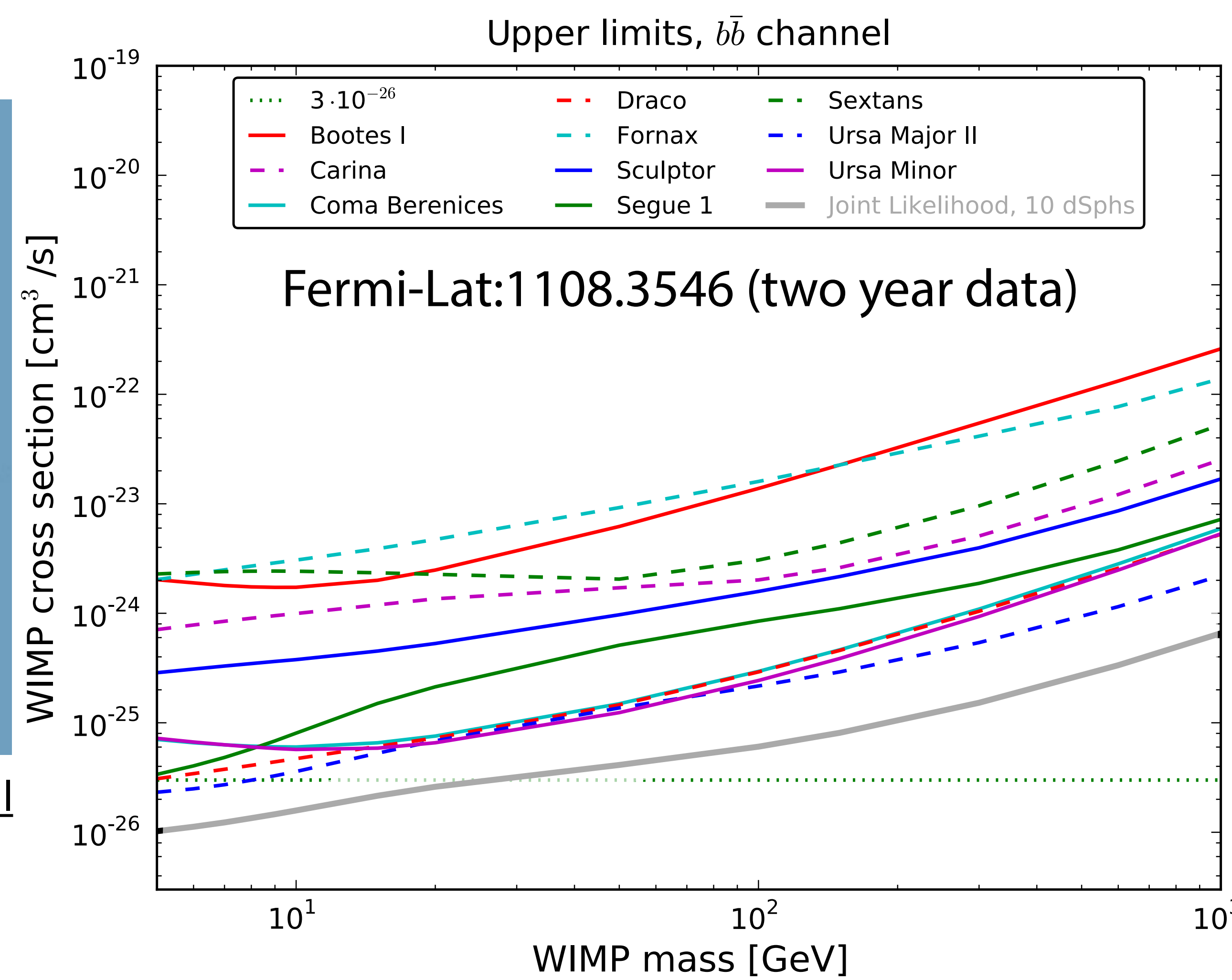
→ Important Hints on the DM?

Dark Matter indirect detection

The gamma ray flux from the dwarf Spheroidal galaxies puts rather severe constraints on the DM annihilation!



http://astronomy.nmsu.edu/tharriso/ast110/class24.html



$$\frac{\mathrm{d}\Phi_\gamma}{\mathrm{d}E_\gamma}(E_\gamma, \Delta\Omega) = \frac{1}{4\pi}\frac{\langle\sigma v\rangle}{2{m_\chi}^2}\frac{\mathrm{d}N_\gamma}{\mathrm{d}E_\gamma} \times \boxed{\int_{\Delta\Omega}\int_{l.o.s}\rho^2_{\mathrm{DM}}(l,\Omega)dld\Omega}$$

J-factor : DM profile

Dark Matter indirect detection

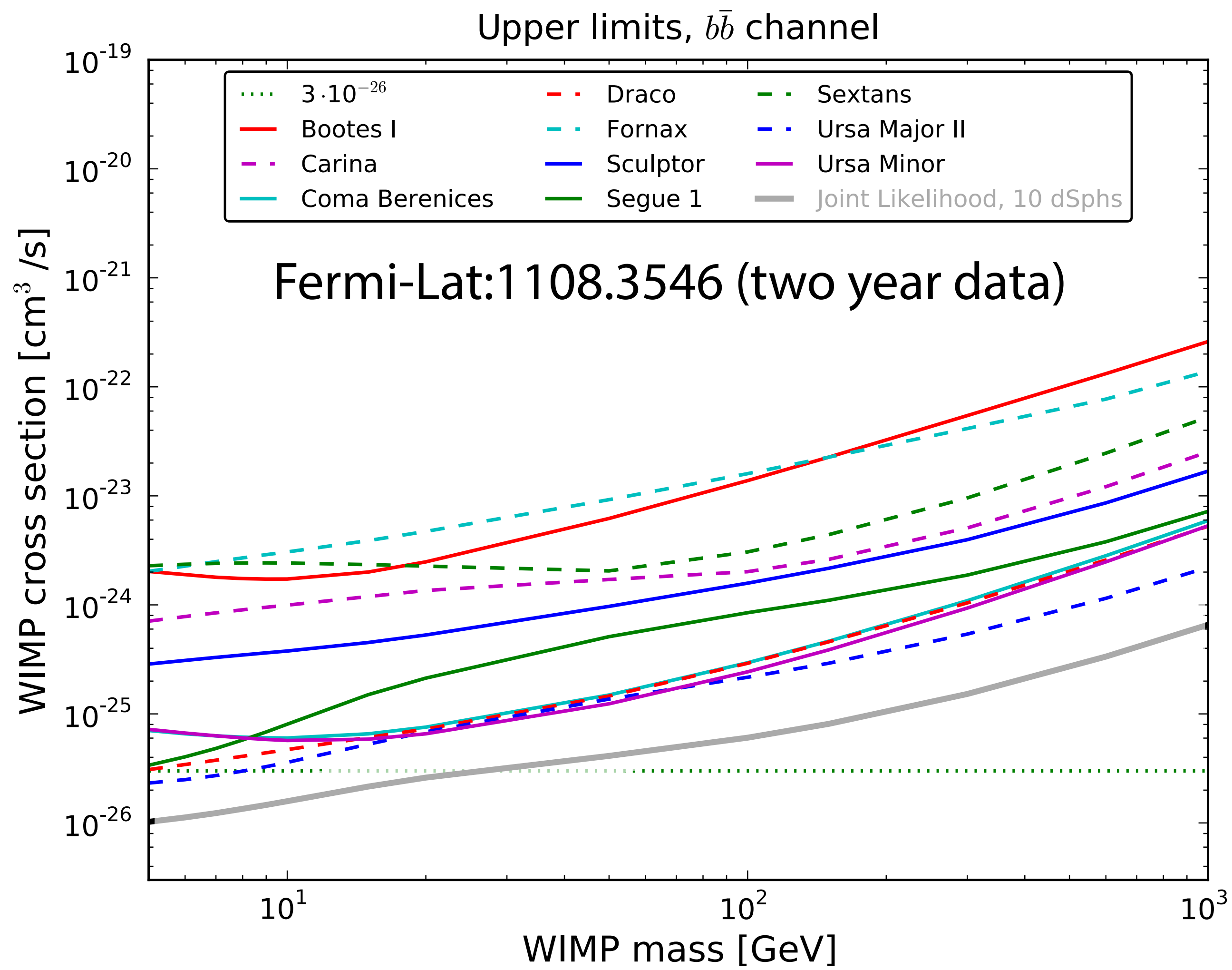
The gamma ray flux from the dwarf Spheroidal galaxies puts rather severe constraints on the DM annihilation!

classical

dSph	long. [deg]	lat. [deg]	d [kpc]	$\log_{10}[J(0.5^\circ)]$ $[\mathrm{GeV}^2\mathrm{cm}^{-5}]$
Ursa Minor	105.0	+44.8	66	18.5 ± 0.18
Sculptor	287.5	-83.2	79	18.4 ± 0.13
Draco	86.4	+34.7	82	18.8 ± 0.13
Sextans	243.5	+42.3	86	17.8 ± 0.23
Carina	260.1	-22.2	101	18.0 ± 0.13
Fornax	237.1	-65.7	138	17.7 ± 0.23

Faint

dSph	long. [deg]	lat. [deg]	d [kpc]	$\log_{10}[J(0.5^\circ)]$ $[\mathrm{GeV}^2\mathrm{cm}^{-5}]$
Bootes I	358.08	+69.62	60	17.7 ± 0.34
Coma Berenices	241.9	+83.6	44	19.0 ± 0.37
Segue 1	220.48	+50.42	23	19.6 ± 0.53
Ursa Major II	152.46	+37.44	32	19.6 ± 0.40



Constraint from faint dSg has a large ambiguities...

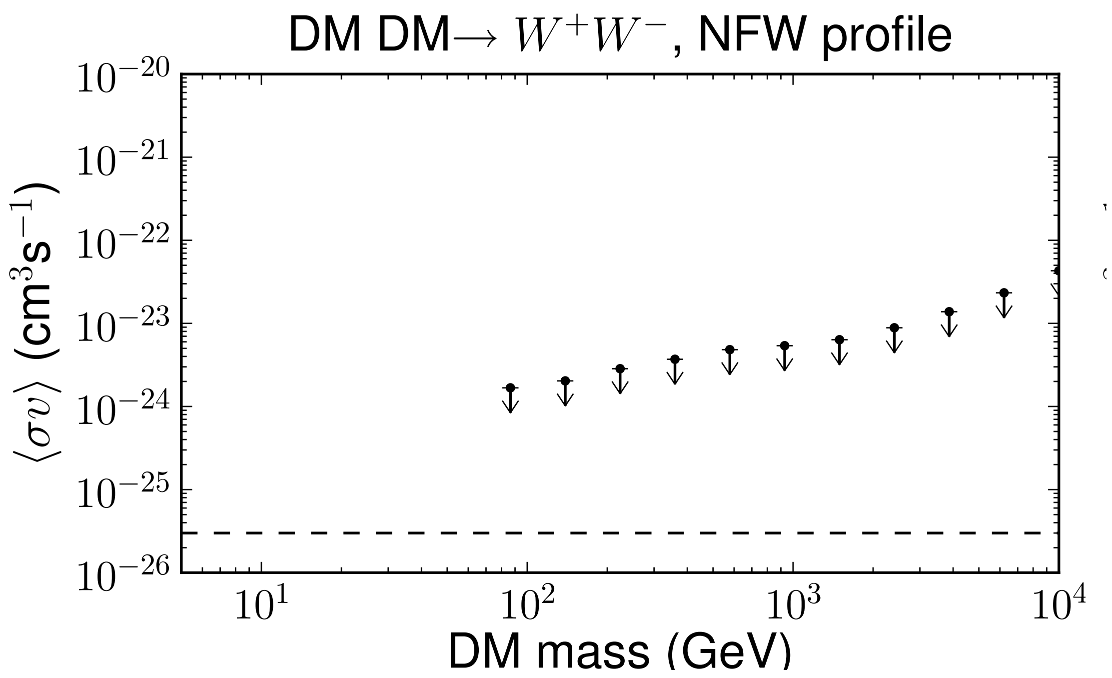
$$\frac{d\Phi_\gamma}{dE_\gamma}(E_\gamma, \Delta\Omega) = \frac{1}{4\pi}\frac{\langle\sigma v\rangle}{2{m_\chi}^2}\frac{dN_\gamma}{dE_\gamma} \times \boxed{\int_{\Delta\Omega}\int_{l.o.s}\rho^2_{\mathrm{DM}}(l,\Omega)dld\Omega}$$

J-factor : DM profile

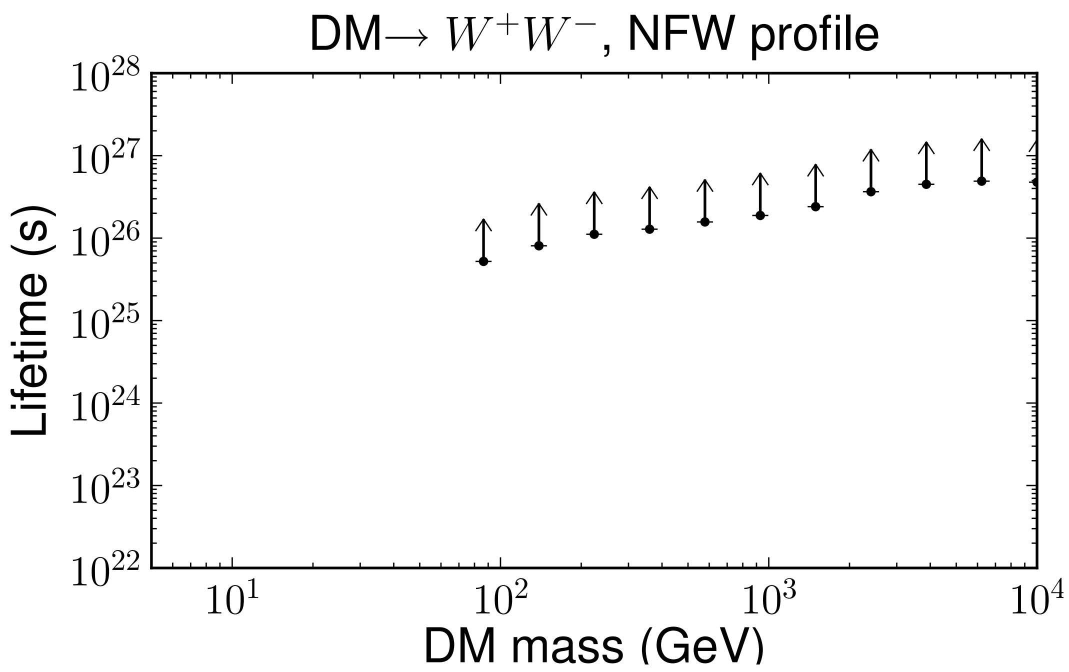
Dark Matter indirect detection

All sky survey of the gamma ray flux also puts constraints on the DM properties.

ex) annihilating DM



decaying DM



[Fermi-Lat 1205.2739, two year data]

Further constraints or hints on the DM properties will be provided with more data taking!

(More precise estimation of the J-factors are also important)

Summary

- ✓ Supersymmetric Standard Model is now more motivated by the discovery of the Higgs boson.

- ✓ In the MSSM, the Higgs boson is an elementary scalar particles whose mass parameters are controlled by a “chiral” symmetry!

- ✓ The MSSM gives us an calculable model all the way up to the GUT scale!

- ✓ So far, no SUSY events were observed at the LHC...

  SUSY particles could be a little heavier than the naive expectations.

  About 126GeV Higgs boson mass also suggests heavy SUSY particles.

- ✓ DM detection experiments may give us hints on SUSY before collider experiments...?