

# **Focus Point Assisted by Right-handed Neutrinos**

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“Focus Point Assisted by Right-Handed Neutrinos”  
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Masaki Asano, Takeo Moroi, RS and Tsutomu T. Yanagida

@ 2012.2.21 Toyama

# Naturalness

- **Standard Model**

$$V = \frac{\lambda}{2}|H|^4 - m^2|H|^2 \longrightarrow m_Z^2 = \frac{g^2 + g'^2}{2\lambda}m^2$$

$$\delta m^2 \sim \Lambda^2$$

Cancellation between  $m_0^2$  and  $\delta m^2$   $\longrightarrow \mathcal{O}\left(\frac{m_Z^2}{\Lambda^2}\right)$  tuning

- **Supersymmetry (MSSM)**

$$\frac{m_Z^2}{2} = -m_{H_u}^2 - \mu^2 + \mathcal{O}\left(\frac{m_H^2}{\tan^2 \beta}\right)$$

Cancellation between  $m_{H_u}^2$  and  $\mu^2$   $\longrightarrow \mathcal{O}\left(\frac{m_Z^2}{\mu^2}\right)$  tuning

# Finetuning Measure

The parameters in UV theory determines EW scale ( $m_Z$ ).

e.g) mSUGRA case  $\longrightarrow m_Z^2 = m_Z^2(m_0, m_{1/2}, A_0, \mu_0, B_0)$

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**Finetuning measure :**

$$\Delta = \max[\Delta(a)] \quad \Delta(a) = \left| \frac{a}{m_Z^2} \frac{\partial m_Z^2}{\partial a} \right| = \left| \frac{\partial \log m_Z^2}{\partial \log a} \right|$$

[Ellis, Enqvist, Nanopoulos and Zwirner (1986)]

$$a \longrightarrow (1 + \epsilon)a \quad \Delta = 10 \longrightarrow 10 \% \text{ tuning required}$$

$$\Downarrow \quad \Delta = 30 \longrightarrow 3.3 \% \text{ tuning required}$$

$$m_Z^2 \longrightarrow (1 + \Delta \times \epsilon)m_Z^2 \quad \Delta = 100 \longrightarrow 1 \% \text{ tuning required}$$

# Finetuning Measure in SUSY

Naïve estimation of

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$$\mu_0 \xrightarrow[\text{Mediation scale}]{\text{RGE running}} \mu \sim \mu_0$$

EW scale

$$\Delta(\mu_0) = \left| \frac{\mu_0}{m_Z^2} \frac{\partial m_Z^2}{\partial \mu_0} \right| \sim \frac{4\mu_0^2}{m_Z^2}$$

Heavy Higgsino -> finetuning...

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$$m_{H_u,0}^2, m_{Q,0}^2, \dots \xrightarrow[\text{Mediation scale}]{} m_{H_u}^2, m_Q^2, \dots$$

RGE running  
EW scale

$$\text{If } |m_{H_u}^2| \sim \mathcal{O}(1) \times m_Q^2 \longrightarrow \Delta(m_{Q,0}) \sim \frac{m_{Q,0}^2}{m_Z^2}$$

Heavy squark -> finetuning ???

# Focus Point

[Feng, Matchev and Moroi (1999)]

Heavy Squark is not so bad!

When  $m_0 \gg M_{1/2}, A_0$

$$\frac{d}{d \log \mu} \begin{pmatrix} m_{Q_3}^2 \\ m_{U_3}^2 \\ m_{H_u}^2 \end{pmatrix} \simeq \frac{y_t^2}{16\pi^2} \begin{pmatrix} 2 & 2 & 2 \\ 4 & 4 & 4 \\ 6 & 6 & 6 \end{pmatrix} \begin{pmatrix} m_{Q_3}^2 \\ m_{U_3}^2 \\ m_{H_u}^2 \end{pmatrix}$$

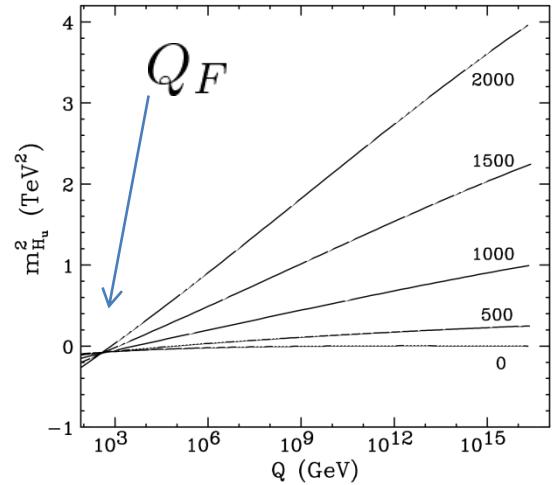
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[Feng, Mathcev and Moroi; PRL84 (2000) 2322]

Solution of RGEs is proportional to  $m_0$

The scale  $Q_F$  with  $m_{H_u}^2(Q_F) = 0$  is almost independent on  $m_0$ .

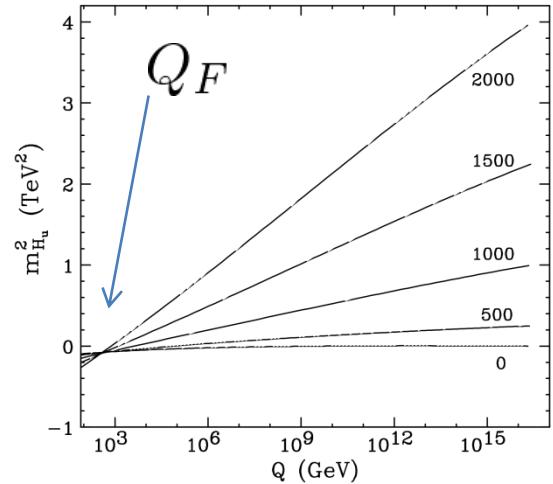
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Solution of RGEs is proportional to  $m_0$

The scale  $Q_F$  with  $m_{H_u}^2(Q_F) = 0$  is **almost independent on  $m_0$** .

$Q_F$  is determined by

- gauge coupling
- Yukawa coupling
- Universal scalar mass at GUT scale



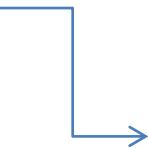
$Q_F \sim \mathcal{O}(100)$  GeV

**mSUGRA**

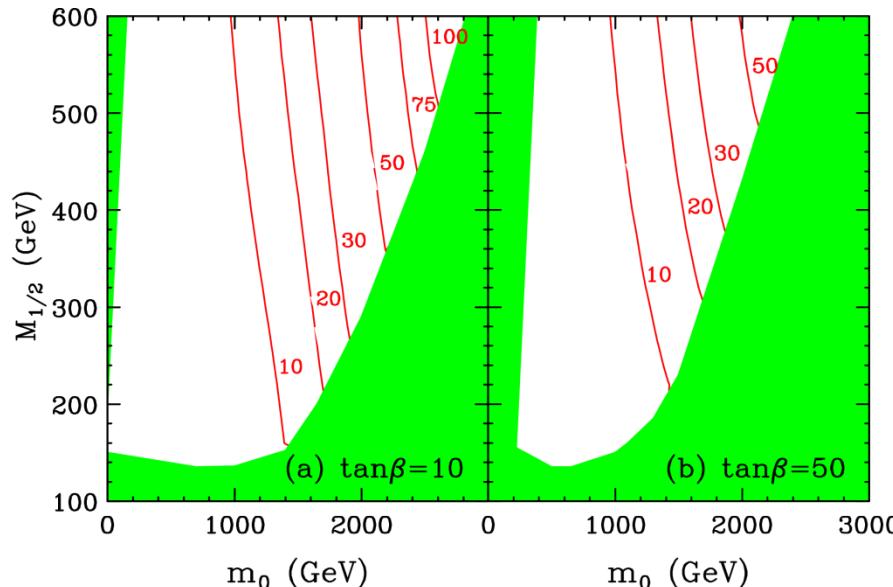
# Focus Point

Focus point behavior  
helps **NATURALNESS.**

$$\frac{\partial m_{H_u}^2}{\partial m_0^2} \ll 1$$



$$\Delta(m_0) = \left| \frac{m_0}{m_Z^2} \frac{\partial m_Z^2}{\partial m_0} \right| \ll \frac{m_0^2}{m_Z^2}$$



[Feng, Mathcev and Moroi; PRD61 (2000) 075005]

Stop loop correction is **HARMFUL** to naturalness !!

$$\begin{aligned} \frac{m_Z^2}{2} &\simeq -\mu^2 - m_{H_u}^2(Q) - \delta m_{H_u}^2(Q) \\ &\sim -\mu^2 - m_{H_u}^2(Q = m_{\tilde{t}}) \end{aligned}$$

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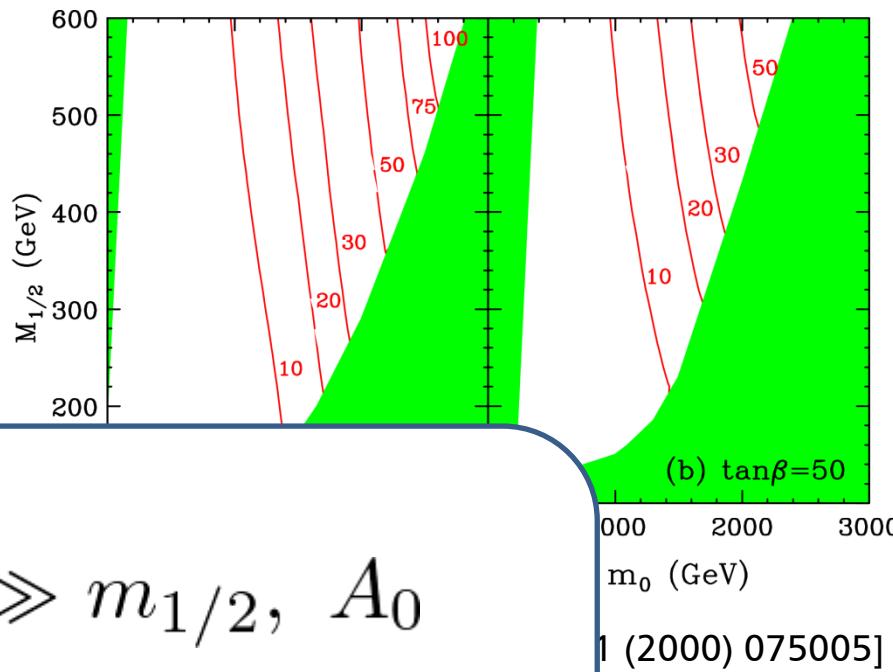
$$\frac{\partial m_{H_u}^2}{\partial m_0^2} \ll 1$$

$\Delta$

In mSUGRA with  $m_0 \gg m_{1/2}$ ,  $A_0$   
Finetuning can be "**mild**", but....  
  
**Not completely FREE !!**

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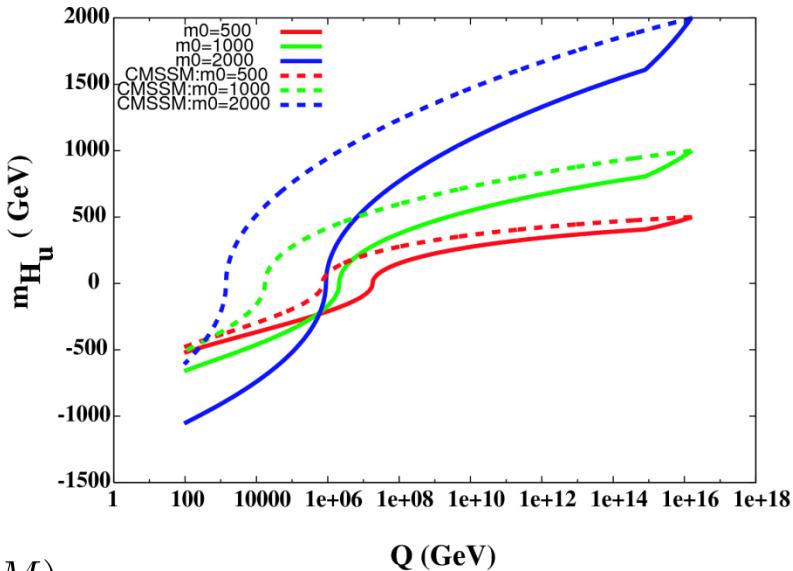
# Focus Point with vR

[Kadota and Olive (2009)]

Large Yukawa coupling modifies  
Running of soft mass.  $y_\nu \sim \mathcal{O}(1)$

$$W = y_\nu N_i L_i H_u + \frac{M}{2} N_i^2$$

$$\frac{dm_{H_u}^2}{d \log \mu} = \left. \frac{dm_{H_u}^2}{d \log \mu} \right|_{\text{MSSM}} + \frac{y_\nu^2}{16\pi^2} (m_{N_3}^2 + m_{L_3}^2 + m_{H_u}^2) \theta(\mu - M)$$

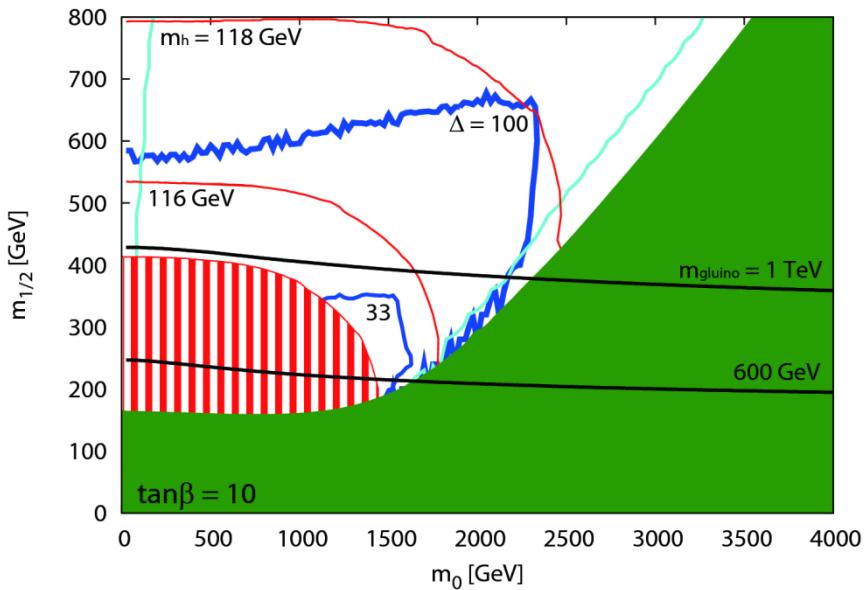


[Kadota and Olive; PRD80 (2009) 095015]

We calculate finetune measure in mSUGRA framework with vR

# Result

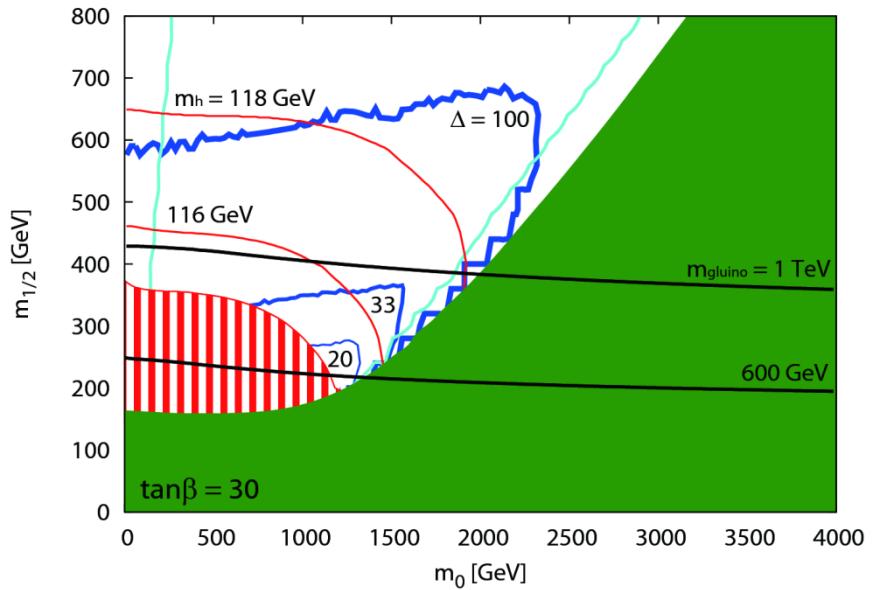
mSUGRA



Green :  $\mu < 104$  GeV

Red Stripe :  $m_{\text{higgs}} < 114.4$  GeV

Turquoise :  $\Omega_{\text{DM}} h^2 = 0.112$



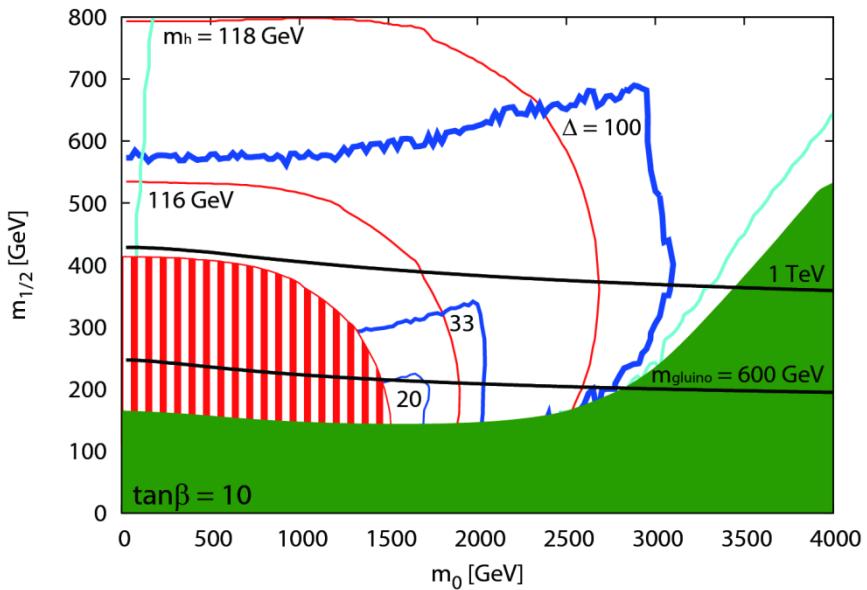
$$\Delta = \max_{a \in m_0, m_{1/2}, A_0, \mu, B_0} [\Delta(a)]$$

[Asano, Moroi, RS and Yanagida; PLB708 (2012) 107]

# Result

mSUGRA with vR

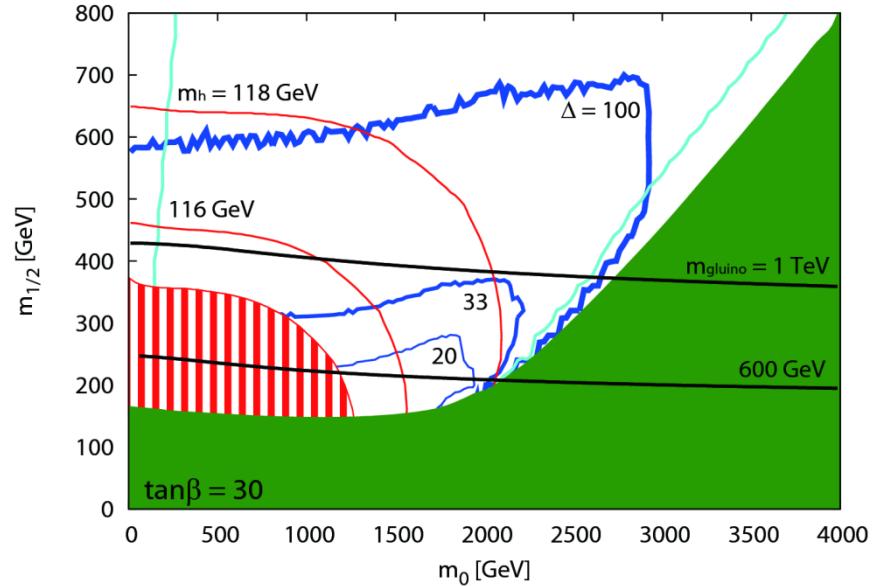
$$M_N = 2 \times 10^{14} \text{ GeV}$$



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# Conclusion

Large  $Y_V$  softens fine tuning  
In multi TeV squark region.

In focus point region,  
Higgs mass is relatively small.

