Imprints of Cosmic Phase Transition on Gravitational Waves (GWs)

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Refs:

Jinno, TM and Nakayama, arXiv:1112.0084 [hep-ph]

1. Introduction

Early universe: environment with high-energy particles

High temperature $\sim$ high energy

How “deep” can we probe with various objects?

- Last scattering of photon: ~ 1 eV
- Last scattering of neutrino: ~ 1 MeV
- “Last scattering” of GWs: inflation

The history of our universe is imprinted in GWs

$\Rightarrow$ We may be able to extract information about high energy physics which cannot be probed by colliders

$\Rightarrow$ GW spectrum may be precisely measured in (far) future by, for e.g., DECIGO

Today's subject: phase transition (or SSB)

- QCD phase transition
- EW symmetry breaking
- Peccei-Quinn symmetry
- GUT
- $\cdots$

In models with SSB, cosmic phase transition may occur

$\Rightarrow$ Is there any effect on observables?

$\Rightarrow$ If yes, what can we learn?

The spectrum of GWs is affected by cosmic phase transition

1. Primordial GWs are produced during inflation (via quantum fluctuation)
2. Spectrum of GWs is deformed during the cosmic phase transition

Outline

2. GWs: Production and Evolution

Story:

1. Primordial GWs are produced during inflation (via quantum fluctuation)
2. Evolution of the amplitudes of GWs depends how the universe expands
3. Spectrum of GWs is deformed during the cosmic phase transition

Gravitational wave:

- Fluctuation of the metric (propagating mode)
- Its evolution is governed by the Einstein equation

Metric: $ds^2 = -dt^2 + a^2(t)(\delta_{ij} + 2h_{ij})dx^i dx^j$

Physical mode: transverse and traceless $(h^i_i = h_{ij}{}^{,j} = 0)$

Fourier amplitude (using comoving wave-number $\vec{k}$)

$$h_{ij}(t,\vec{x}) = \frac{1}{M_{\mathsf{Pl}}} \sum_{\lambda=+,\times} \int \frac{d^3\vec{k}}{(2\pi)^3} \tilde{h}^{(\lambda)}_{\vec{k}}(t) \epsilon^{(\lambda)}_{ij} e^{i\vec{k}\vec{x}}$$

$M_{\mathsf{Pl}} \simeq 2.4 \times 10^{18}$ GeV: Reduced Planck scale

$\epsilon^{(\lambda)}_{ij}$: polarization tensor (transverse & traceless)

$\tilde{h}^{(\lambda)}_{\vec{k}}(t)$: Canonically normalized

$$L^{(\mathsf{flat})} = \frac{1}{2} M^2_{\mathsf{Pl}} \int d^3x R + \cdots \simeq \int \frac{d^3\vec{k}}{(2\pi)^3} \sum_{\lambda} \left[\frac{1}{2} \dot{\tilde{h}}^{(\lambda)}_{\vec{k}} \dot{\tilde{h}}^{(\lambda)}_{-\vec{k}} + \cdots \right]$$

Gravitational wave in de Sitter background: $a \propto e^{H_{\mathsf{inf}} t}$

$$L = \int \frac{d^3\vec{k}}{(2\pi)^3} a^3 \sum_\lambda \left[\frac{1}{2} \dot{\tilde{h}}_{\vec{k}}^{(\lambda)} \dot{\tilde{h}}_{-\vec{k}}^{(\lambda)} - \frac{1}{2} \left(\frac{k}{a} \right)^2 \tilde{h}_{\vec{k}}^{(\lambda)} \tilde{h}_{-\vec{k}}^{(\lambda)} \right]$$

$\Rightarrow$ $\tilde{h}_{\vec{k}}^{(\lambda)}$ behaves as massless scalar field

Quantum fluctuation generated during inflation

$$\Delta_h^2 \equiv \frac{1}{V} \left(\frac{k^3}{2\pi^2} \right) \times \frac{1}{M_{\mathsf{Pl}}^2} \sum_\lambda \left\langle |\tilde{h}_{\vec{k}}^{(\lambda)}|^2 \right\rangle_{\mathsf{inflation}} \simeq \frac{1}{M_{\mathsf{Pl}}^2} \sum_\lambda \left(\frac{H_{\mathsf{inf}}}{2\pi} \right)^2$$

The primordial GW amplitude is proportional to H_{inf}

$\Rightarrow$ The effects of GWs become observable when the energy scale of the inflation is high

The tensor-to-scalar ratio

$$r \equiv \frac{\Delta_h^2}{\Delta_{\mathcal{R}}^2} (= 16\epsilon)$$

$\mathcal{R}$: curvature perturbation ($\Delta_{\mathcal{R}}^2 \simeq 2.42 \times 10^{-9}$)

ϵ: Slow-roll parameter

- WMAP 7 years

  $\Rightarrow r < 0.24$
- PLANCK / Future CMB interferometric observations

  $\Rightarrow r$ as small as $0.1 - 0.01$ will be detected
- Future experiments to detect GWs (DECIGO, $\cdots$)

  $\Rightarrow$ GW spectrum will be observed if $r \gtrsim \sim 10^{-3}$

GW evolution after inflation

$$\ddot{\tilde{h}}^{(\lambda)}_{\vec{k}} + 3H\dot{\tilde{h}}^{(\lambda)}_{\vec{k}} + \frac{k^2}{a^2(t)}\tilde{h}^{(\lambda)}_{\vec{k}} = 0 \quad \text{with} \quad H = \frac{\dot{a}}{a}$$

Before the horizon-in: $k \ll aH$

$$\tilde{h}_{\vec{k}} \sim \text{const.}$$

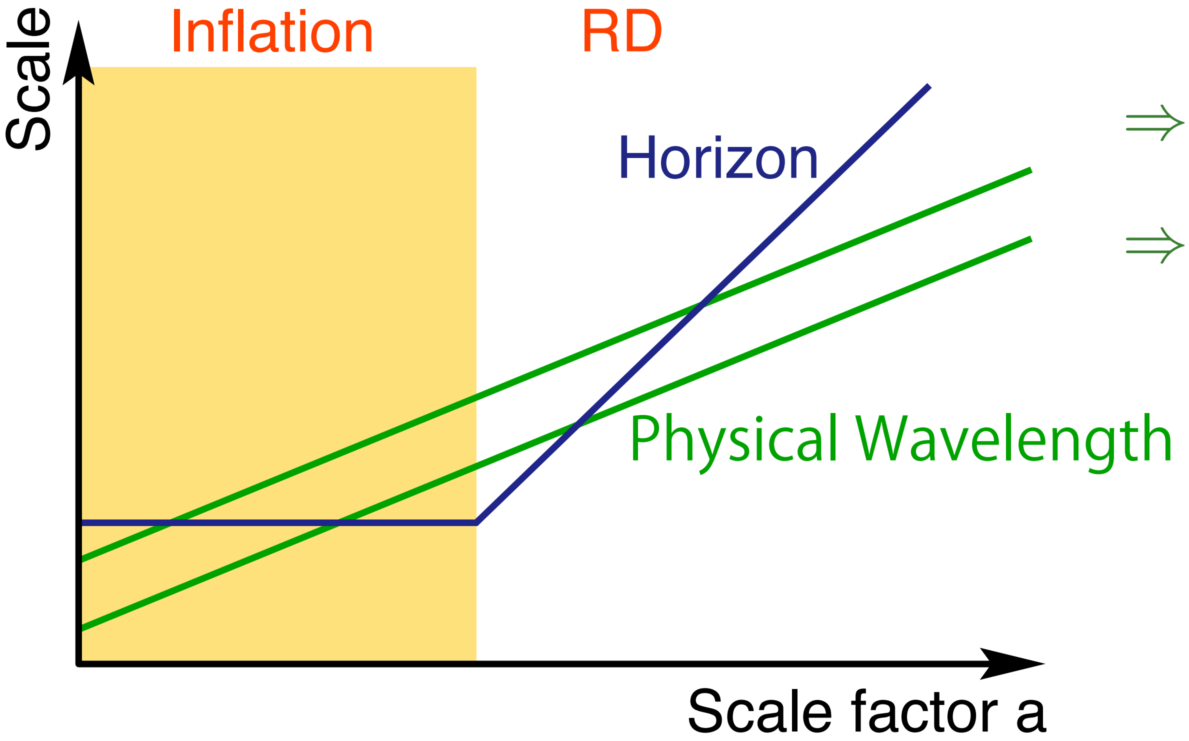
After the horizon-in: $k \gg aH$

$$-3H\dot{\tilde{h}}^2_{\vec{k}} = \dot{\tilde{h}}_{\vec{k}}\ddot{\tilde{h}}_{\vec{k}} + \frac{k^2}{a^2(t)}\dot{\tilde{h}}_{\vec{k}}\tilde{h}_{\vec{k}} = \frac{d}{dt}\left[\frac{1}{2}\dot{\tilde{h}}^2_{\vec{k}} + \frac{1}{2}\frac{k^2}{a^2}\tilde{h}^2_{\vec{k}}\right] + H\frac{k^2}{a^2}\tilde{h}^2_{\vec{k}}$$

$$\Rightarrow \ \frac{d}{dt}\left\langle \frac{k^2}{a^2}\tilde{h}^2_{\vec{k}} \right\rangle_{\text{osc}} \simeq -4\frac{\dot{a}}{a}\left\langle \frac{k^2}{a^2}\tilde{h}^2_{\vec{k}} \right\rangle_{\text{osc}} \quad \Leftrightarrow \quad \left\langle \dot{\tilde{h}}^2_{\vec{k}} \right\rangle_{\text{osc}} \simeq \left\langle \frac{k^2}{a^2}\tilde{h}^2_{\vec{k}} \right\rangle_{\text{osc}}$$

$$\Rightarrow \ \left\langle \tilde{h}^2_{\vec{k}} \right\rangle_{\text{osc}} \sim a^{-2}$$

Amplitude of GWs



$\Rightarrow k \gtrsim aH$: $\tilde{h}_{\vec{k}} \sim$ const.

$\Rightarrow k \lesssim aH$: $\langle \tilde{h}^2_{\vec{k}} \rangle_{\text{osc}} \sim a^{-2}$

$$\left\langle |\tilde{h}^{(\lambda)}_{\vec{k}}|^2 \right\rangle_{\text{osc}} \simeq \left\langle |\tilde{h}^{(\lambda)}_{\vec{k}}|^2 \right\rangle_{\text{inflation}} \times \left[\frac{a(t)}{a|_{k=aH}} \right]^{-2}$$

Energy density: $\rho_{\mathsf{GW}}(t) \equiv \int d\ln k \;\; \rho_{\mathsf{GW}}(t;k)$

$$\begin{aligned}
\rho_{\mathsf{GW}}(t;k) &= \frac{1}{V}\left(\frac{k^3}{2\pi^2}\right) \times \sum_\lambda \left[\frac{1}{2}\dot{\tilde{h}}^{(\lambda)}_{\vec{k}}\dot{\tilde{h}}^{(\lambda)}_{-\vec{k}} + \frac{1}{2}\left(\frac{k}{a}\right)^2 \tilde{h}^{(\lambda)}_{\vec{k}}\tilde{h}^{(\lambda)}_{-\vec{k}}\right] \\
&\simeq \frac{1}{V}\left(\frac{k^3}{2\pi^2}\right) \times \left(\frac{k}{a}\right)^2 \sum_\lambda \left\langle |\tilde{h}^{(\lambda)}_{\vec{k}}|^2 \right\rangle_{\mathsf{osc}} \\
&\simeq \left(\frac{H_{\mathsf{inf}}}{2\pi M_{\mathsf{Pl}}}\right)^2 \left[\frac{a(t)}{a|_{k=aH}}\right]^{-4} M_{\mathsf{Pl}}^2 H_{k=aH}^2
\end{aligned}$$

For modes which enter the horizon at the RD epoch:

$$\rho_{\mathsf{GW}}(t;k) \simeq \left(\frac{H_{\mathsf{inf}}}{2\pi M_{\mathsf{Pl}}}\right)^2 \rho_{\mathsf{rad}}(t)$$

Present GW spectrum:

$$\Omega_{\mathrm{GW}}^{(\mathrm{tot})} = \frac{\rho_{\mathrm{GW}}^{(\mathrm{tot})}(t_{\mathrm{NOW}})}{\rho_{\mathrm{crit}}} \equiv \int d\ln k \;\Omega_{\mathrm{GW}}(k)$$

In the case without phase transition (i.e., standard case):

$$\Omega_{\mathrm{GW}}^{(\mathrm{SM})}(k) \simeq 1.7 \times 10^{-15} r_{0.1}\gamma \;\;:\;\; k_{\mathrm{EW}} \ll k \ll k_{\mathrm{RH}}.$$

$r_{0.1}$: the tensor-to-scalar ratio in units of 0.1

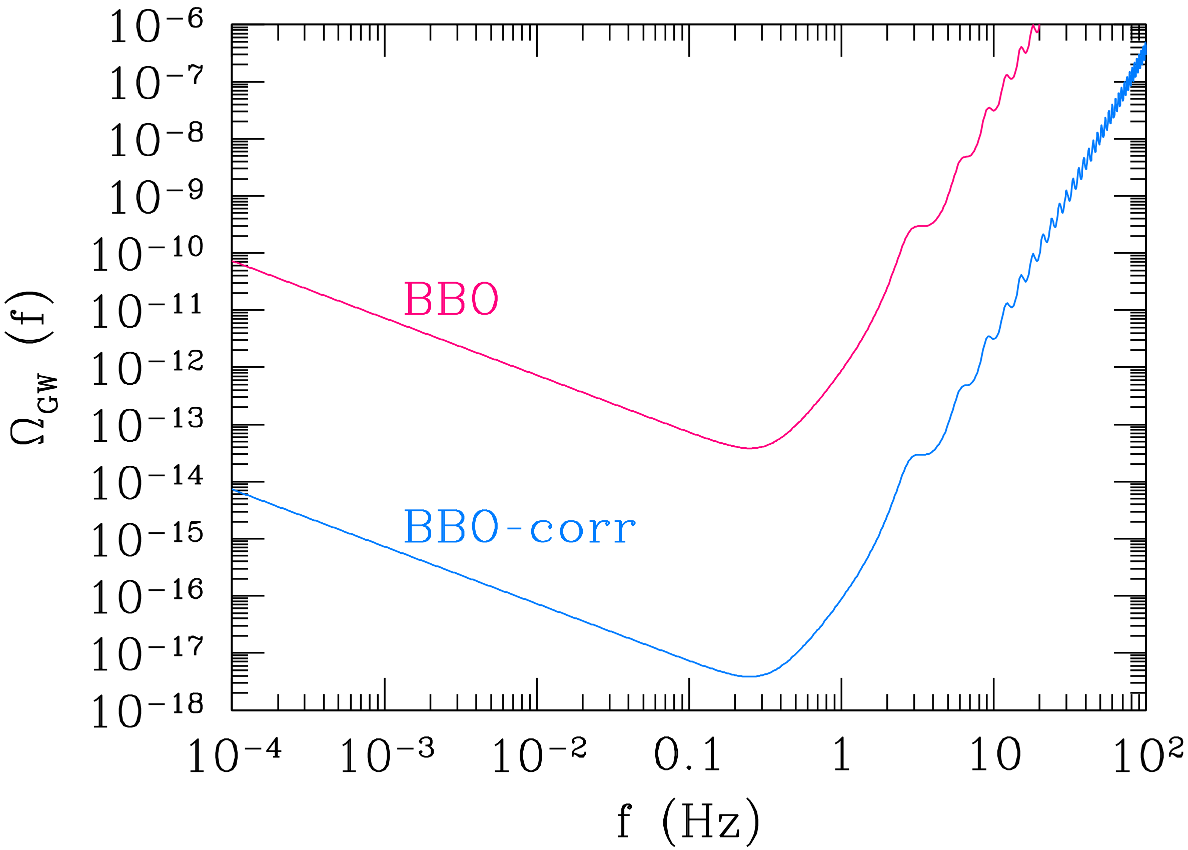
$$\gamma = \left[\frac{g_*(T_{\mathrm{in}}(k))}{g_{*0}}\right]\left[\frac{g_{*s0}}{g_{*s}(T_{\mathrm{in}}(k))}\right]^{4/3}\left(\frac{k}{k_0}\right)^{n_t}$$

$\Omega_{\mathrm{GW}}^{(\mathrm{SM})}(k)$ is insensitive to k

In future, GW spectrum may be measured

$\Rightarrow$ BBO / DECIGO

Expected sensitivity



3. Phase Transition

The spectrum of GWs is affected by phase transitions

$\Leftrightarrow$ There may exist significant entropy production at the time of phase transition

Model: two real scalar fields ϕ and χ

$$V(\phi) = \frac{g}{24}(\phi^2 - v_\phi^2)^2 + \frac{h}{2}\chi^2\phi^2$$

ϕ: scalar field responsible for symmetry breaking

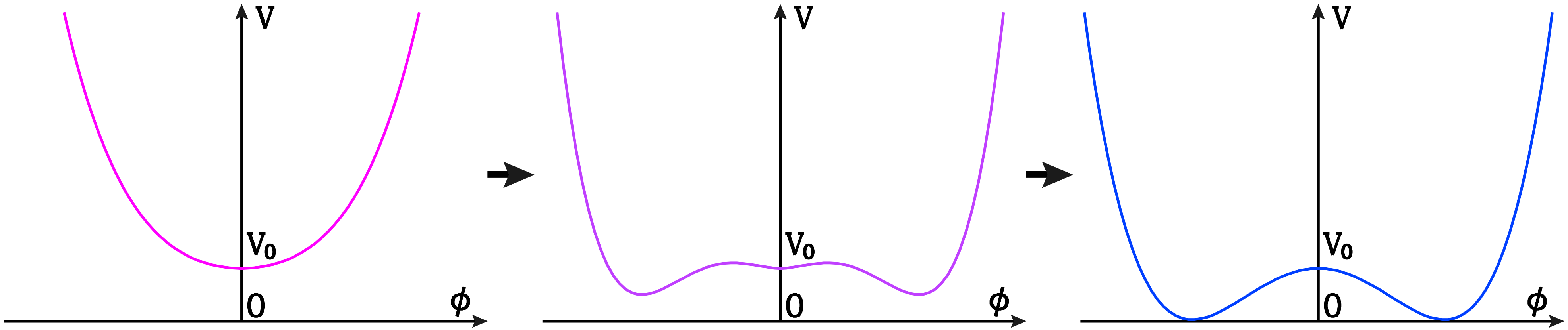
χ: degrees of freedom in thermal bath

"Thermal mass" is generated for ϕ in the thermal bath

$\Rightarrow$ Cosmic phase transition occurs

Potential of ϕ surrounded by the thermal bath (at $\phi \sim 0$)

$$V_T(\phi) = \frac{g}{24}(\phi^2 - v_\phi^2)^2 + \frac{h}{24}T^2\phi^2 + \cdots \equiv V_0 + \frac{h}{24}(T^2 - T_c^2)\phi^2 + \cdots$$



Critical temperature: temperature for $V_T''(\phi = 0) = 0$

$$T_\mathsf{c} = \sqrt{\frac{2g}{h}}v_\phi$$

Approximately, the phase transition occurs when $V_T''(\phi = 0) = 0$

$\Leftrightarrow$ Tunneling rate is suppressed when $g \ll 1$

Expectation value of ϕ:

$$\langle\phi\rangle = \begin{cases} 0 \ : T > T_{\mathsf{c}} \\ v_\phi : T < T_{\mathsf{c}} \end{cases}$$

Entropy is produced due to the phase transition

- Temperature just before the phase transition: T_c
- Temperature just after the phase transition: $T_{\mathsf{PT}} > T_c$

$\Rightarrow \rho_{\mathsf{rad}}(T_{\mathsf{PT}}) = \rho_{\mathsf{rad}}(T_c) + V_0$

Expansion rate at the phase transition:

$$H_{\mathsf{PT}} \equiv \sqrt{\frac{\rho_{\mathsf{rad}}(T_{\mathsf{c}}) + V_0}{3M_{\mathsf{Pl}}^2}}$$

The mode which enters the horizon at the phase transition:

$$k_{\mathsf{PT}} \equiv a(t_{\mathsf{PT}})H_{\mathsf{PT}}$$

$\Rightarrow k < k_{\mathsf{PT}}$: out-of-horizon at $t = t_{\mathsf{PT}}$

Present frequency: $f_{\mathsf{PT}} = k_{\mathsf{PT}}/2\pi a_{\mathsf{NOW}}$

$$f_{\mathsf{PT}} \simeq 2.7 \text{ Hz} \times \left(\frac{T_{\mathsf{PT}}}{10^8 \text{ GeV}}\right)$$

$$\Rightarrow [f_{\mathsf{PT}}]_{g/h^2 \ll 1} \simeq 0.50 \text{ Hz} \times \left(\frac{g^{1/4} v_\phi}{10^8 \text{ GeV}}\right)$$

Relevant equations to be solved (background):

- $H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho_{\mathsf{rad}} + V_0 \theta(t_{\mathsf{PT}} - t)}{3M_{\mathsf{Pl}}^2}$

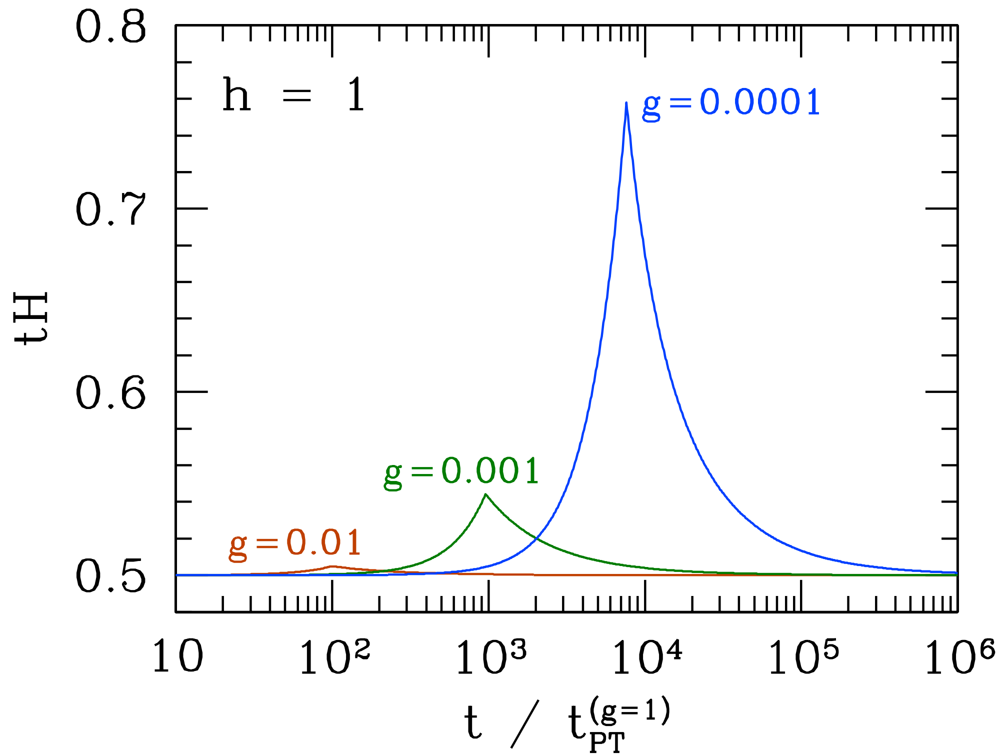
- $\dot{\rho}_{\mathsf{rad}} + 4H\rho_{\mathsf{rad}} = V_0 \delta(t - t_{\mathsf{PT}})$

t_{PT}: time of phase transition (i.e., $T = T_{\mathsf{c}}$)

Effects of ϕ

- Deviation from the radiation-domination at $t \sim t_{\mathsf{PT}}$
- Entropy production due to the phase transition

Evolution of the universe (with $h = 1$):



$$H = \frac{1}{2t} \text{ in RD}$$

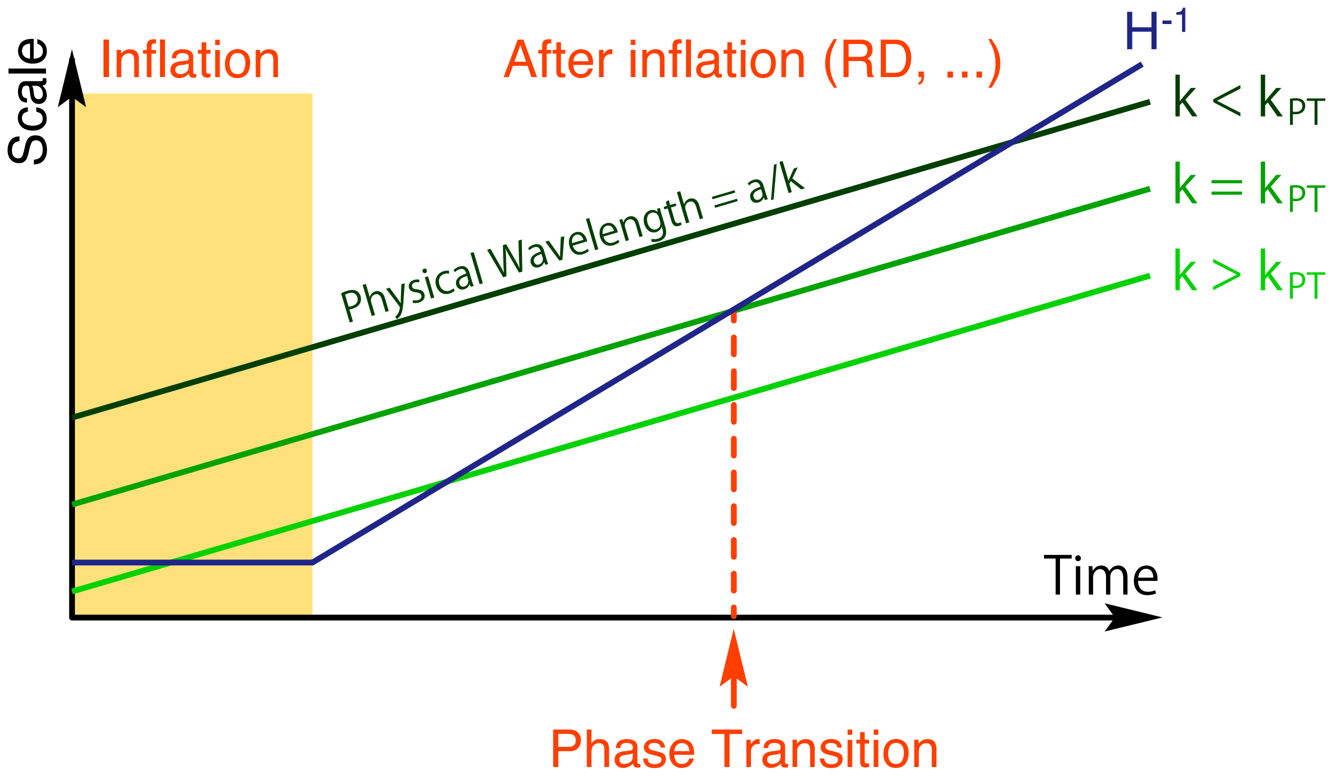
$$\frac{V_0}{\rho_{\text{rad}}(T_c)} \sim O(1) \times \frac{h^2}{g_* g}$$

g_*: Effective number of massless degrees of freedom

4. Imprints of Phase Transition in GWs

Behavior of GW amplitudes:

- $k \leq k_{\mathrm{PT}}$: No effect of phase transition
- $k \geq k_{\mathrm{PT}}$: Density is diluted due to the entropy production



$\Rightarrow \Omega_{\mathrm{GW}}(k \gtrsim k_{\mathrm{PT}})$ is suppressed

“Short wavelength (i.e., high frequency)” mode: $k \geq k_{\mathsf{PT}}$

1. The amplitude is constant until the horizon-reentry

$$[\rho_{\mathsf{GW}}(k)]_{k=aH} \simeq \rho_{\mathsf{GW}}(t=0)$$

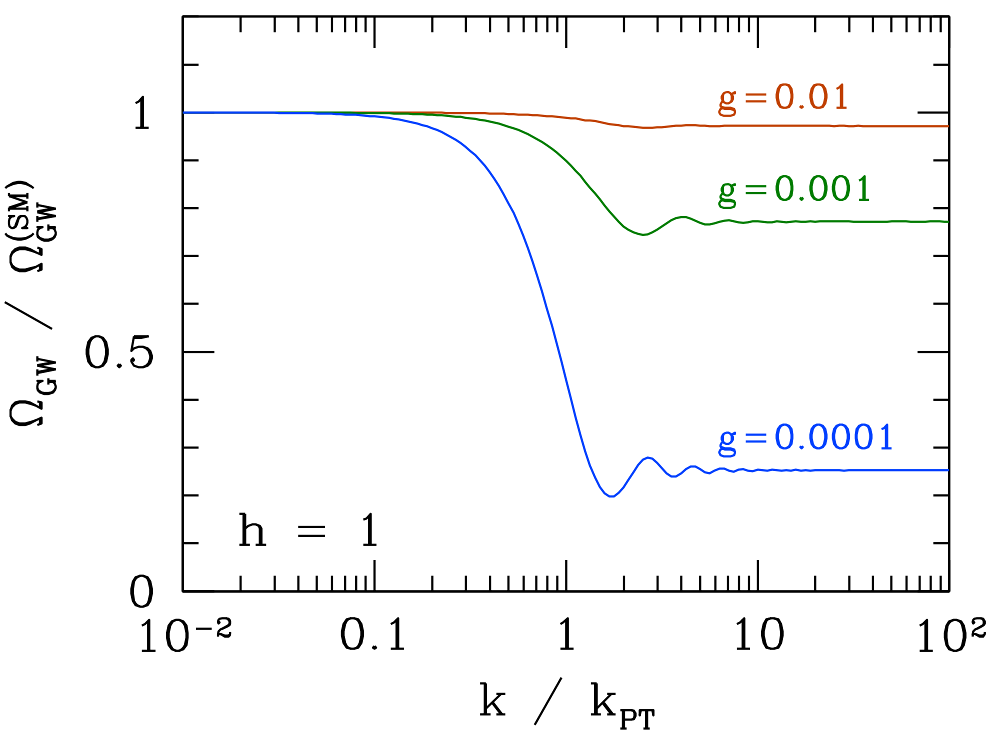
2. $\rho_{\mathsf{GW}}(k) \propto a^{-4}$ once the mode enters the horizon

$$\begin{aligned}
[\rho_{\mathsf{GW}}(k)](t) &\simeq [\rho_{\mathsf{GW}}(k)]_{k=aH} \left(\frac{a_{\text{horizon-in}}}{a_{\mathsf{PT}}}\right)^4 \left(\frac{a_{\mathsf{PT}}}{a(t)}\right)^4 \\
&\simeq [\rho_{\mathsf{GW}}(k)]_{k=aH} \left(\frac{T_{\text{horizon-in}}}{T_c}\right)^{-4} \left(\frac{T_{\mathsf{PT}}}{T(t)}\right)^{-4} \\
&\simeq \left(\frac{T_c}{T_{\mathsf{PT}}}\right)^4 [\rho_{\mathsf{GW}}(k)]_{\text{no phase transition}}
\end{aligned}$$

$\Omega_{\mathsf{GW}}(k \gtrsim k_{\mathsf{PT}})$ becomes suppressed

$$R \equiv \left.\frac{\Omega_{\mathsf{GW}}(k)}{\Omega_{\mathsf{GW}}^{(\mathsf{SM})}(k)}\right|_{k \gg k_{\mathsf{PT}}} = \left(\frac{T_c}{T_{\mathsf{PT}}}\right)^4 = \frac{\rho_{\mathsf{rad}}(T_{\mathsf{c}})}{\rho_{\mathsf{rad}}(T_{\mathsf{c}}) + V_0}$$

Spectrum of GWs: result of numerical calculation



$$f_{\mathsf{PT}} = \frac{k_{\mathsf{PT}}}{2\pi a_{\mathsf{NOW}}}$$

$$\simeq 2.7 \text{ Hz} \left(\frac{T_{\mathsf{PT}}}{10^8 \text{ GeV}}\right)$$

What can we learn from the GW spectrum?

- Position of the drop-off ($\sim f_{\mathsf{PT}}$)

  $\Rightarrow$ "Reheating temperature" after the phase transition

- Magnitude of the drop-off (R)

  $\Rightarrow$ Entropy production

- Slope of the drop-off ($\sim d\Omega_{\mathsf{GW}}/d\ln k$)

  $\Rightarrow$ Time scale of the reheating (instantaneous or ?)

GWs from white dwarf binaries are significant for small-f

$\Rightarrow$ It will be difficult to extract the signal of cosmic phase transition in the GW spectrum if $f_{\rm PT} \lesssim 0.1$ Hz

[Farmer & Phinney]

Detectability of the "drop-off" signal

- Drop-off of Ω_{GW} should be bigger than the sensitivity

  $\Rightarrow$ Lower bound on R

- $\Omega_{\text{GW}}(k \gtrsim k_{\text{pt}})$ should be observable

  $\Rightarrow$ Upper bound on R

Comparison with the BBO-corr sensitivity:

- $(r, f_{\text{PT}}) = (0.1, 0.1\ \text{Hz})$

  $$0.005 < R < 0.98 \Rightarrow 1.5 \times 10^{-6} < g < 0.014 \text{ (for } h = 1)$$

- $(r, f_{\text{PT}}) = (0.1, 1\ \text{Hz})$

  $$0.17 < R < 0.83 \Rightarrow 6.0 \times 10^{-5} < g < 0.0014 \text{ (for } h = 1)$$

5. Summary

GW spectrum contains information about the early universe

Example: Cosmic phase transition

$\Rightarrow$ If a cosmic phase transition occurred, its effect may be imprinted in the spectrum of GWs

Message: GWs are interesting because

- Various information about the early universe is imprinted in GWs
- In a future, precise determination of the GW spectrum may be performed by satellite experiments

$\Rightarrow$ If a non-vanishing value of r is confirmed, DECIGO (or anything else) is strongly suggested as the next project