Boltzmann Equation for Non-equilibrium Particles & Its Application to Non-thermal DM Production

Koichi Hamaguchi (Tokyo U. Phys. Dept. & IPMU)

based on,..... **KH, T.Moroi, and K.Mukaida,** **arXiv: 1111.4594** (JHEP1201)

@ Toyama ~~BURI~~ phenomenology and cosmology workshop, February 20, 2012

Since this is the last talk of today's hard program....

SUMMARY

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(and BURI is waiting for us,.....)

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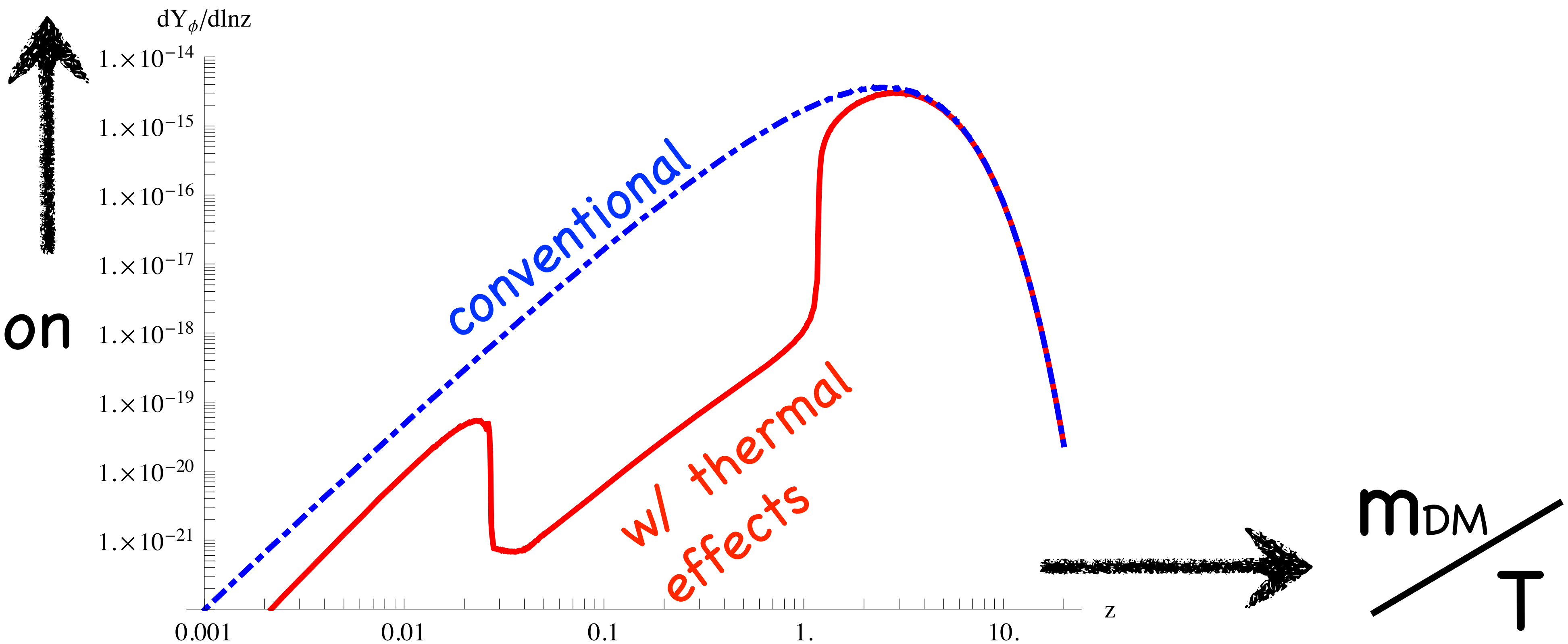
- We have studied the "thermal effects" on a non-thermal DM production scenario ("FIMP" scenario)
  (by using Kadanoff-Baym eqs. in closed-time-path formalism)
  which was not taken into account in previous works.

- We found that the "thermal effects" can significantly change the conventional picture, and the resultant DM abundance.

DM production rate



PLAN

1. motivation

2. setup

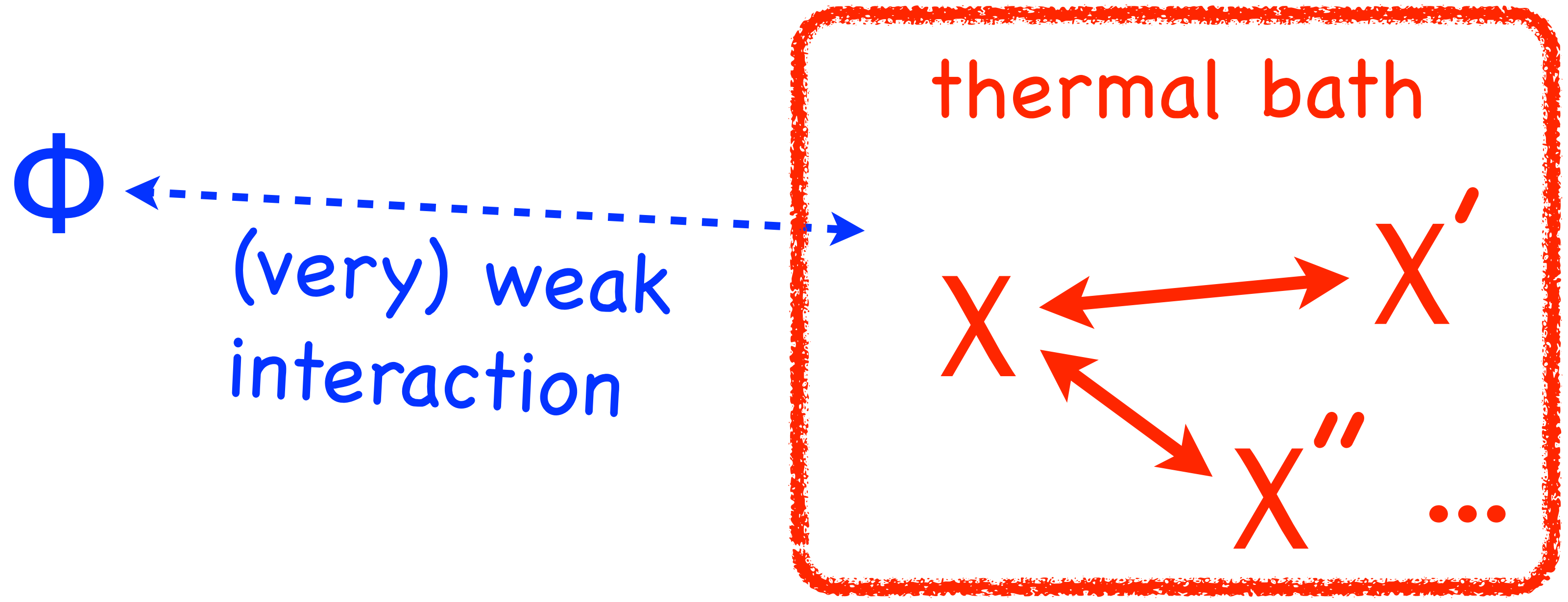
3. thermal effect

4. result

1. Motivation

Motivation (1)

In general, we want to study the following setup:



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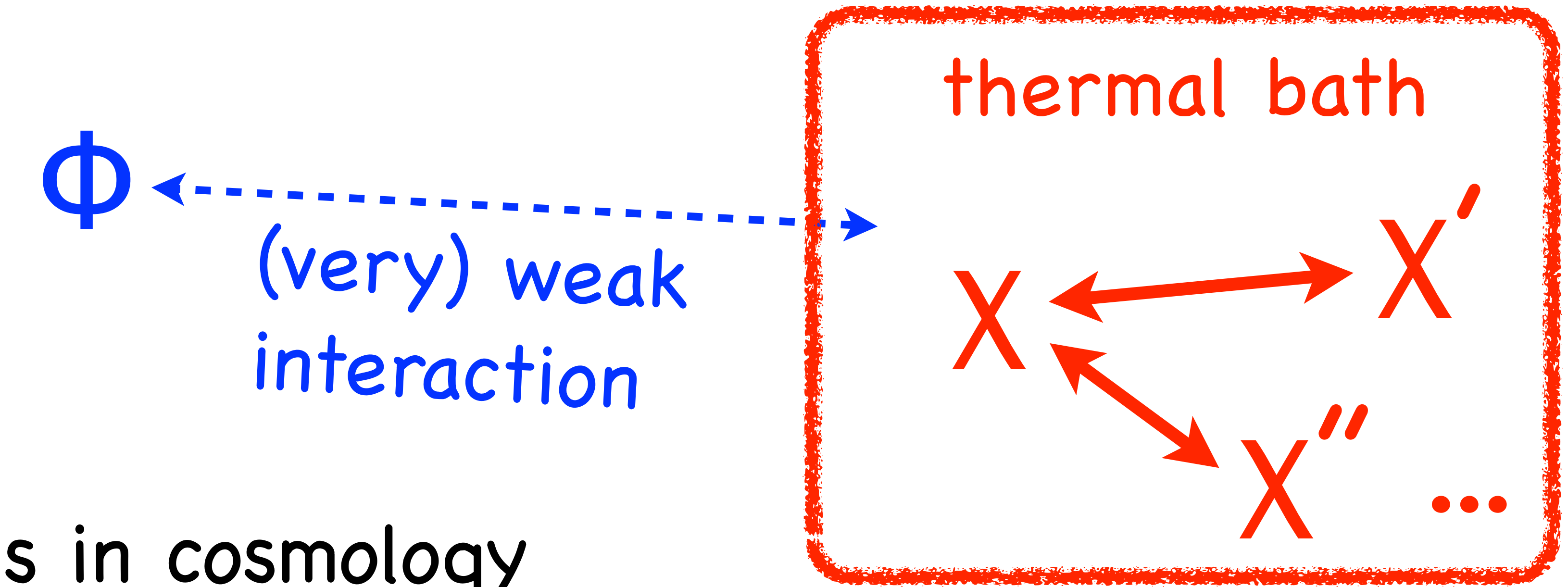

many examples in cosmology

- inflaton/moduli decay
- dynamics of scalar fields (inflaton/moduli/Affleck-Dine...)
- production of gravitino /axino /(Dirac) R.H.sneutrino DM...(not mixed)

[note: many early works... cf. Refs. of our paper.]

As an example, we study a non-thermal DM production.

Asaka, Ishiwata, Moroi, '05,'06

Hall, Jedamzik, March-Russell, West, '09

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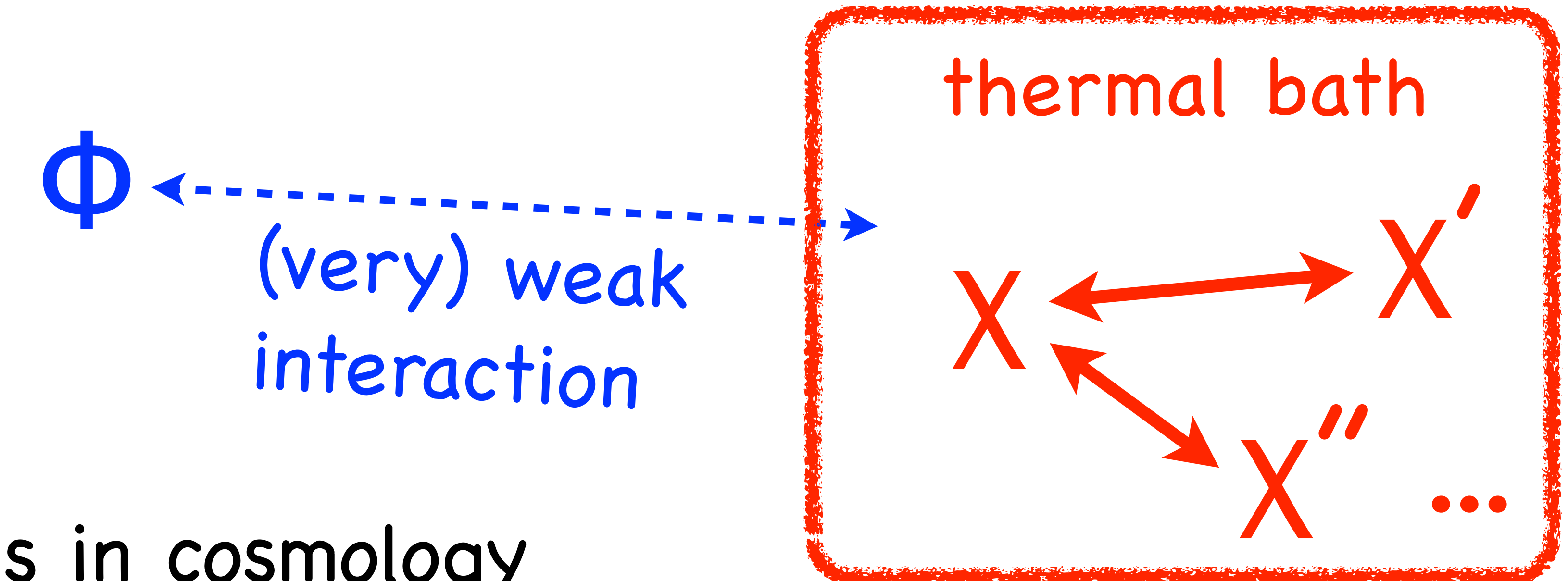

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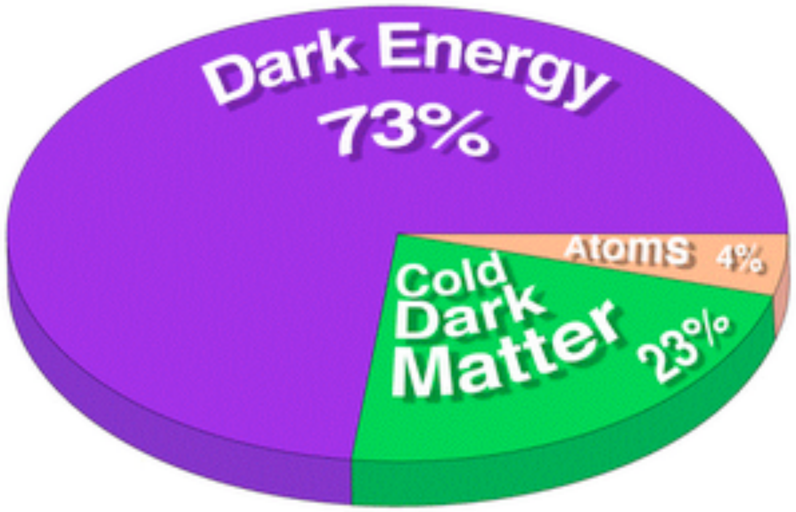
Motivation (2)

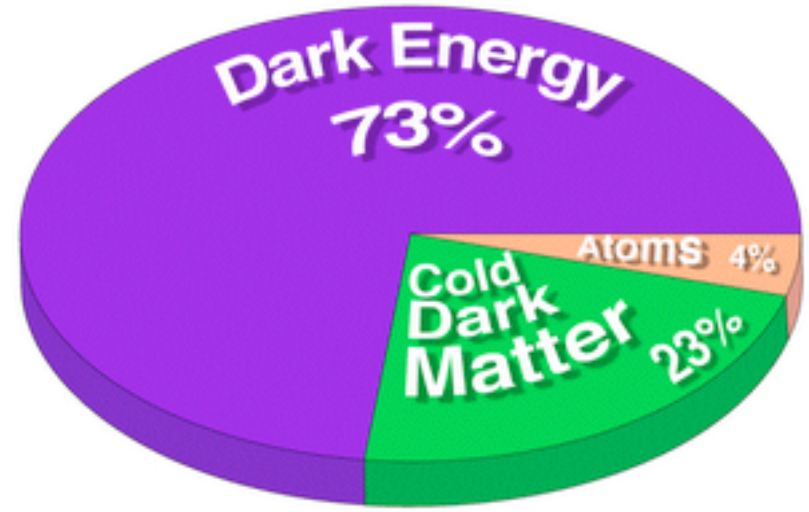


Composition of the Universe Today

Dark Matter density

$$\Omega_{CDM}h^2 = 0.1126 \pm 0.0036$$

Motivation (2)



Composition of the Universe Today

Dark Matter density

$\Omega_{CDM}h^2 = 0.1126 \pm 0.0036$

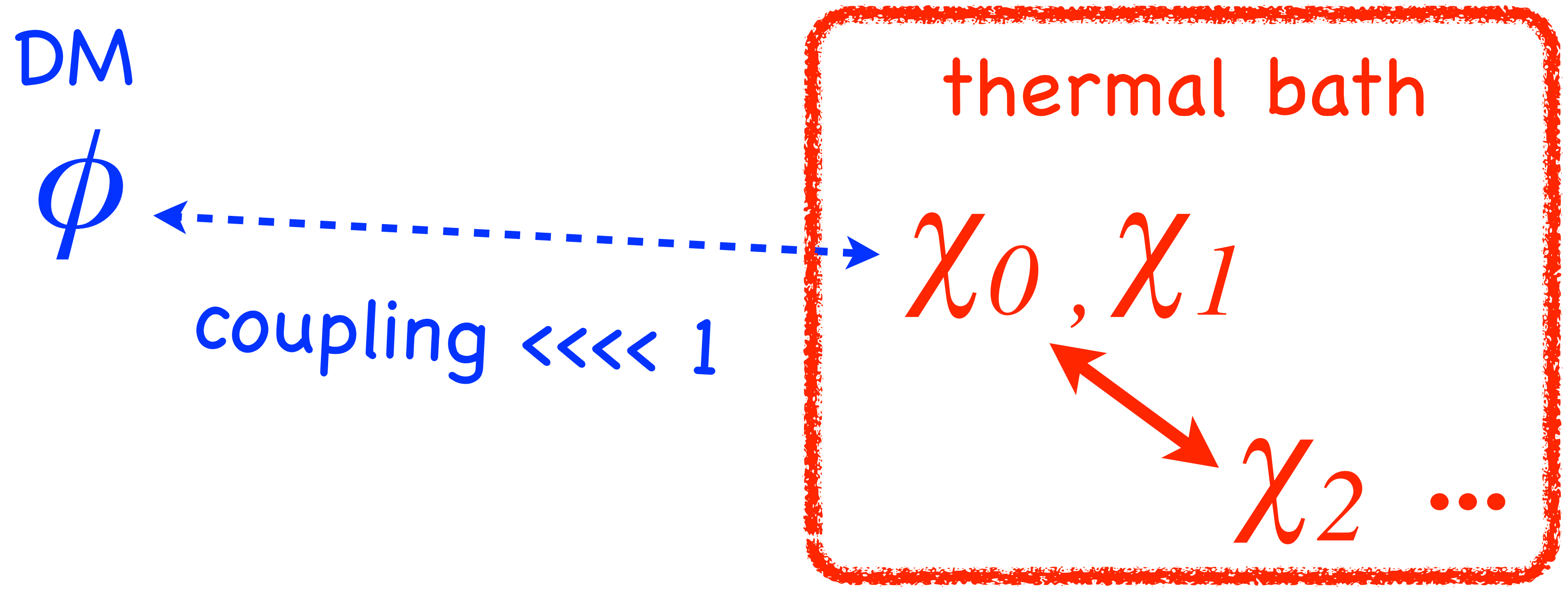
measured with an accuracy of O(1%).

→ requiring comparable precision in theoretical calculation (depending on the DM scenario).

→ important to include the "thermal effects", which was not taken into account in the previous studies.

2. Setup

Setup

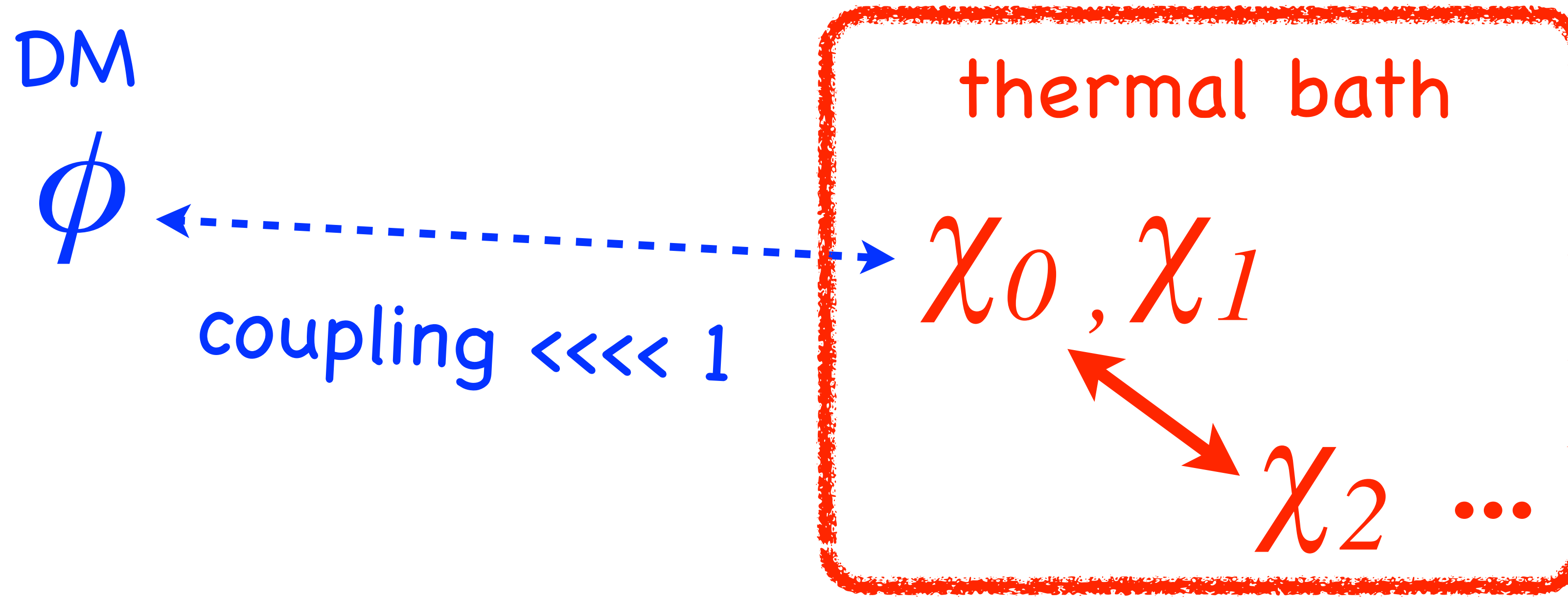


$$L_{int} = \varepsilon \, \phi \, \chi_0 \chi_1$$

DM
very weakly interacting:
(out of equilibrium)

within thermal bath

Setup



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DM
very weakly interacting:
(out of equilibrium)

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- For simplicity, assume these particles are **scalars.**
- Dimensionless **coupling**: $\mathcal{E}/m_\phi$ **<<<<< 1. (typically ~ 10^{-13})**

→ **DM ϕ is never in thermal bath.**

Asaka, Ishiwata, Moroi, '05,'06
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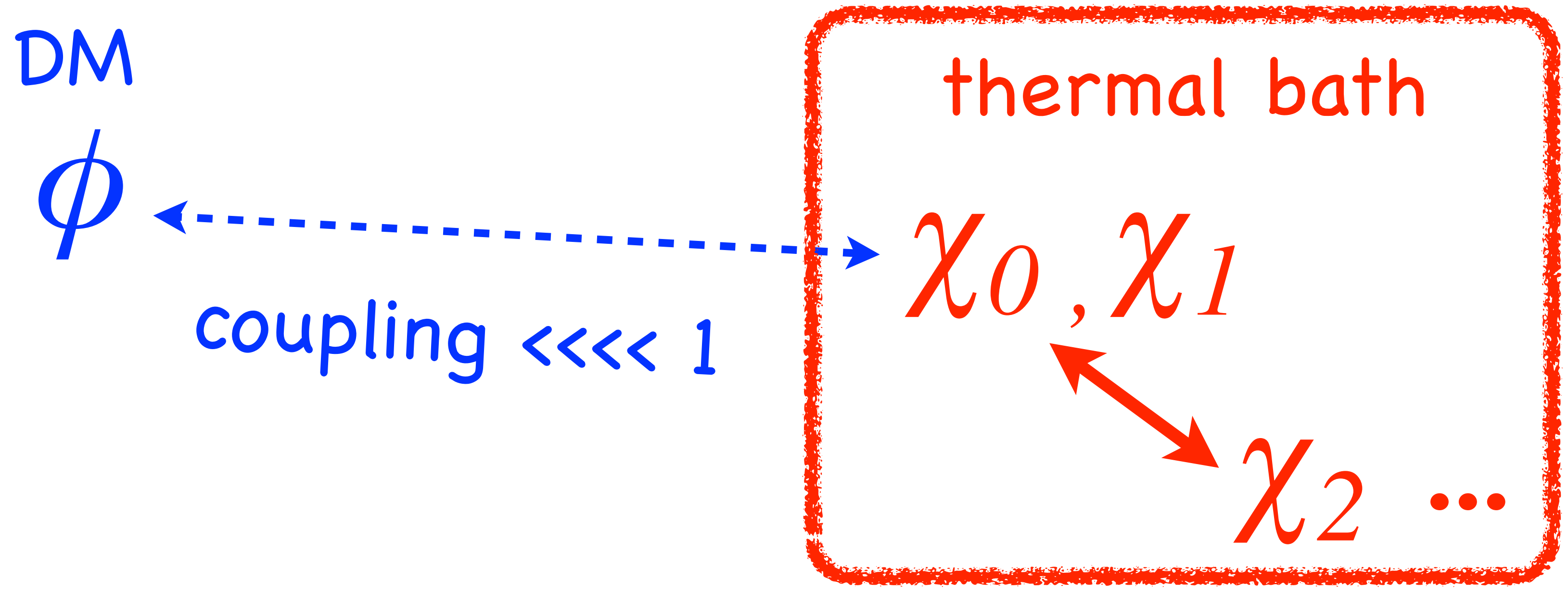
Setup



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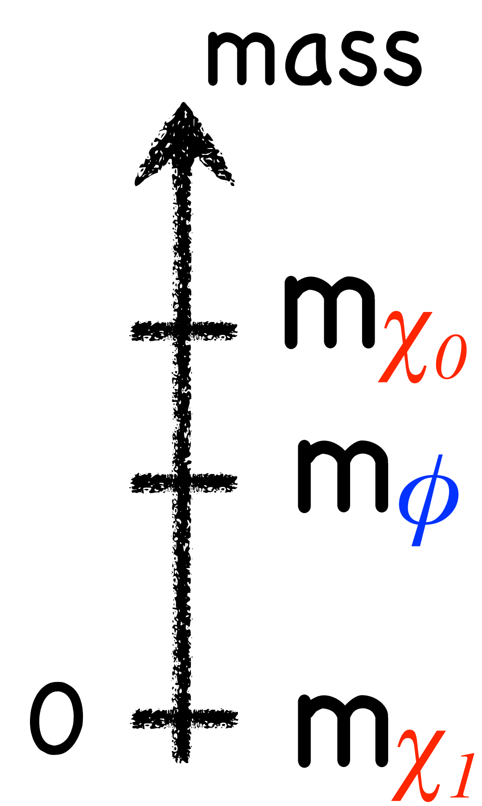
- Assume a mass hierarchy like this: →



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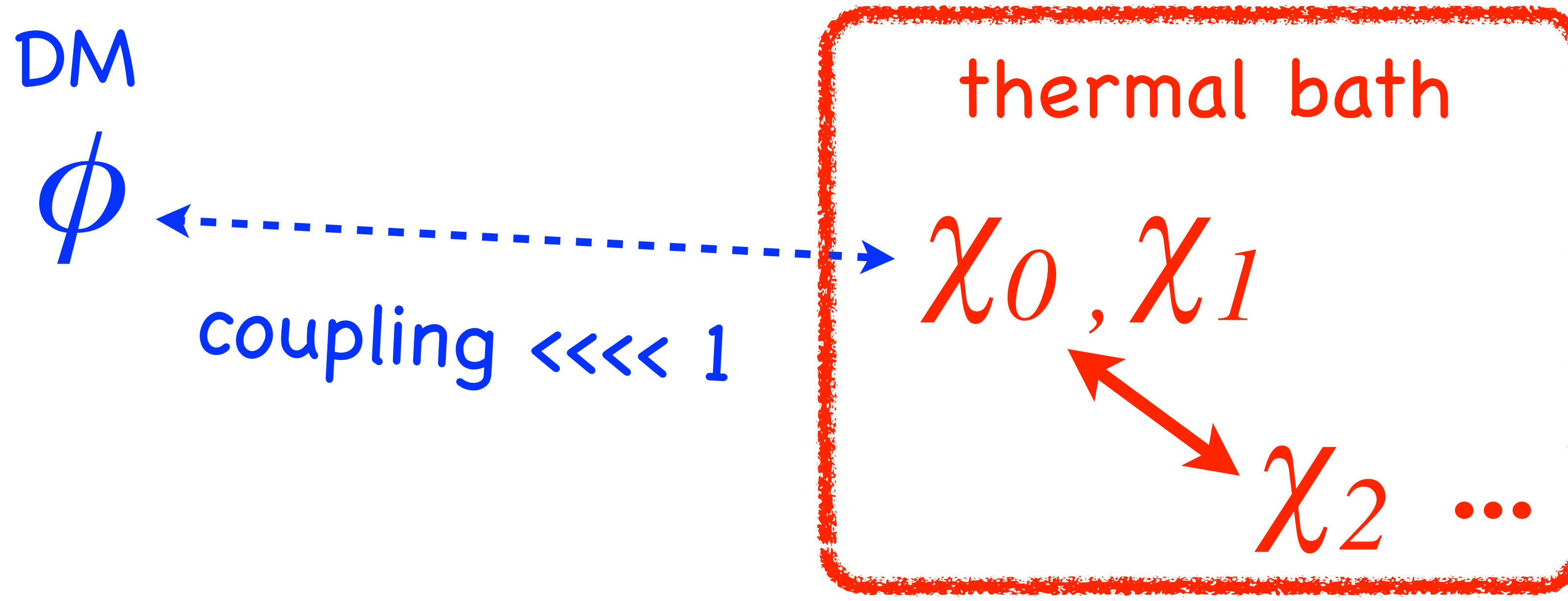

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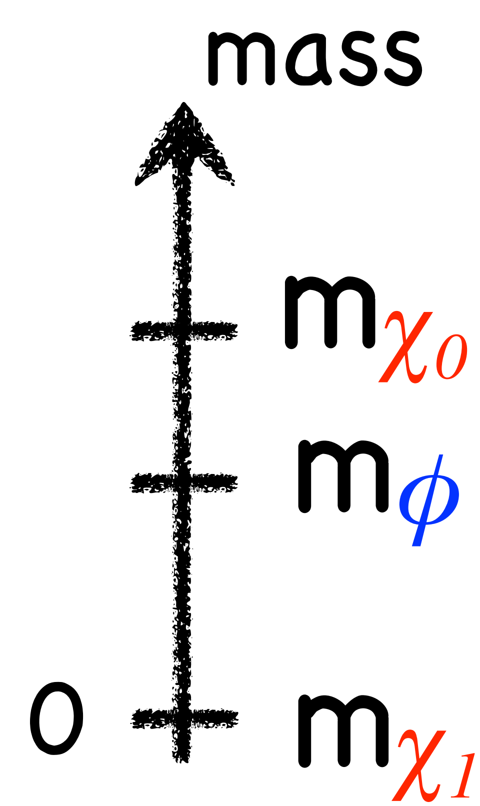
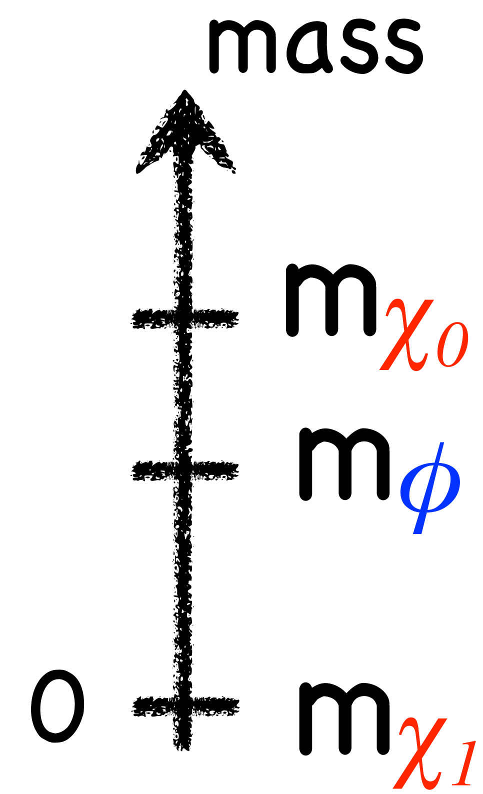


Then, DM ϕ is produced via χ_0 decay

$$\chi_0 \rightarrow \phi + \chi_1$$

Setup

DM production: $\chi_0 \rightarrow \phi + \chi_1$



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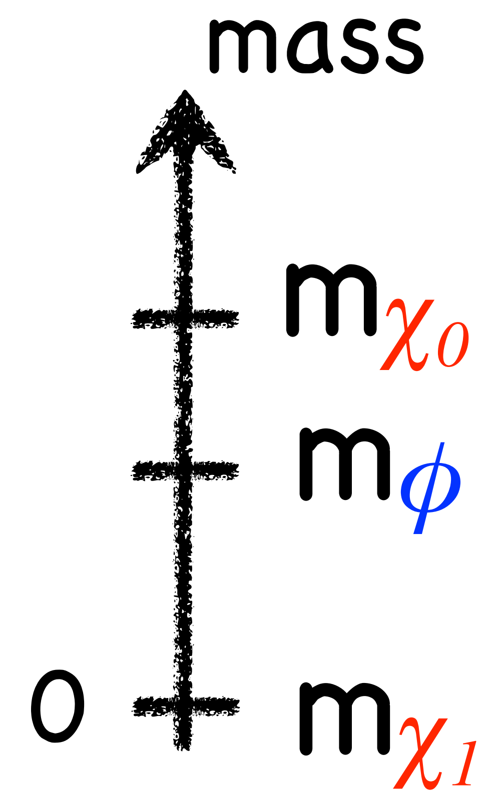

Conventional calculation (cf. Kolb-Turner textbook)

$$\dot{n}_\varphi + 3Hn_\varphi \simeq \int d\Pi\, (2\pi)^4 \delta(p_{\rm tot}) \left|M(\chi_0 \to \varphi\chi_1 \quad)\right|^2 f_{\chi_0}(1 + f_{\chi_1}) \cdots$$

$$\text{where } d\Pi = \prod_{i=\chi_0,\varphi,\chi_1} \deg(i) \frac{d^3p}{(2\pi)^3 2\omega_{i,\mathbf{p}}}; \quad f_{\chi_i} = \frac{1}{e^{\omega_{i,\mathbf{p}}/T} - 1} \qquad \omega_{i,\mathbf{p}} = \sqrt{m_i^2 + \mathbf{p}^2}$$

equilibrium distribution

temperature

Setup

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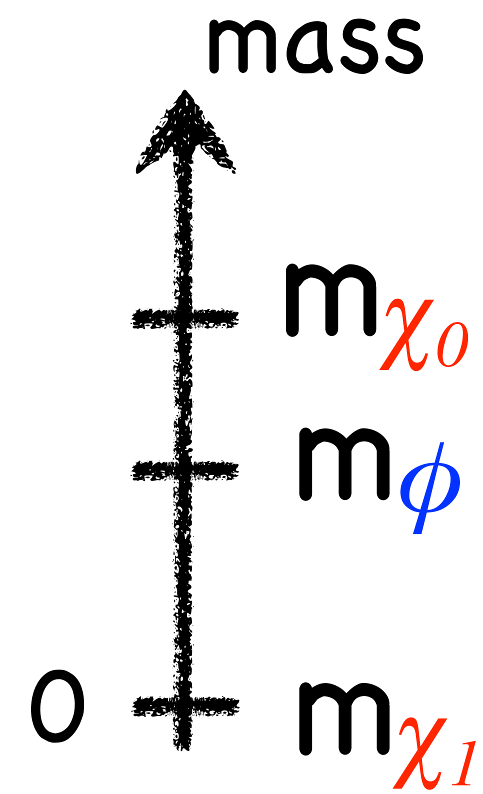

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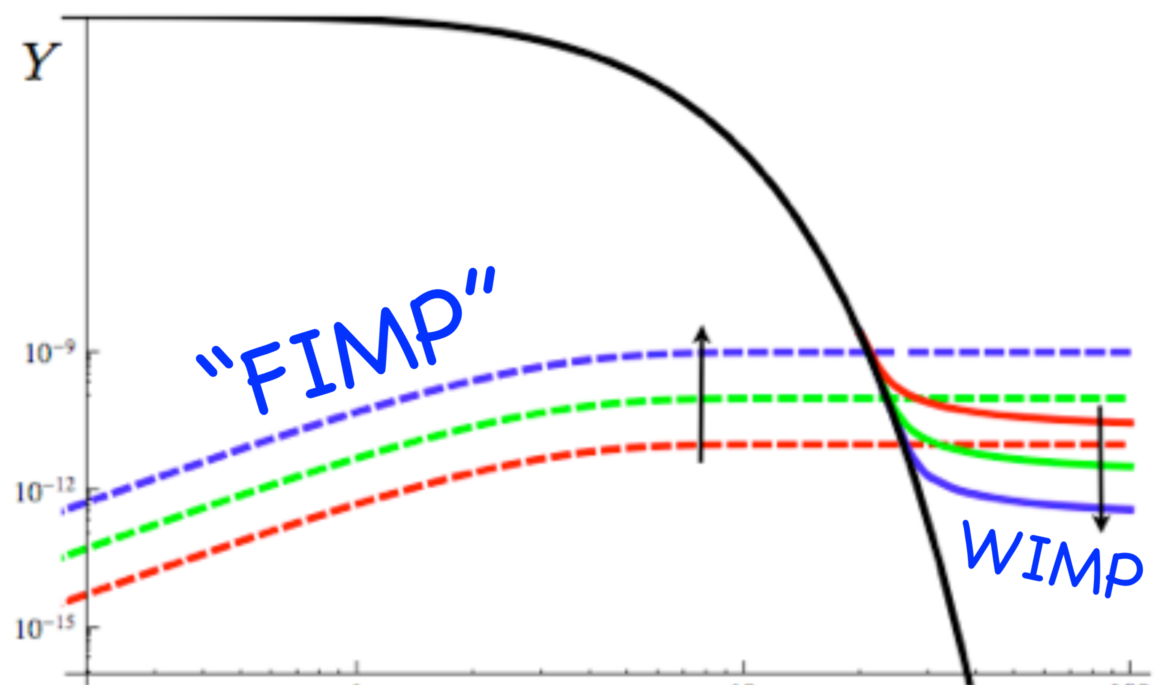

Fig. from Hall et.al.

In this non-thermal DM scenario ("FIMP" scenario), the DM density is insensitive to high T history, as in the case of WIMP.

Asaka, Ishiwata, Moroi, '05,'06
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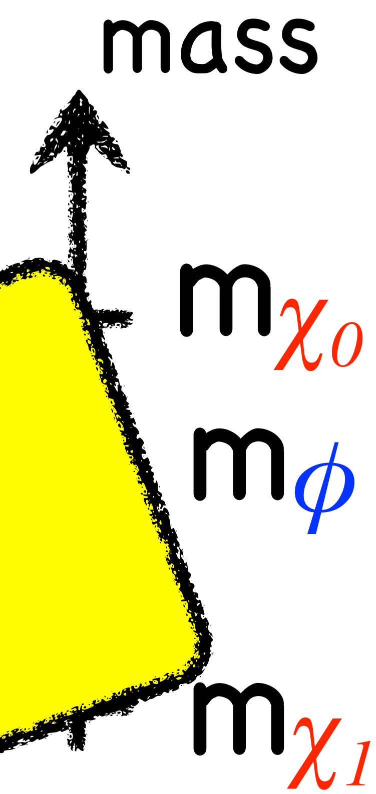

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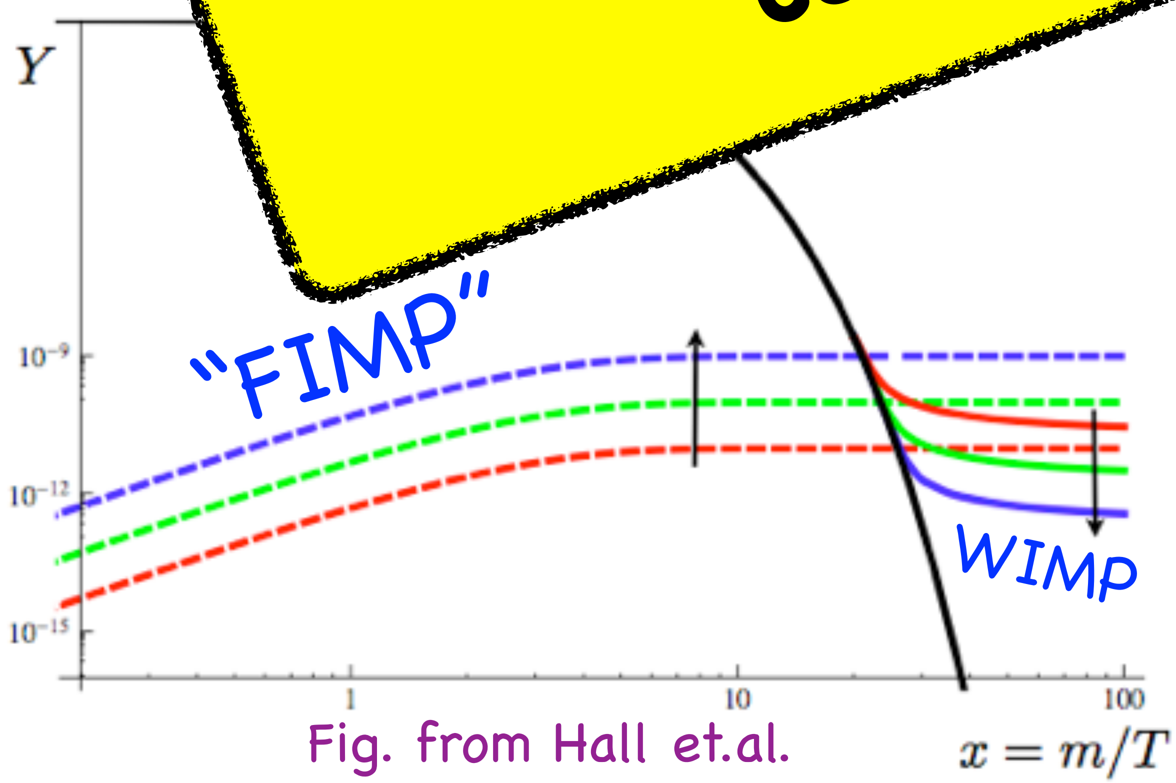
$$|M(\chi ...$$

$$... \frac{...}{(2\pi)^3 2\omega_{i,\mathbf{p}}}; \quad f_{\chi_i} = \frac{1}{e^{\omega_{i,\mathbf{p}}/T} - 1} \qquad \omega_{i,\mathbf{p}} = \sqrt{m_i^2 + \mathbf{p}^2}$$

Is this naive Boltzmann-equation calculation correct ??



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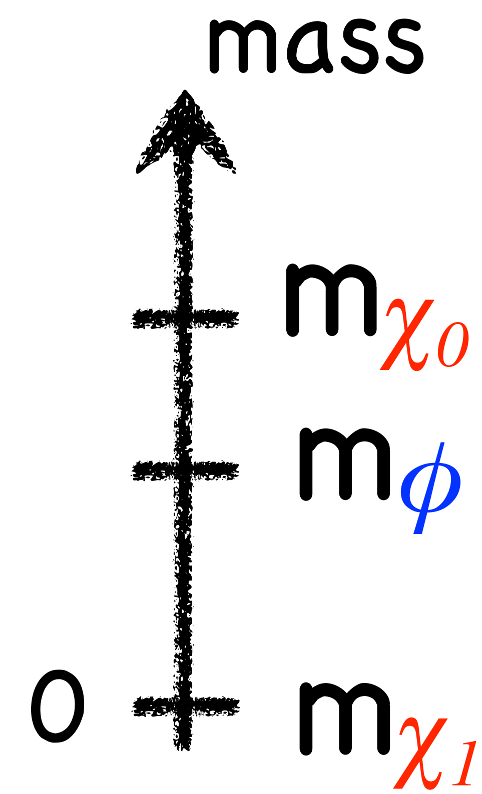
3. thermal effects

Thermal Effects

DM production: $\chi_0 \to \phi + \chi_1$



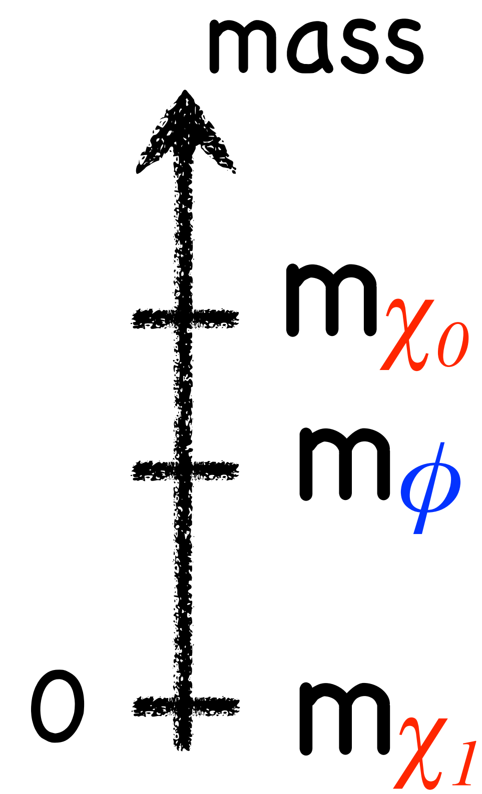
At T>0, the masses of χ_0 and χ_1 are modified ("thermal masses").

$$m_{\chi_i}^{\text{thermal}} = m_{\chi_i} + \delta m_i(T)$$

determined by the interactions of thermal bath

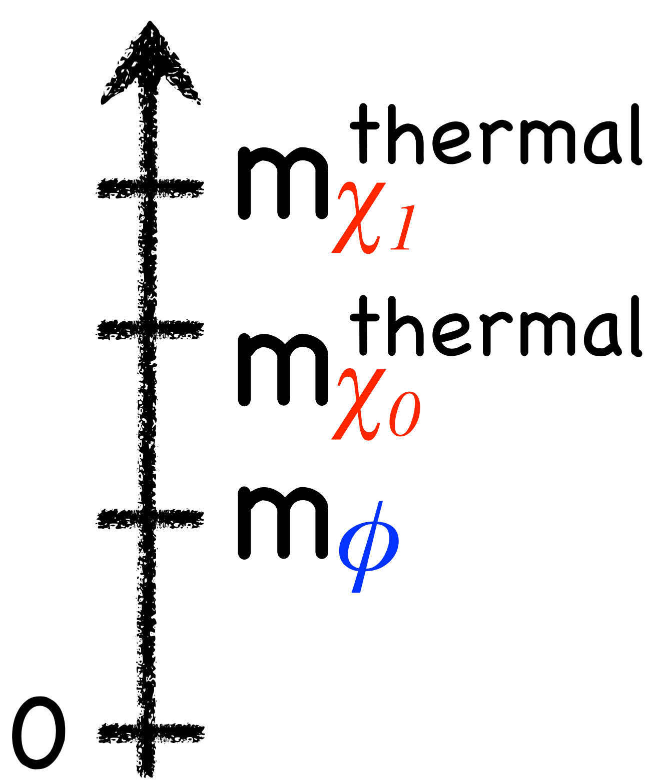
Thermal Effects

DM production: $\chi_0 \rightarrow \phi + \chi_1$



$m_{\chi_i}^{thermal} = m_{\chi_i} + \delta m_i(T)$

thermal mass



The mass hierarchy is changed:

- $\chi_0 \rightarrow \phi + \chi_1$ may be **blocked** at high T.
- even $\chi_1 \rightarrow \phi + \chi_0$ may occur ?!

Wait a second.....

Conventional calculation

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$$\left|M(\chi_0 \to \varphi\chi_1 \quad)\right|^2 f_{\chi_0}(1 + f_{\chi_1})\cdots$$

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mass is here

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here

here

mass is here

2012年2月21日火曜日

Conventional calculation

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here

here

here

here

here

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mass is here

Which "mass" should be replaced with "thermal mass" ?

Conventional calculation

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..... After all, what is the appropriate formalism?

What is the 1st principle?

formalism

- density matrix: $\hat{\rho}$
- expectation value: $\langle \hat{A} \rangle \equiv \mathrm{tr}[\hat{\rho}\hat{A}]$

- We assume $\hat{\rho} = \hat{\rho}_{\phi,i} \otimes \hat{\rho}_{\chi}$

initial distribution of Φ
(spatially homogeneous)

$$\hat{\rho}_{\chi} = e^{-\hat{H}_{\chi}/T}$$

thermal bath with a temperature T

- What we want to know is: **the time evolution of the expectation value of Φ's number density operator.**

$$n_\phi(t) = \int \frac{d^3k}{(2\pi)^3} \langle \hat{N}^\phi_{\mathbf{k}}(t) \rangle \quad \text{where} \quad \hat{N}^\phi_{\mathbf{k}}(t) = \frac{1}{V}\frac{1}{2\omega_{\phi,\mathbf{k}}} : \left[\dot{\hat{\phi}}(t,\mathbf{k})\dot{\hat{\phi}}(t,-\mathbf{k}) + \omega^2_{\phi,\mathbf{k}}\hat{\phi}(t,\mathbf{k})\hat{\phi}(t,-\mathbf{k}) \right] :$$

formalism ..... Let's skip all the details.....

(See our paper and refs. therein.)

Strategy:

We want to know

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→ want to know 2-point function $\langle \hat{\phi}(x)\hat{\phi}(y) \rangle \; = \; \mathrm{tr}[\,\hat{\rho}\;\hat{\phi}(x)\hat{\phi}(y)\,]$

→ Diff. eqs. of $\langle \hat{\phi}(x)\hat{\phi}(y) \rangle$ can be obtained from Kadanoff-Baym eqs.

→ After all, one obtains, at leading order in Φ's coupling,...

$$\dot{n}_\phi + 3Hn_\phi = \int \frac{d^3k}{(2\pi)^3 2\omega_{\phi,\mathbf{k}}} \int d^4x \; e^{i(\omega_{\phi,\mathbf{k}}t - \mathbf{k}\cdot\mathbf{x})} \frac{\mathrm{tr}\left[e^{-H_\chi/T} \cdot \epsilon\hat{\chi}_0\hat{\chi}_1(0) \cdot \epsilon\hat{\chi}_0\hat{\chi}_1(x) \right]}{\mathrm{tr}\left[e^{-H_\chi/T} \right]}$$

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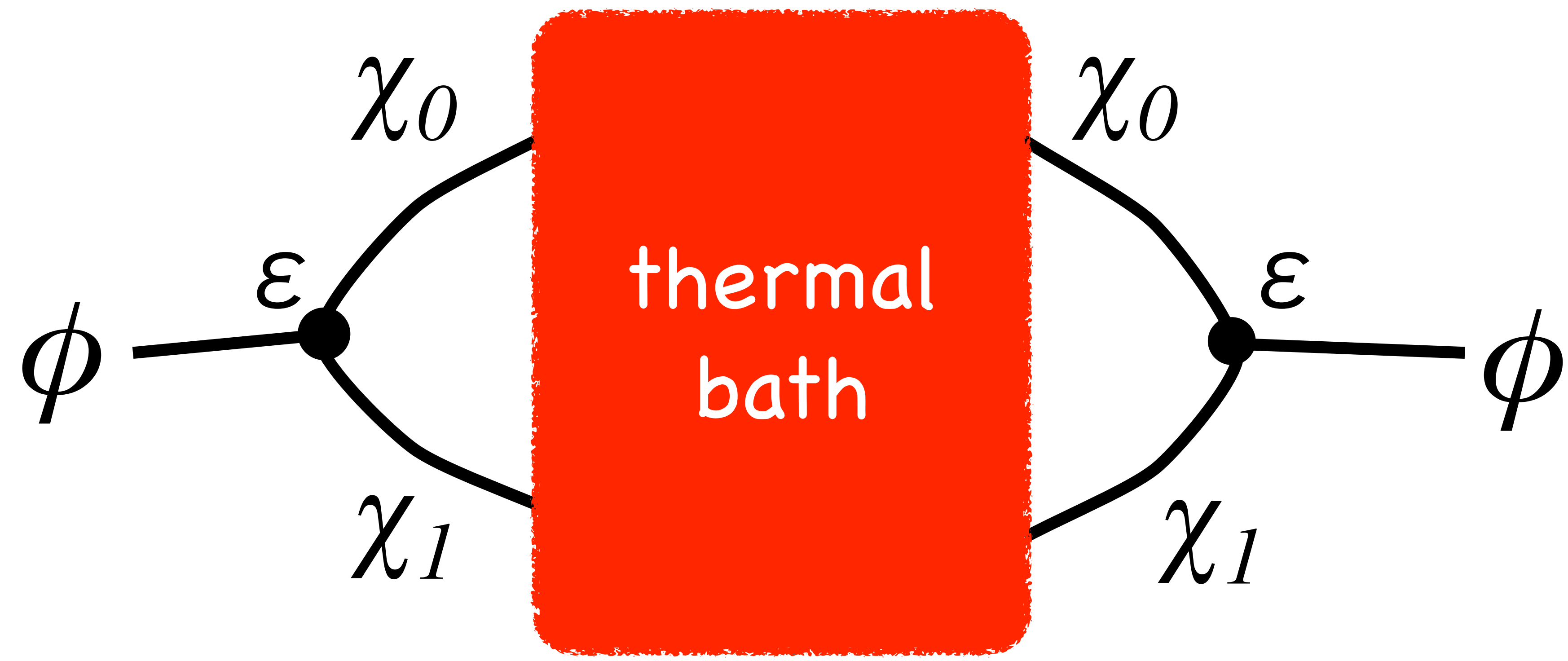
New Boltzmann equation !!

formalism ..... Let's skip all the details.....

Str

We v

n_ϕ



→ W

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Str

We w

n_ϕ

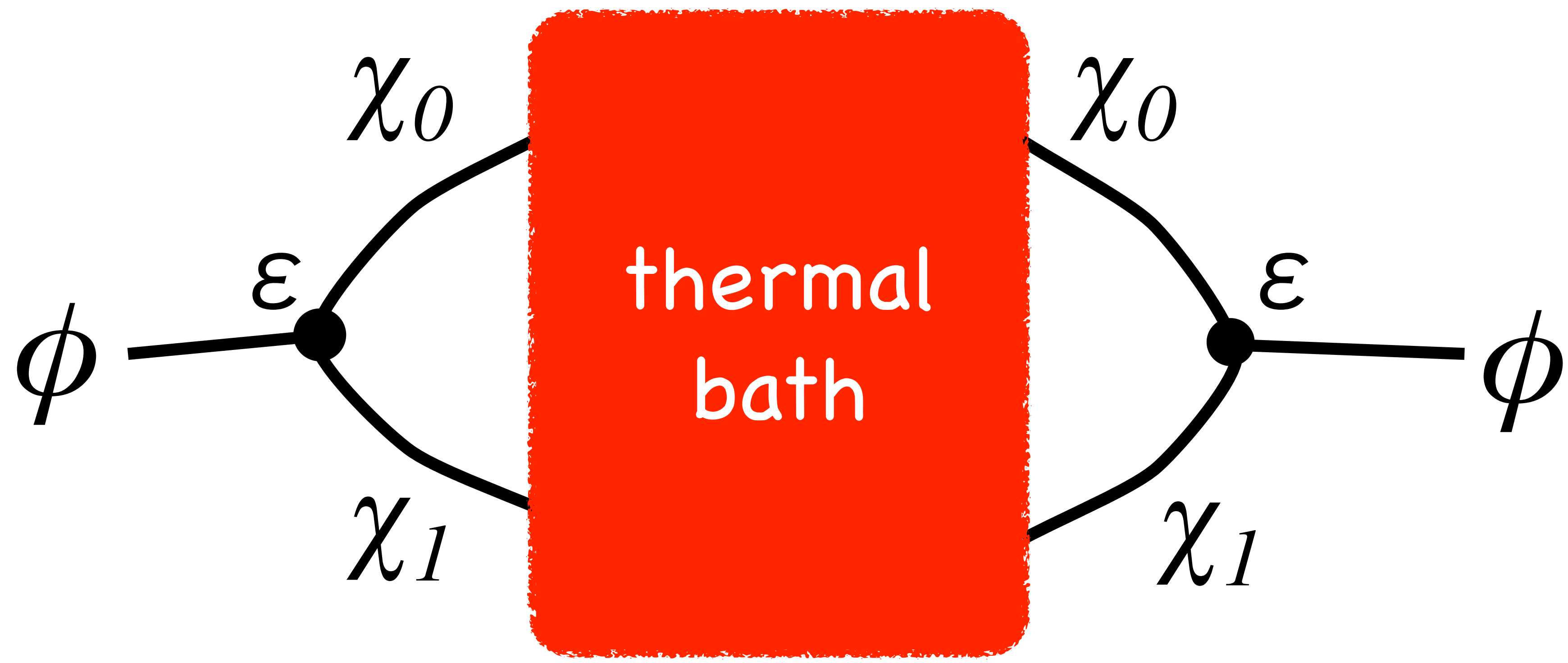
→ w

→ Di

Assume that the dominant interaction is

$$\mathcal{L}_{\rm int}(\chi) = -\frac{g_{\chi_0}^2}{4!}\chi_0^4 - \frac{g_{\chi_1}^2}{4!}\chi_1^4$$

Then,....



→ After all, one obtains, at leading order in Φ's coupling....

$$\dot{n}_\phi + 3Hn_\phi = \int \frac{d^3k}{(2\pi)^3 2\omega_{\phi,\mathbf{k}}} \int d^4x\, e^{i(\omega_{\phi,\mathbf{k}}t - \mathbf{k}\cdot\mathbf{x})} \frac{\mathrm{tr}\left[e^{-H_\chi/T}\cdot \epsilon\hat{\chi}_0\hat{\chi}_1(0)\cdot\epsilon\hat{\chi}_0\hat{\chi}_1(x)\right]}{\mathrm{tr}\left[e^{-H_\chi/T}\right]}$$

New Boltzmann equation !!

formalism ..... Let's skip all the details.....

Str

We w

n_ϕ

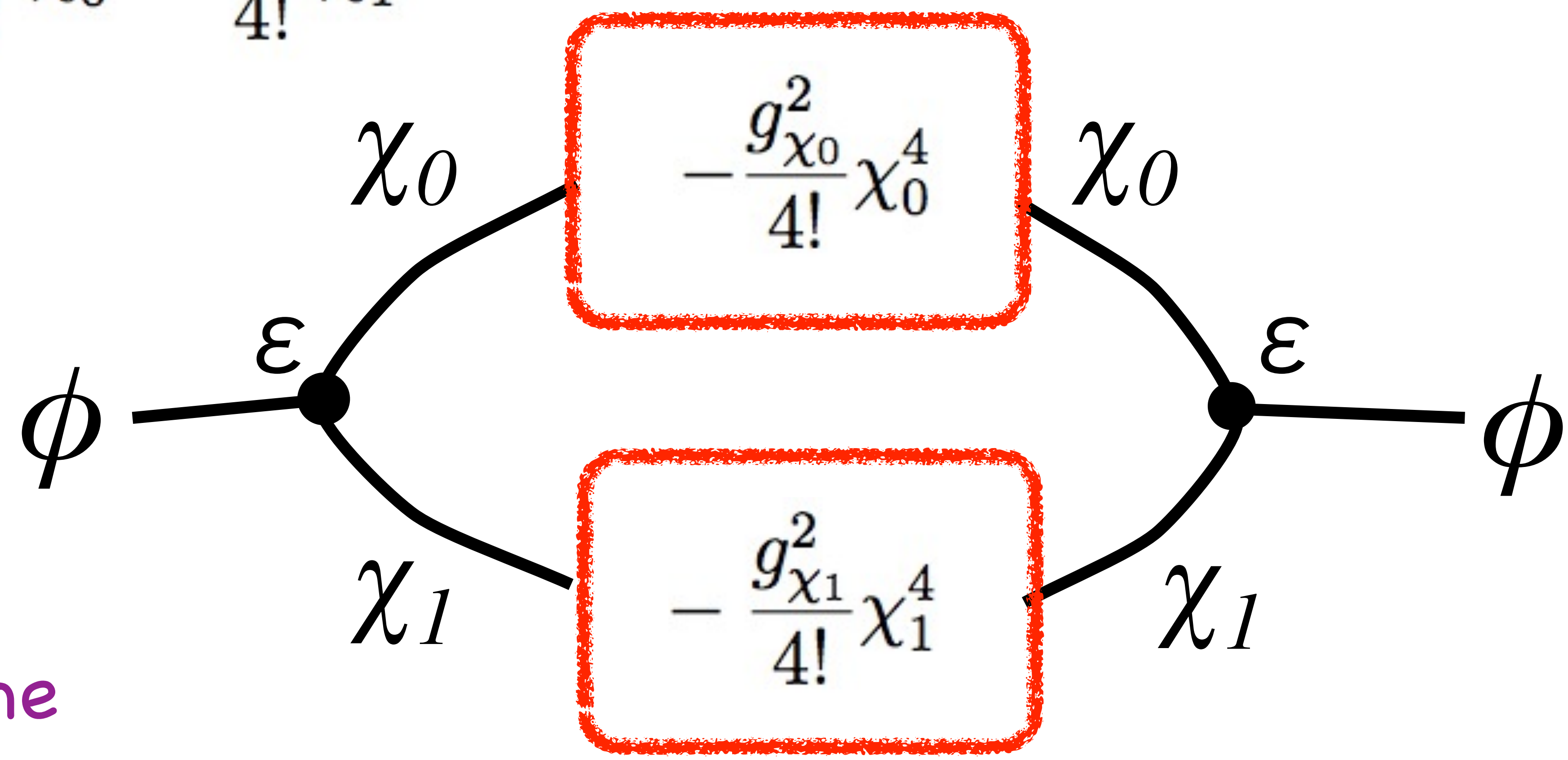
→ w

→ Di

Assume that the dominant interaction is

$$\mathcal{L}_{\rm int}(\chi) = -\frac{g_{\chi_0}^2}{4!}\chi_0^4 - \frac{g_{\chi_1}^2}{4!}\chi_1^4$$

Then,....



We can use the

results of Φ^4 thermal field theory.

→ After all, one obtains, at leading order in Φ's coupling....

$$\dot{n}_\phi + 3Hn_\phi = \int \frac{d^3k}{(2\pi)^3 2\omega_{\phi,\mathbf{k}}} \int d^4x\, e^{i(\omega_{\phi,\mathbf{k}}t - \mathbf{k}\cdot\mathbf{x})} \frac{\mathrm{tr}\left[e^{-H_\chi/T}\cdot \epsilon\hat{\chi}_0\hat{\chi}_1(0)\cdot\epsilon\hat{\chi}_0\hat{\chi}_1(x)\right]}{\mathrm{tr}\left[e^{-H_\chi/T}\right]}$$

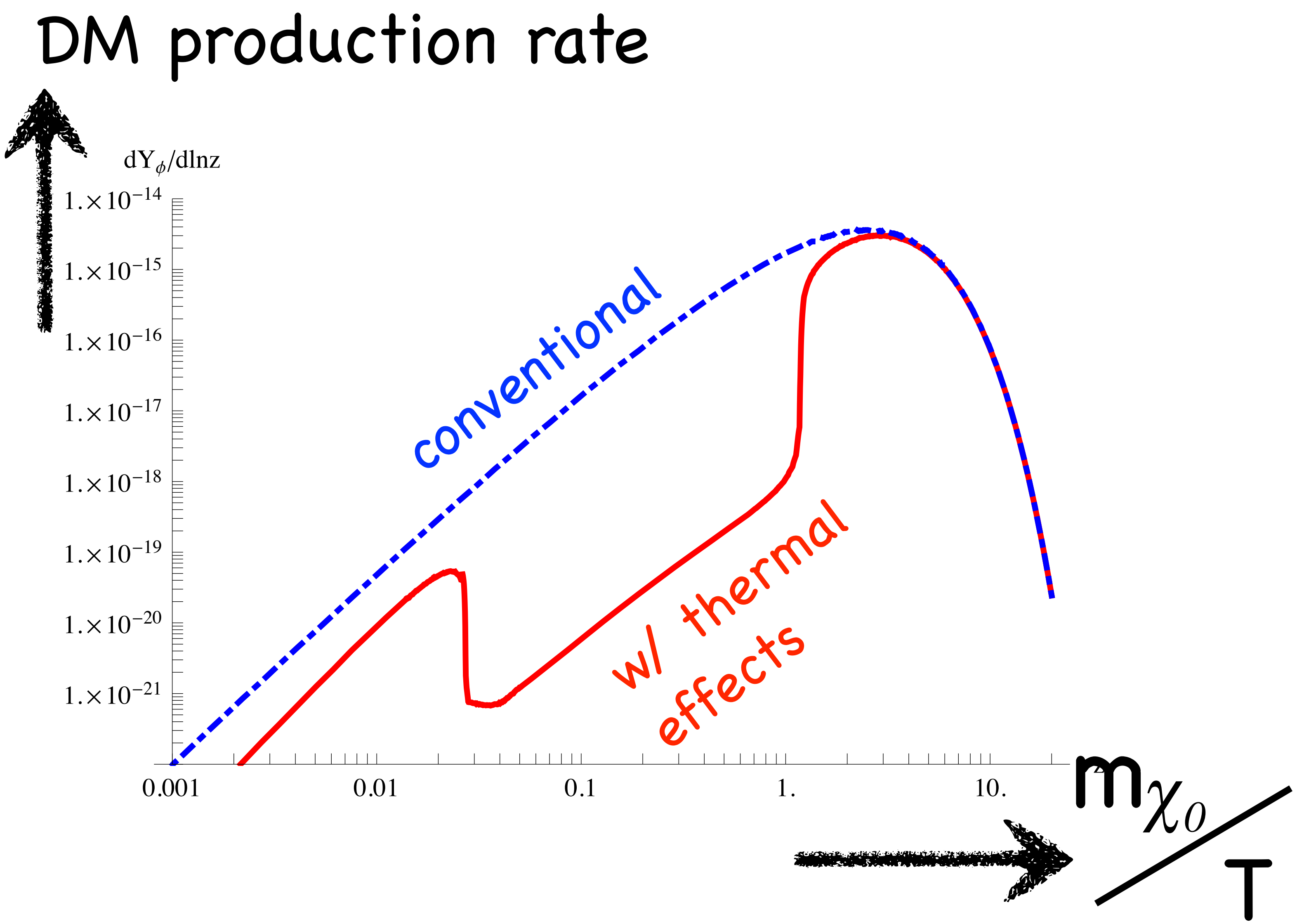
New Boltzmann equation !!

3. Result

$$\varepsilon/m_\phi = 10^{-13},\ g_{\chi_0} = 0.1,\ g_{\chi_1} = 0.3$$

$$m_{\chi_0} : m_\phi : m_{\chi_1} = 1 : 0.95 : 0$$

$$\left(\mathcal{L}_{\rm int} = \varepsilon\ \phi \chi_0 \chi_1 - \frac{g_{\chi_0}^2}{4!}\chi_0^4 - \frac{g_{\chi_1}^2}{4!}\chi_1^4 \right)$$



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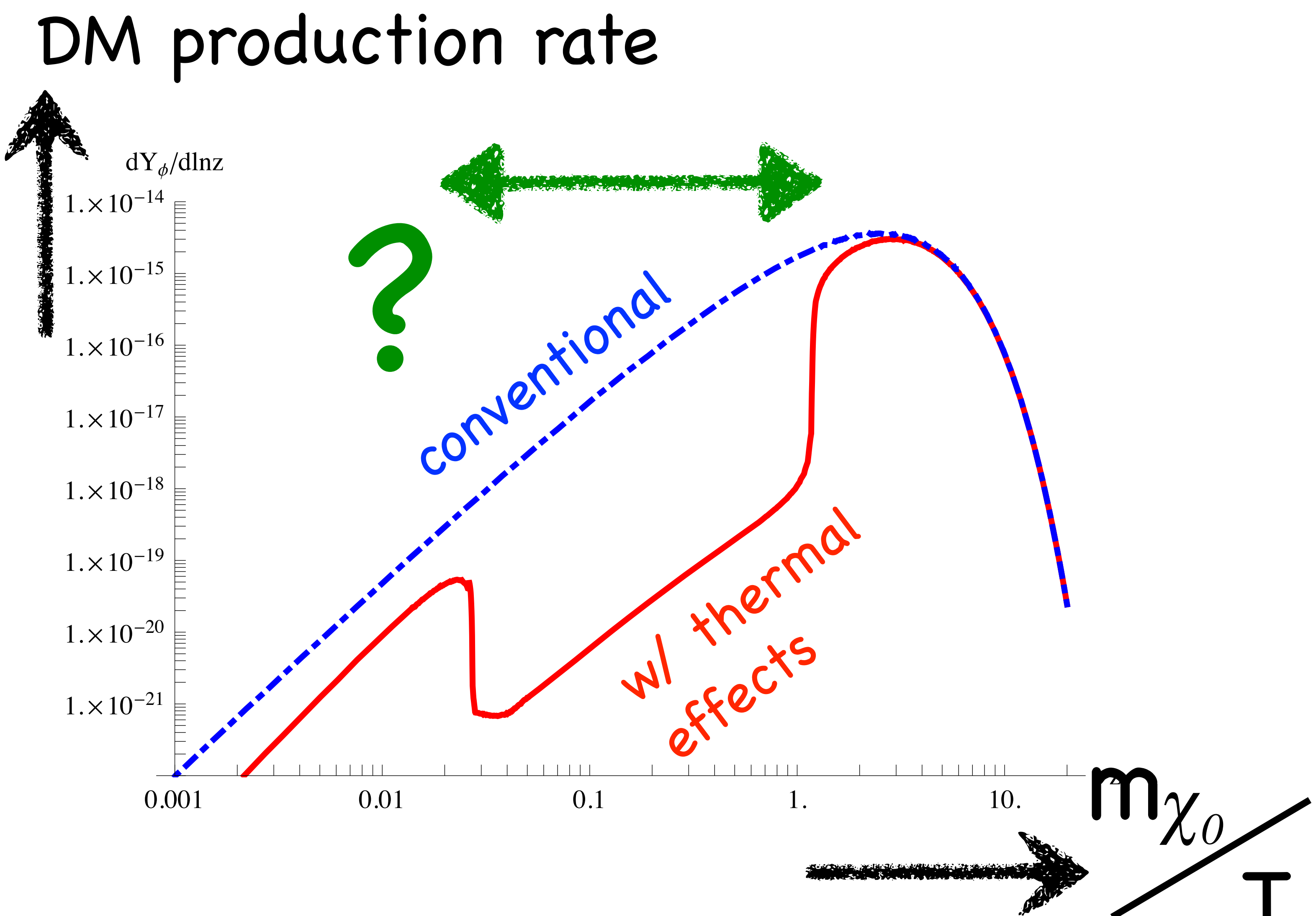

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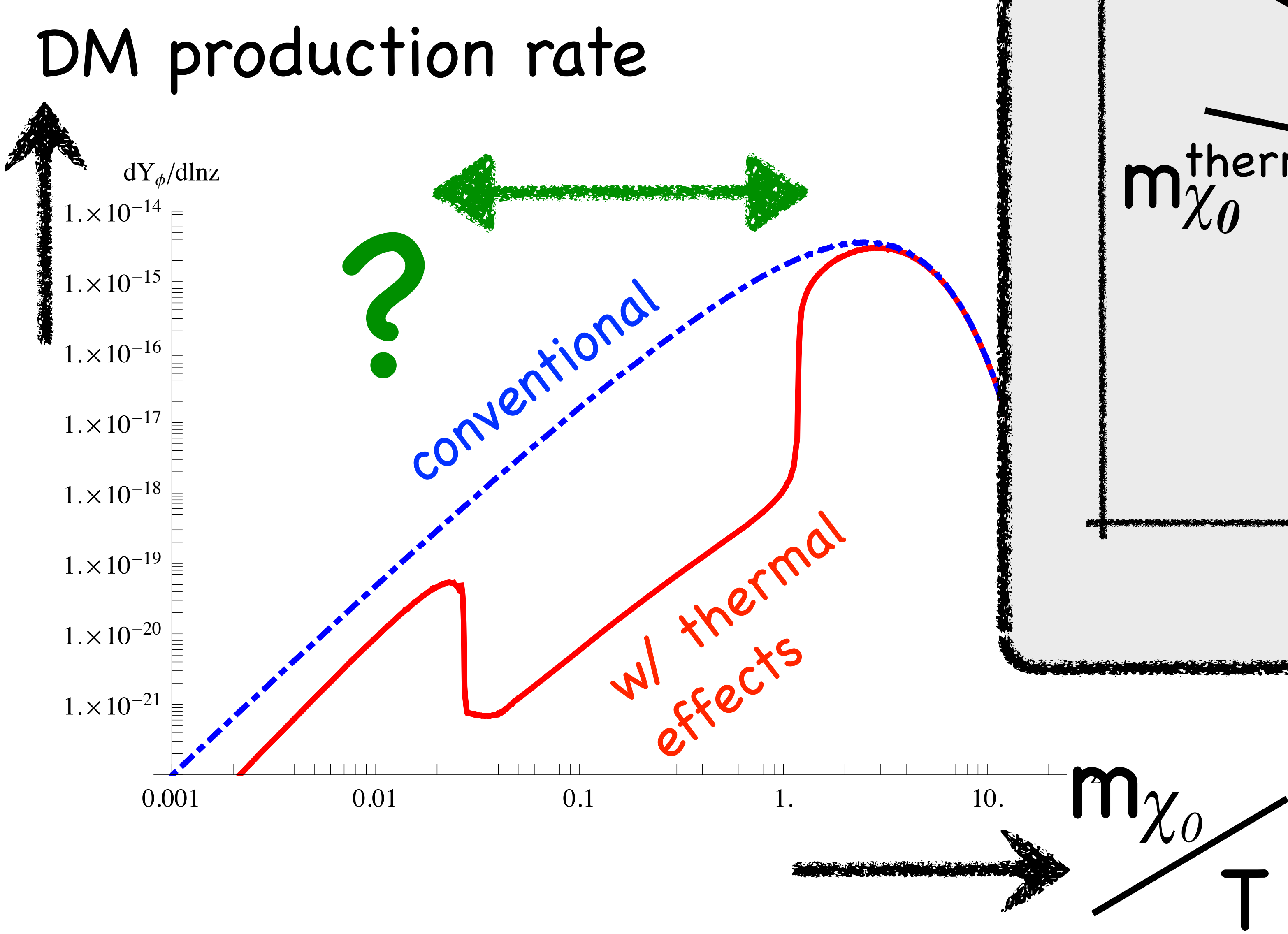

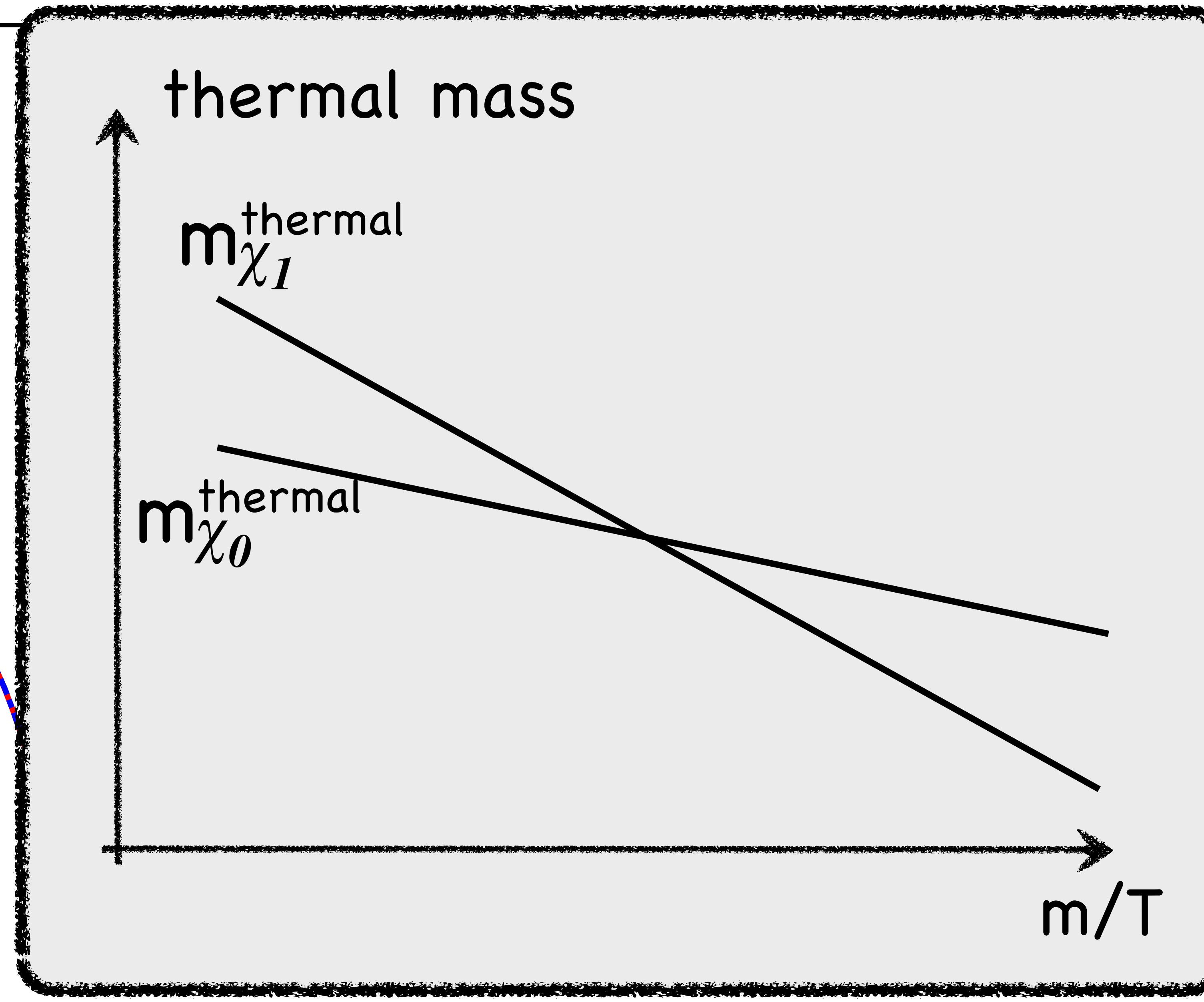

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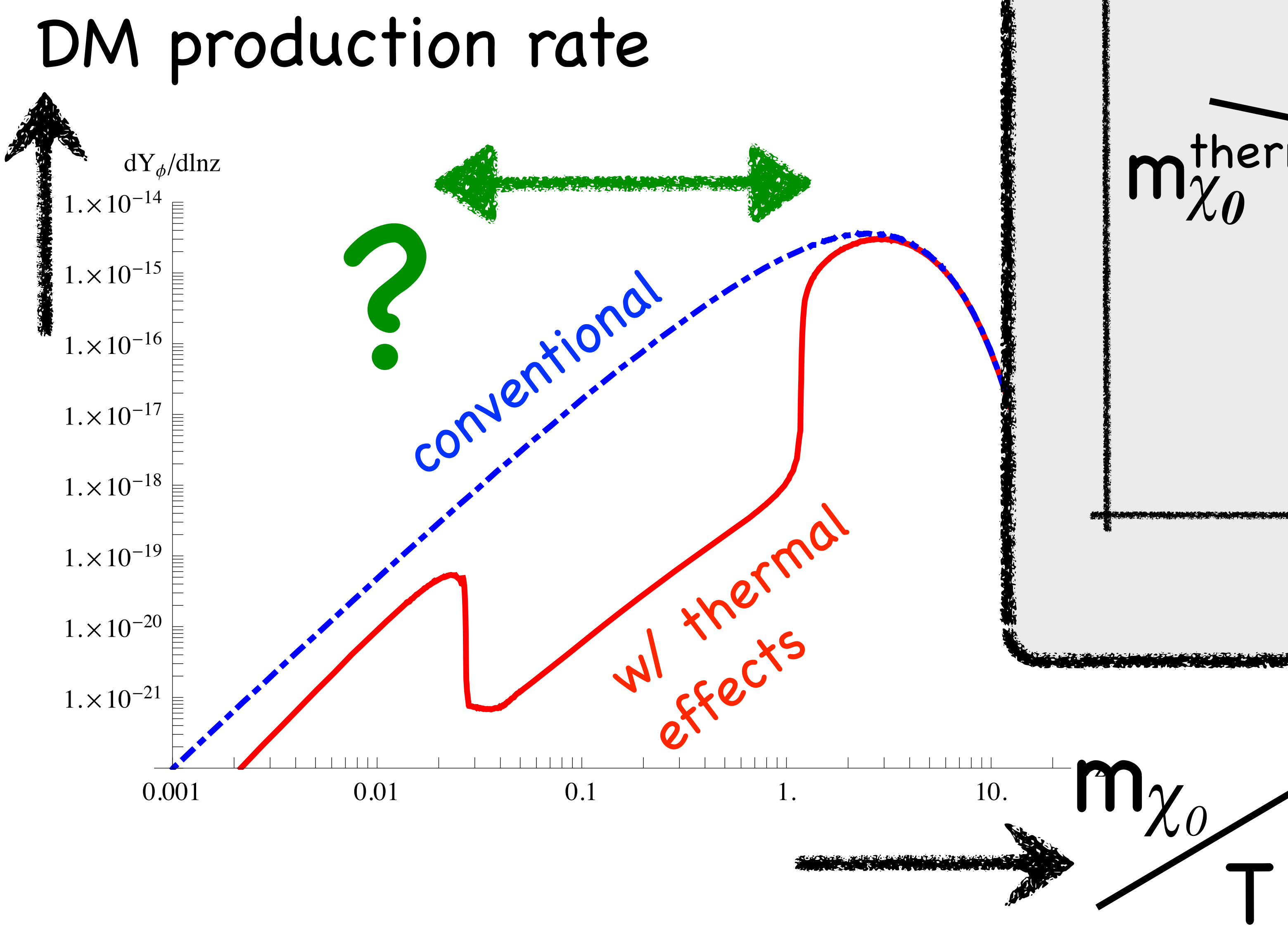

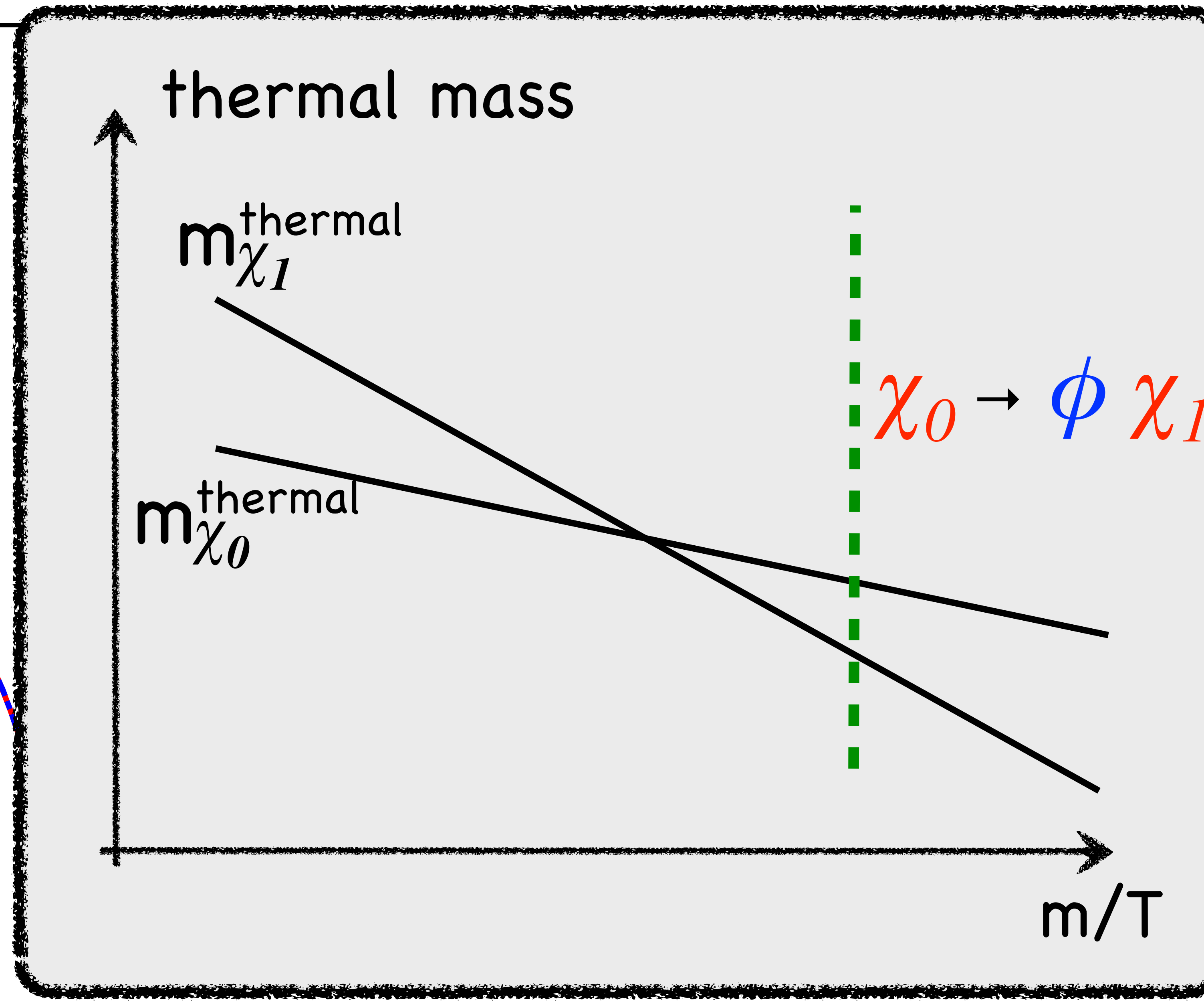

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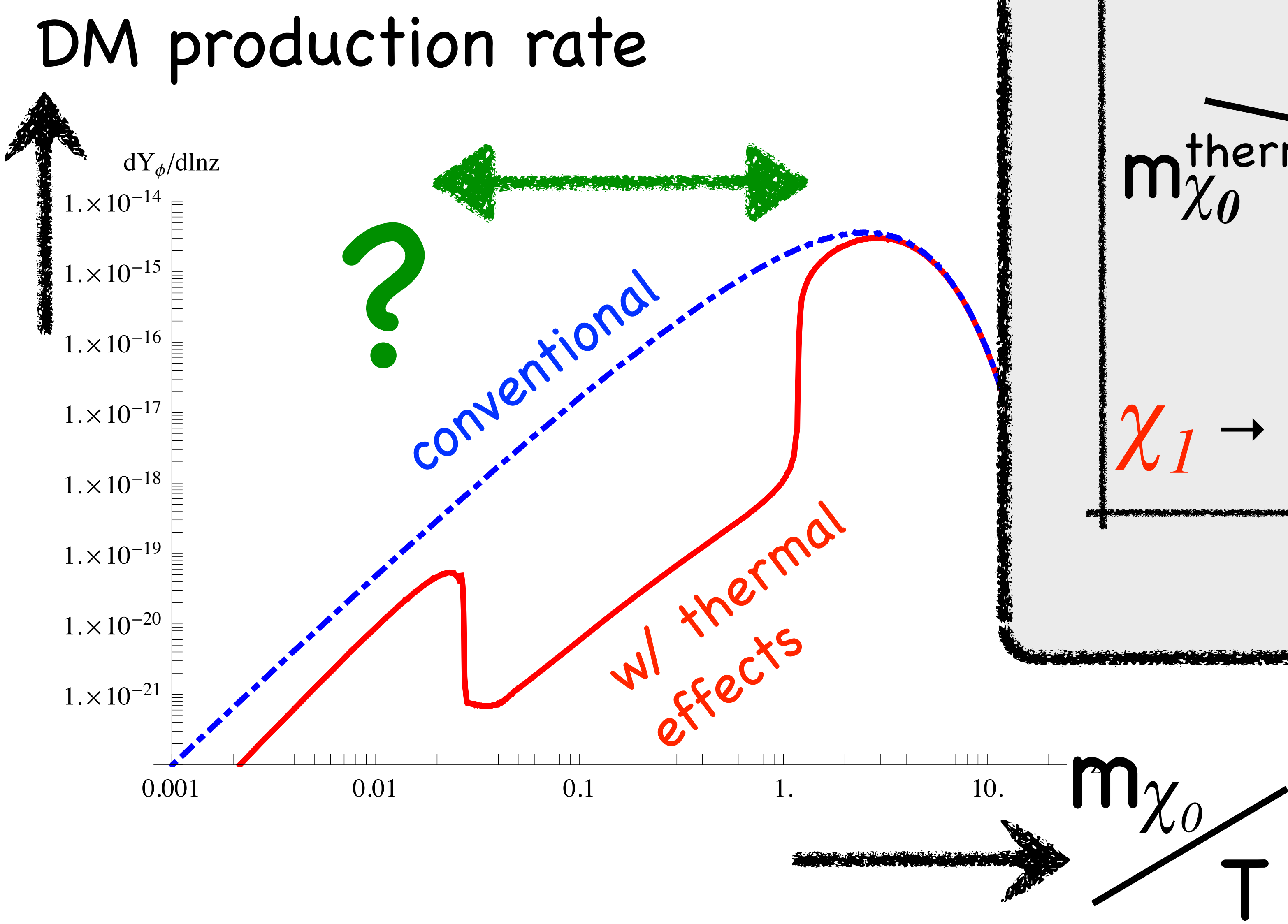

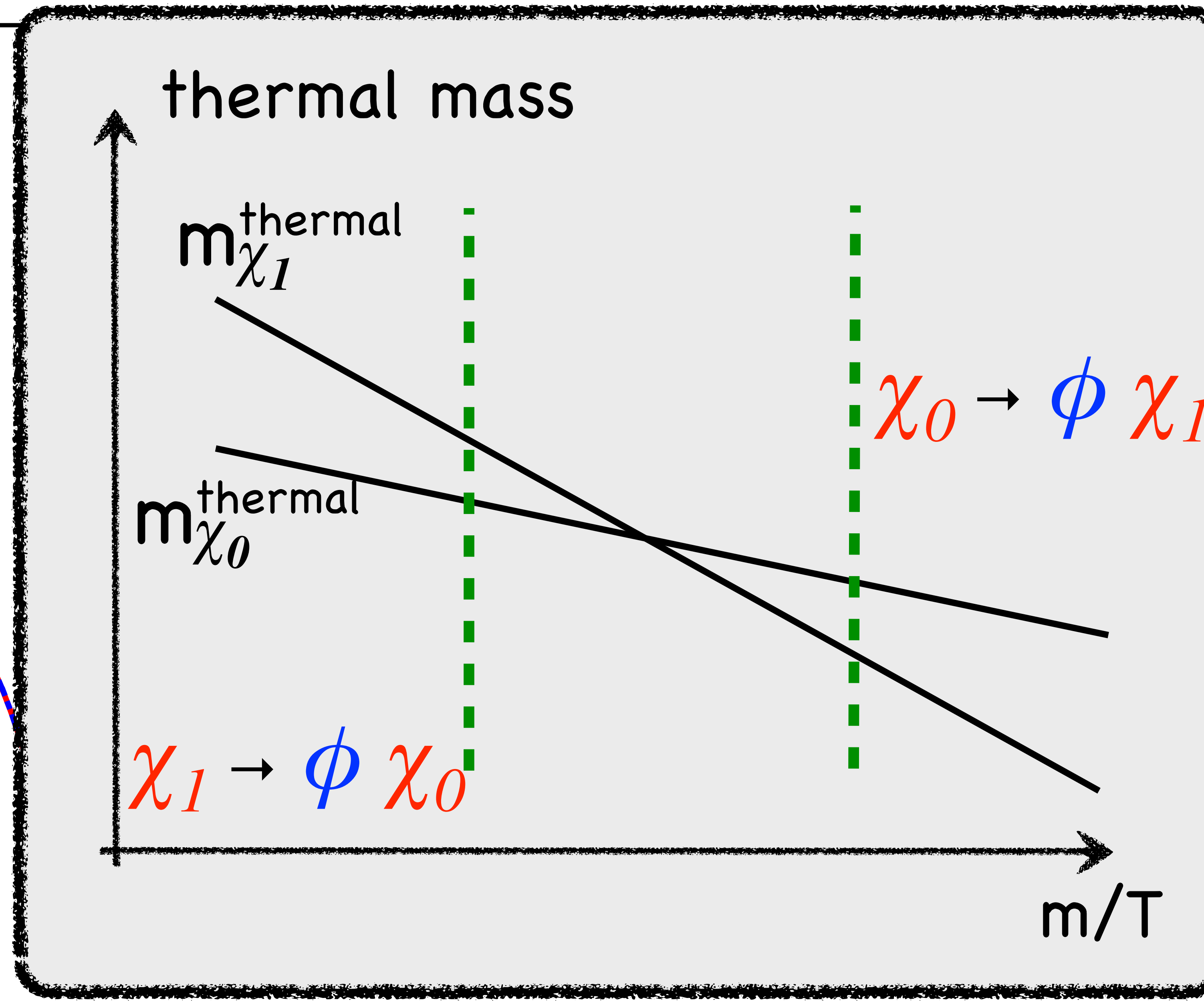

$$\varepsilon/m_\phi = 10^{-13},\ g_{\chi 0} = 0.1,\ g_{\chi 1} = 0.3 \qquad \left(\mathcal{L}_{\rm int} = \varepsilon\, \phi \chi_0 \chi_1 - \frac{g_{\chi 0}^2}{4!}\chi_0^4 - \frac{g_{\chi 1}^2}{4!}\chi_1^4 \right)$$

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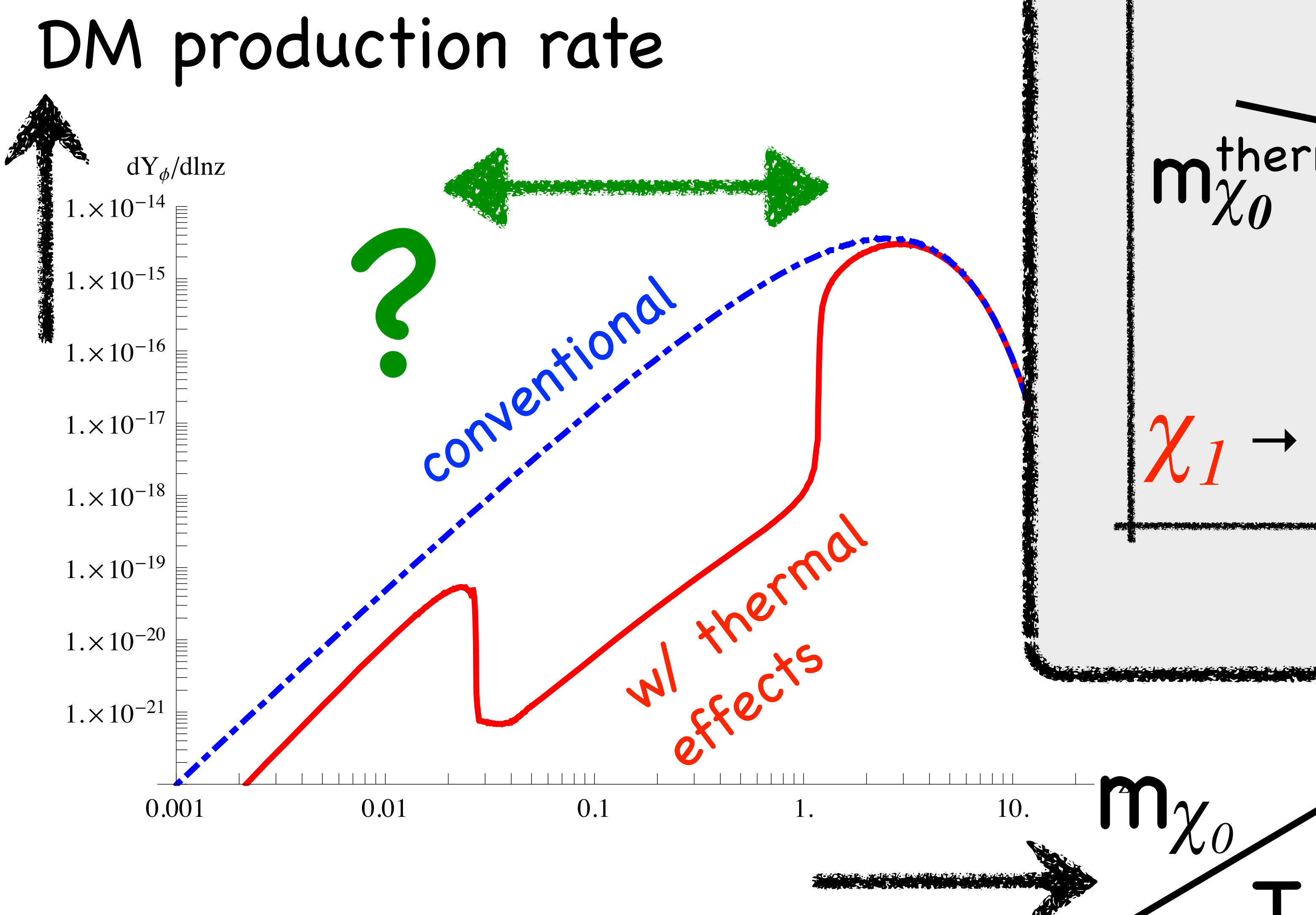

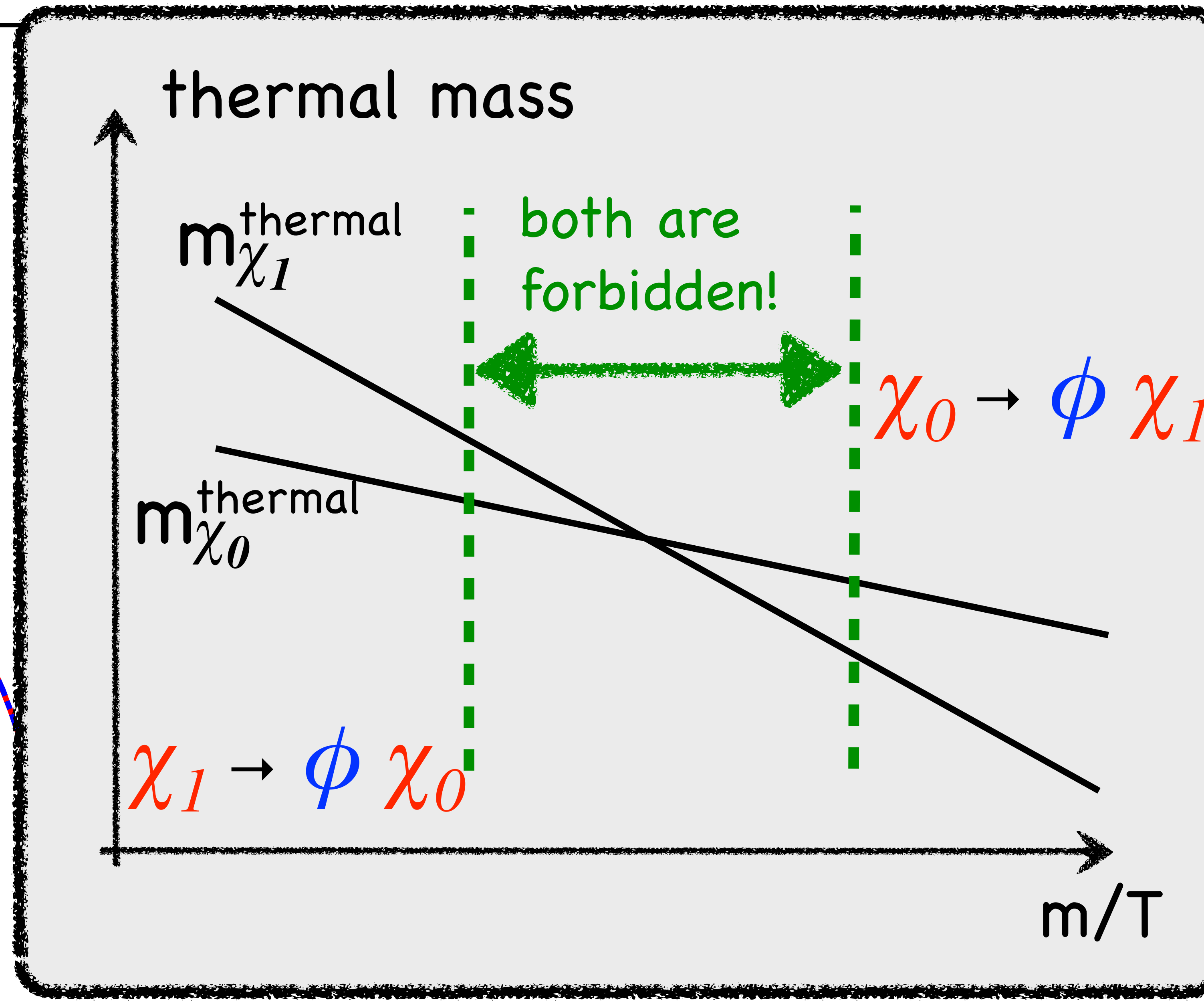

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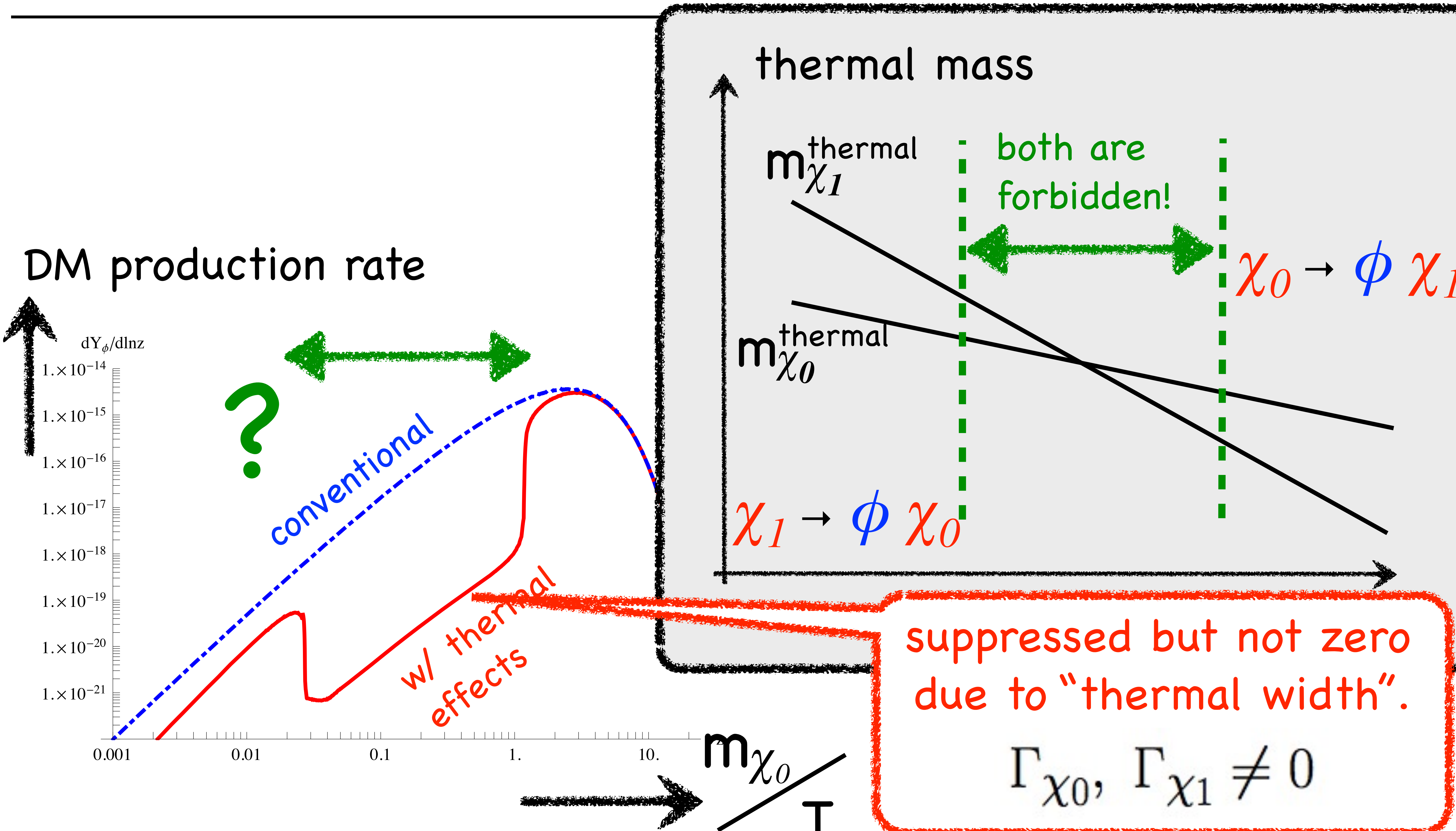

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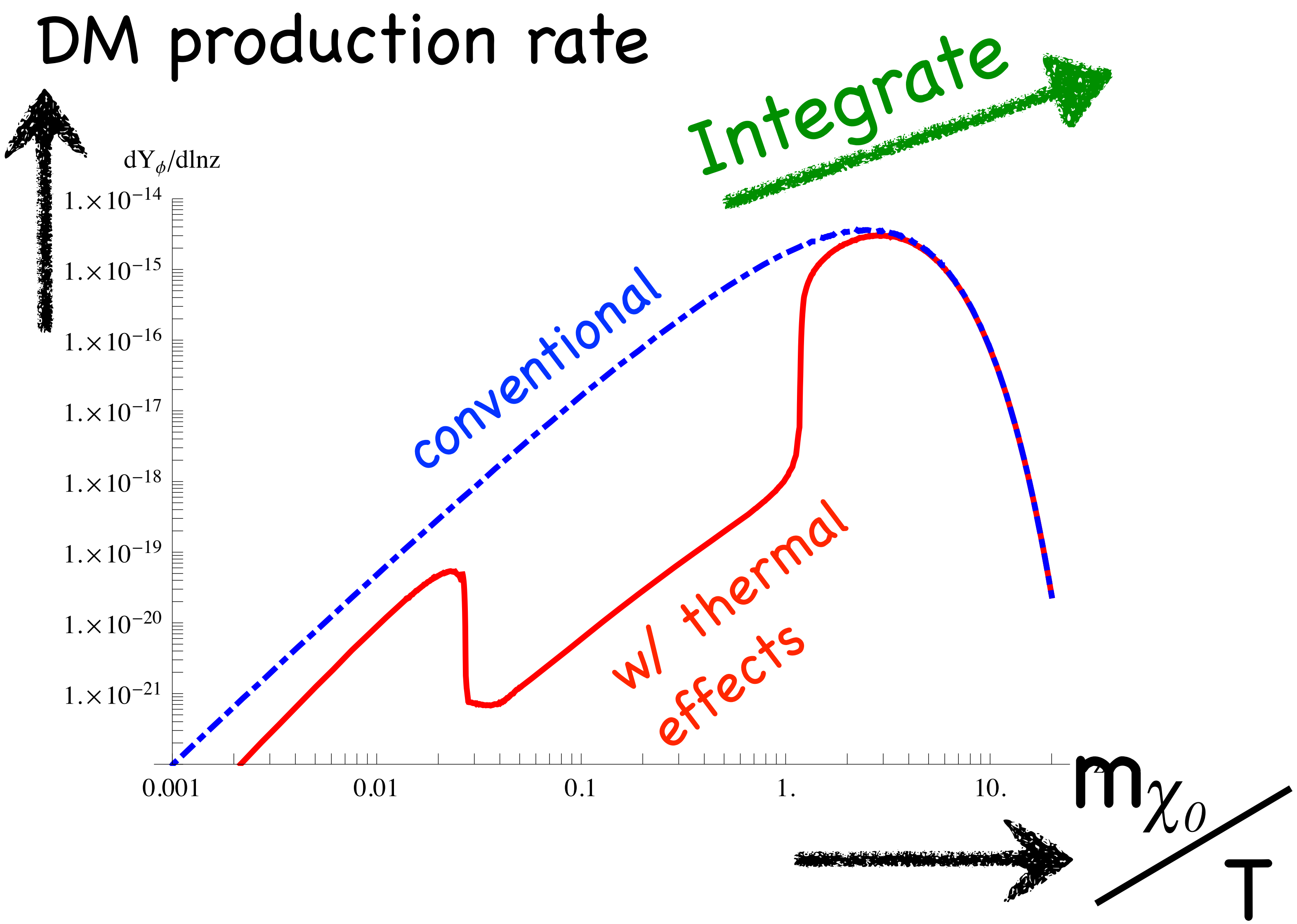

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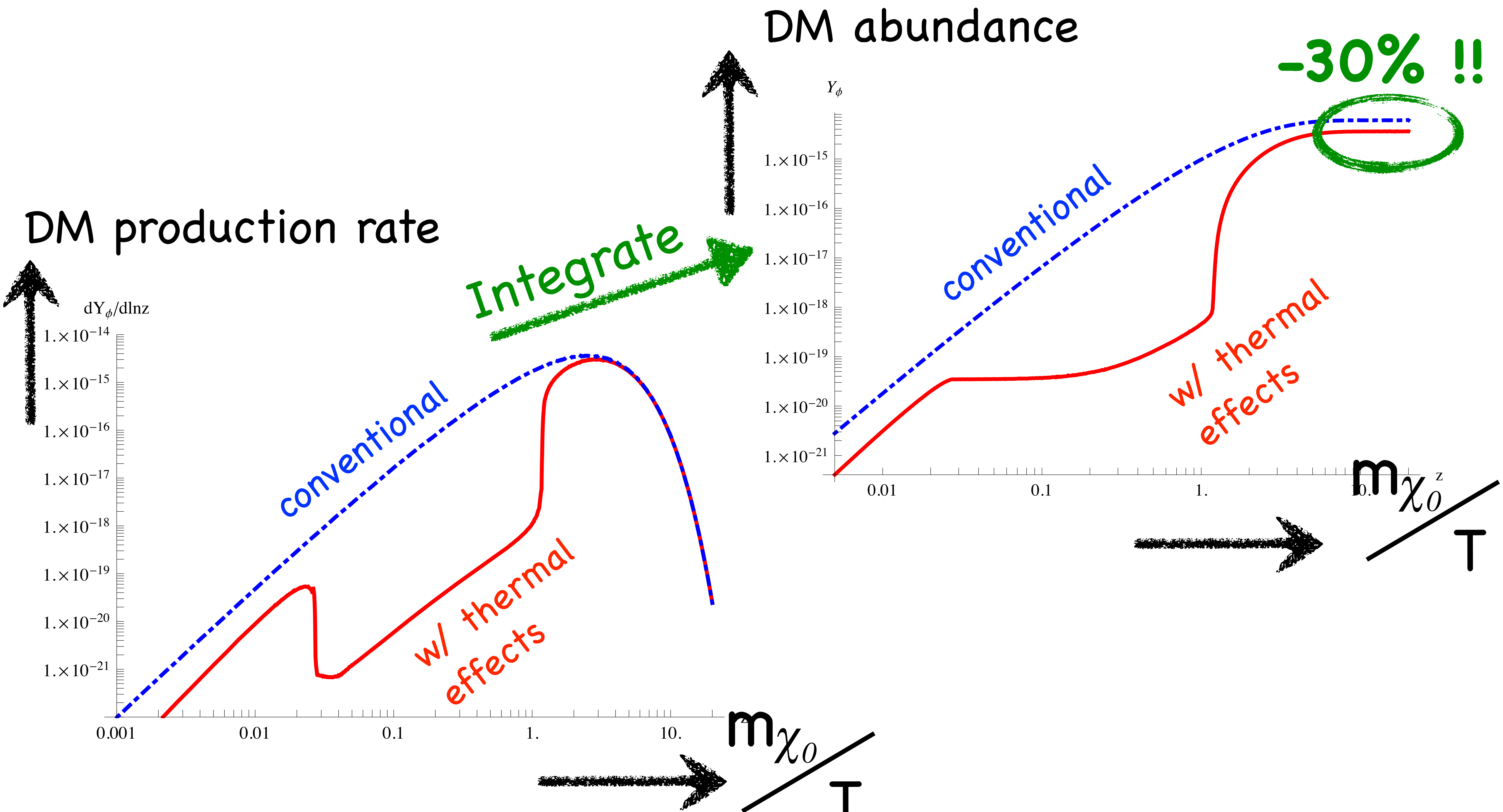
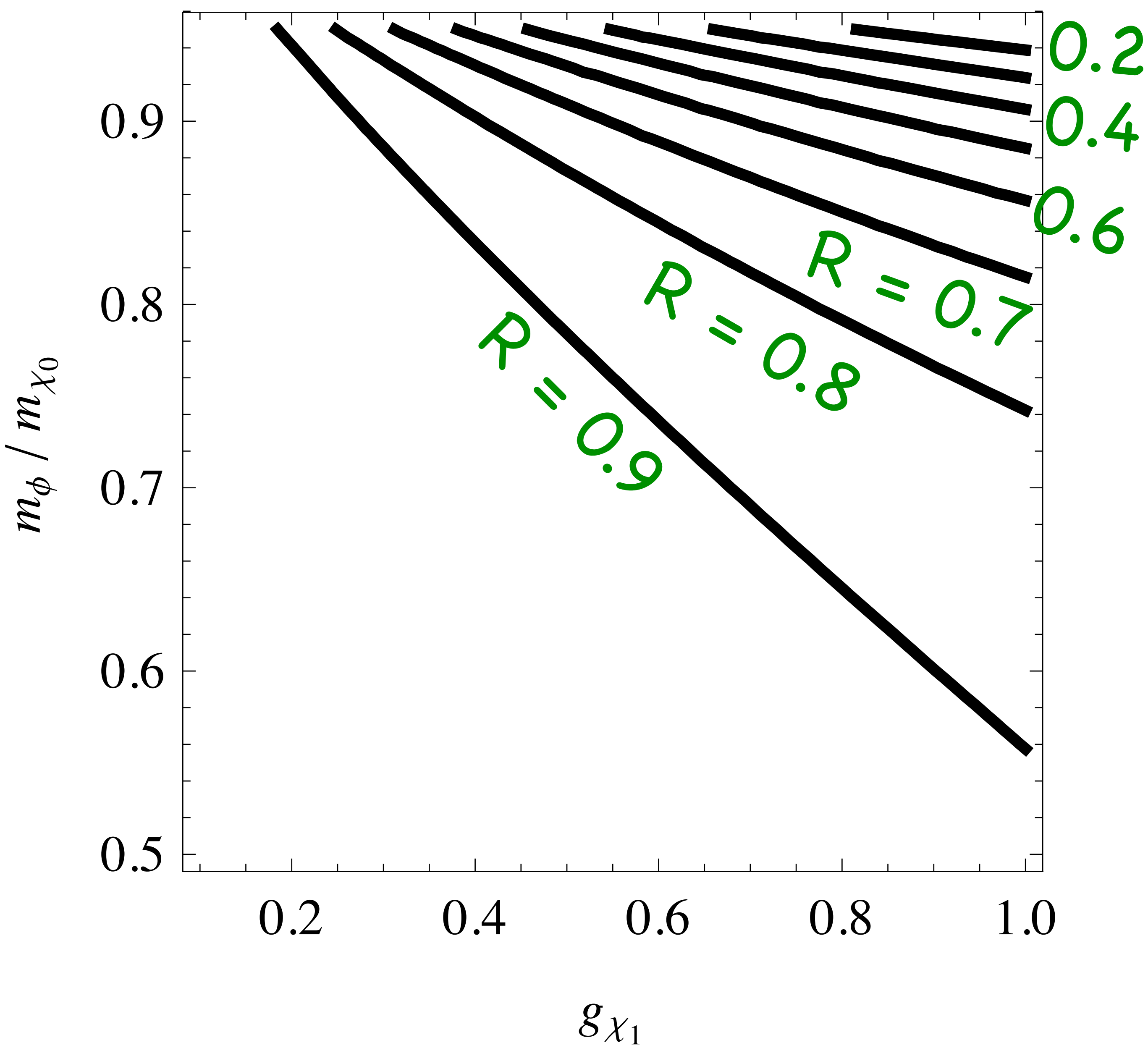


$$\varepsilon/m_\phi = 10^{-13},\ g_{\chi 0} = 0.1,\ m_{\chi 1} = 0 \quad \left(\mathcal{L}_{\rm int} = \varepsilon\, \phi \chi_0 \chi_1 - \frac{g_{\chi 0}^2}{4!}\chi_0^4 - \frac{g_{\chi 1}^2}{4!}\chi_1^4 \right)$$

$$R \equiv \left. \frac{Y_\phi}{Y_\phi(\text{conventional Boltzmann equation})} \right|_{\rm now}$$

mass degeneracy ↑



coupling in thermal bath →

Comment: we have also shown that......

$$\dot{n}_\phi + 3Hn_\phi = \int \frac{d^3k}{(2\pi)^3 2\omega_{\phi,\mathbf{k}}} \int d^4x \, e^{i(\omega_{\phi,\mathbf{k}}t - \mathbf{k}\cdot\mathbf{x})} \frac{\mathrm{tr}\left[e^{-H_\chi/T} \cdot \epsilon\hat{\chi}_0\hat{\chi}_1(0) \cdot \epsilon\hat{\chi}_0\hat{\chi}_1(x)\right]}{\mathrm{tr}\left[e^{-H_\chi/T}\right]}$$

(new Boltzmann eq.)

↓

in the limit of

zero thermal width

$$\Gamma_{\chi_0},\ \Gamma_{\chi_1} \to 0$$

Comment: we have also shown that......

$$\dot{n}_\phi + 3Hn_\phi = \int \frac{d^3k}{(2\pi)^3 2\omega_{\phi,\mathbf{k}}} \int d^4x \, e^{i(\omega_{\phi,\mathbf{k}}t - \mathbf{k}\cdot\mathbf{x})} \frac{\mathrm{tr}\left[e^{-H_\chi/T} \cdot \epsilon\hat{\chi}_0\hat{\chi}_1(0) \cdot \epsilon\hat{\chi}_0\hat{\chi}_1(x)\right]}{\mathrm{tr}\left[e^{-H_\chi/T}\right]}$$

(new Boltzmann eq.)

in the limit of

zero thermal width

$$\Gamma_{\chi_0},\ \Gamma_{\chi_1} \to 0$$

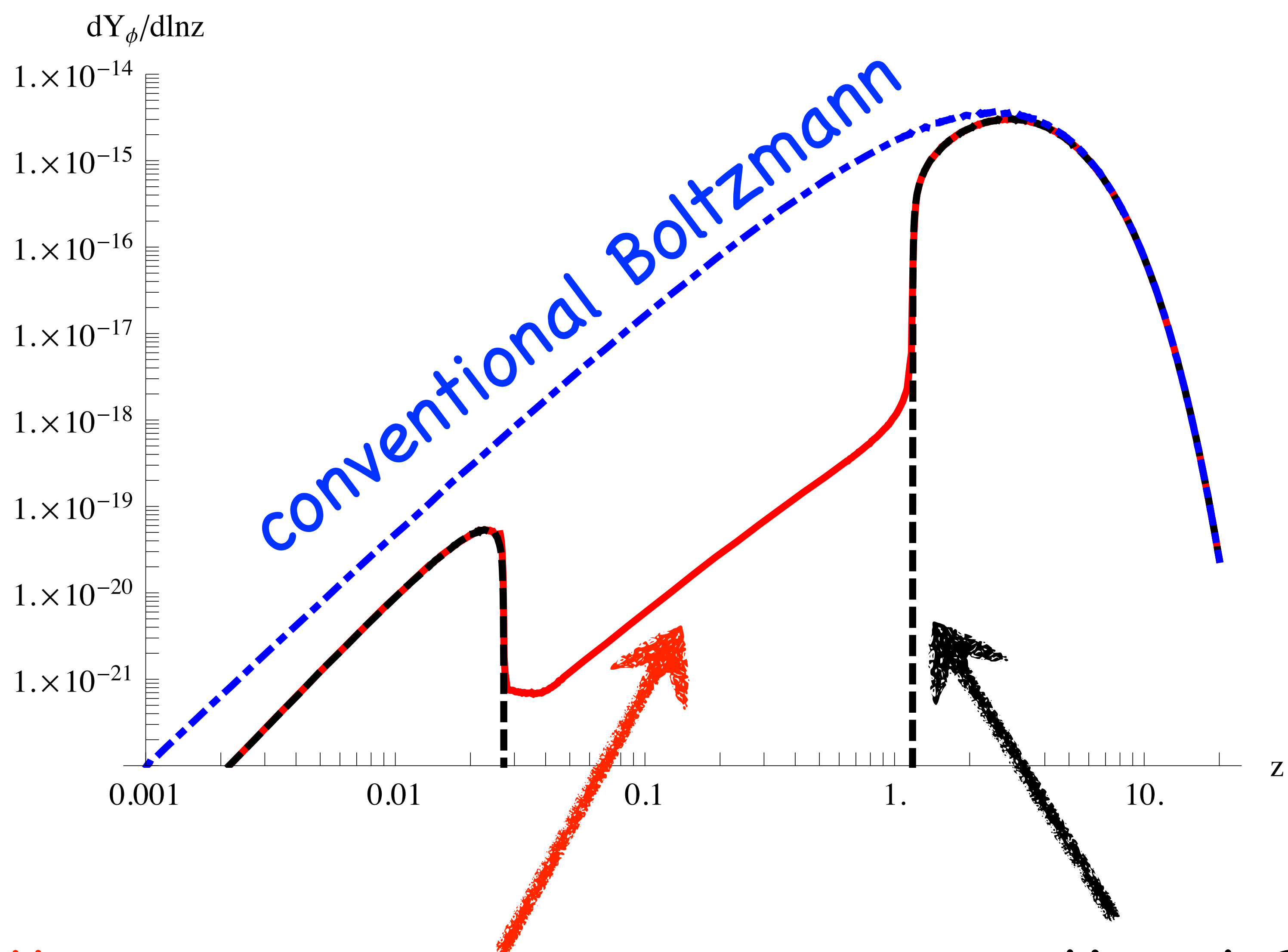
$$\dot{n}_\varphi + 3Hn_\varphi \simeq \int d\Pi \, (2\pi)^4 \delta(p_{\mathrm{tot}}) \, |M(\chi_0 \to \varphi\chi_1)|^2 f_{\chi_0}(1 + f_{\chi_1}) \cdots$$

where $d\Pi = \prod_{i=\chi_0,\varphi,\chi_1} \deg(i) \dfrac{d^3p}{(2\pi)^3 2\omega_{i,\mathbf{p}}}$; $\quad f_{\chi_i} = \dfrac{1}{e^{\omega_{i,\mathbf{p}}/T} - 1} \quad \omega_{i,\mathbf{p}} = \sqrt{m_i^2 + \mathbf{p}^2}$

The Conventional Boltzmann eq. but with

all the masses replaced with thermal masses !!

Comment: we have also shown that.....



new Boltzmann eq.
(with thermal effects)

conventional Boltzmann eq.
+ thermal masses

The "just thermal mass" approximation works well
if kinematically allowed by thermal masses.

SUMMARY

- We have studied the "thermal effects" on a non-thermal DM production scenario ("FIMP" scenario)

- We found that the "thermal effects" can significantly change the conventional picture, and the resultant DM abundance.