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Harrodian Dynamics and Hoover Curve:

Japanese case

Yasutaka Niisato

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FACULTY OF ECONOMICS
UNIVERSITY OF TOYAMA

3190 Gofuku, Toyama
930-8555 JAPAN

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Yasutaka Niisato*

Abstract

Hoover (2008):Was Harrod Right? gives an empirical test with respect to an implication of Harrod(1939)'s dynamic theory. That is, the GDP gap should be inversely related to the difference between the natural and the proper warranted rate of growth. I call this hypothesis as Hoover curve. He gets a downward-sloping regression line of the U.S. economy during 1930-2005. In this paper I show the similar estimation with a structural change in the Japanese economy during 1957-2007.

Keywords: Harrod, warranted rate of growth, natural rate of growth, GDP gap

* Faculty of Economics, University of Toyama, 3190 Gofuku, Toyama 930-8555, Japan, e-mail: niisato@eco.u-toyama,ac.jp

I. Introduction

Hoover (2008) discussed why Roy Harrod's growth theory has been discarded. He examined Harrod (1939) and pointed out that Harrodian instability does not depend upon a fixed-factor production function, although Solow (1956) says it does.[1]

Hoover gives an empirical test regarding to an implication of Harrod's dynamic theory. That is, the GDP gap should be inversely related to the difference between the natural and the proper warranted rate of growth. I call this hypothesis as Hoover curve. He gets a downward-sloping regression line in the case of the U.S. economy during 1930-2005. In this paper I show the similar estimation with a structural change in the case of Japanese economy during 1957-2007. And I find trends and change in the Japanese economy since 1957.

II. Harrodian dynamics

G and G_w

Harrod (1939) stated a "dynamic" theory as "thinking in terms of trends of increase." (p.15) It is not a log theory. He put the rate of growth of income or output, *G*. "A rate of growth is unknown variable." (p.17)

Harrodian dynamics of the growth rate can be shortly described as the following differential equation:

$$dG/dt = \alpha\,(G - G_w), \qquad (1)$$

where α *is a positive constant* and *t* is time, G_w is called the warranted rate of growth. If

[1] See Bioanosky and Hoover (2009) on recent discussion of Harrod-Domar and Solow.

G_w is a constant, then the stationary point is unstable.

Harrod defined the warranted rate of growth, G_w , as follows: "The warranted rate of growth is taken to be that rate of growth which, if it occurs, will leave all parties satisfied that they have produced neither more nor less than the right amount. Or to state the matter others, it will put them into a frame of mind which will cause them to give such orders as will maintain the same rate of growth. "(p.16)

He named the following equation the fundamental equation:

$$G_w=s/C, \qquad (2)$$

where s is saving ratio, and C is the value of the capital required for the production of a unit increment of output. G_w is "the value of which is determined by certain "fundamental conditions " namely, the propensity to save and the state of technology, etc. "(p.17) "s may be expected to vary, with the size of income, the phase of the trade cycle, institutional changes, etc."(p.16). "C may be expected to vary as income grows and in different phase of trade cycle; it may be somewhat dependent on the rate of interest."(p.17)

He called the following equation a truism:

$$G=s/C_p. \qquad (3)$$

"It is a truism, depending on the proposition that actual saving in a period⋯is equal to the additional to the capital stock. Total saving is equal to sx_0. The addition to the capital stock is equal to $C_p(x_1-x_0)$. This follows from the definition of C_p.⋯G is the rate of increase in total output which actually occurs; C_p is the increment of the stock of capital divided by the increment of total output which actually occurs. "(p.18)

He proceeded the instability principle as follows:"Now suppose that there is a departure from the warranted rate of growth. Suppose excessive output, so that G exceeds G_w. The consequence will be that G_p, the actual increase of capital goods per unit increment of output, falls below C, that which is desired. There will be, in fact, an undue depletion of stock or shortage of equipment, and the system will be stimulated to further expansion. G, instead of returning to G_w, will move farther from it in an upward direction, and farther. It diverges, the greater the stimulus to expansion will be. Similarly, if G falls below G_w, there will be a redundance of capital goods, and a depression influence will be exerted; this will cause a further divergence and a still stronger depression influence; and so on. Thus in the dynamic field we have a condition opposite to that which holds in the static field. A departure from equilibrium, instead of being self- righting, will self-aggravating. G_w presents a moving equilibrium, but a highly unstable. Of interest this for trade-cycle analysis! "(p.22)

Equation (1) can be used for trade-cycle analysis but not for long-term or trend analysis. Although he said s and C are variables depending upon the phase of trade cycles and the interest rate, Harrod did not give a complete dynamic system of the economy. He proposed an idea of dynamic thinking.

Okisho (1964) gave a formal proof of instability of Harrod-Domar's steady state, with attention of the utilization rate and the investment function both in the cases of fixed coefficient and flexible production functions. His system in the case of fixed coefficient is as follows:

$$sY = I, \qquad \text{(a-1)}$$

$$\delta = Y/\sigma K, \qquad \text{(a-2)}$$

$$dK/dt = I, \qquad \text{(a-3)}$$

$$g = I/K, \qquad \text{(a-4)}$$

$$dg/dt = \alpha(\delta - 1), \qquad \text{(a-5)}$$

where Y: output, I: investment demand, s: saving ratio(constant, s<1), δ: the rate of utilization, σ: normal output-capital (constant), K: capital stock, g: growth rate of capital, α: a constant. (a-1) is an equilibrium condition of goods market.(a-2) is a definition of the utilization rate. (a-3) is a capital accumulation equation. (a-4) is a definition of g. (a-5) is an investment function of Harrod-Okishio type. The utilization rate indicates the current level of capital shortage or excess capacity. The investment function means that if the utilization rate is greater than 1, i.e. capital shortage, then capitalists increase the growth rate of capital more than the level of the previous year and *vice versa*. The system is reduced to the following equation.

$$dg/dt = \alpha(g/s\sigma - 1). \qquad \text{(a-6)}$$

This stationary point is clearly unstable and has a "knife-edge" property.[2]

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[2] In the case of production function, his system is as follows:

$$\sigma = f(n), \qquad f'>0, f''<0 \qquad \text{(b-1)}$$
$$f'(n) = w, \qquad \text{(b-2)}$$
$$\delta = h(r), \qquad h'>0 \qquad \text{(b-3)}$$
$$r = \delta\sigma - w\eta n, \qquad \text{(b-4)}$$
$$\eta = \eta(\delta), \qquad \eta'>0, \eta(1)=1 \qquad \text{(b-5)}$$
$$\beta r = g, \qquad \text{(b-6)}$$
$$dg/dt = \alpha(\delta - 1), \qquad \text{(b-7)}$$

where n: normal labour-capital ratio, $f(n)$: production function, w: real wage rate, r: rate of profits, $h(r)$: utilization rate function of profit rate. η: actual - normal ratio of employment, $\eta(\delta)$: employment function, β: saving rate from profits. (b-2) is condition of profit maximization on choice of techniques. (b-3) is an another expression of a short run supply function. (b-4) is a definition of the profit rate. (b-5) is a technical condition of actual employment and output. (b-6) is a condition of saving – investment equality under the assumption that capitalists save some of his profits and workers consume all their income. The dynamics of the system is reduced to:

G_N and G_{PW}

Harrod proposed the concept of natural rate of growth:"This[the natural rate of growth] is the maximum rate of growth allowed by the increase of population, accumulation of capital, technological improvement and the work/leisure preference schedule, supposing that there is always full employment in some sense." (p.30)

And he introduced the concept of the proper warranted rate of growth:"Indeed, there is no unique warranted rate; the value of warranted rate depends upon the phase of the trade cycle and the level of activity. Consideration may be given to that warranted rate which would obtain in conditions of full employment; this may be regarded as the warranted rate "proper" to the economy. "(p.30)

Then he put forward his long-run or trend analysis:"If the proper warranted rate is above this[the natural rate], there will be a chronic tendency to depression; the depressions drag down the warranted rate below its proper level, and so keep its average value over a term of years down to the natural rate. But this reduction of the warranted rate is only achieved by having chronic unemployment. The warranted rate is dragged down by depression; it may be twisted upwards by an inflation of prices and profits. If the proper rate is below the natural rate, the average value of the warranted rate may be sustained above its proper level over a term of years by a succession of profits booms."(p.30)

Harrod's reasoning of long-run analysis is based on his short-run instability

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$$dg/dt = \alpha(h(g/\beta) - 1). \qquad \text{(b-8)}$$

The stationary point is clearly unstable.

principle. Even in the long-run he does not postulate full employment of labour and capital, whereas Solow and no-classical growth theory assume it.[3]

Let us now that we are going on the full employment growth path and that at a time, the natural rate of growth is less than the proper warranted rate of growth because of an increase in saving ratio, then an decrease in consumption demand occurs and an actual rate of growth will decrease because the actual rate (= the natural rate) is less than the warranted rate(=the proper rate) . We enter to be under employment for the some periods. The growth rate decreases as long as it is smaller than the warranted rate at the time. In time when the warranted rate declines because of decrease of s or increase of C, an economic recover will begin.

III. Hoover Curve of USA

Hoover (2008) took Harrod' analysis as a empirically testable proposition: "Harrod's conjecture says that an economy in which the proper warranted rate of growth(G_{PW}) stands above the natural rate of growth will display "a chronic tendency to depression" and an economy in which the proper warranted rate stands below the natural rate may be frequently driven towards full employment."(p.21) "One implication of Harrod's hypothesis is that the output gap should be inversely related to the difference between the natural and the warranted rate (G_N-G_{PW})."(p.22)

[3] Nikaido (1975, 1980) discussed about instability of the equilibrium growth path with Harrodian investment function under neoclassical production function. Fanti and Mansfreddi (2009), Sportelelli (2000) and Yoshida (1999, 2007) are recent elaborations of Harrodian dynamic models. Moudud(2009) discusses the role of the fiscal and taxation policy of the warranted growth. And see Dalgaard and Erickson.(2006) and Shaikh (2009) on policy issues.

Hoover constructed a potential output series (Y_t^{pot}) for the United States 1929-2005. The time series is generated from a Cobb-Douglas production function in which the inputs are the available labor force and the capital stock and the level of total factor productivity is estimated to grow smoothly along the upper bounds of actual total-factor productivity. From the time series for potential output and the capital stock, estimates of C and the net savings rate (s) can be constructed, finally yielding an estimate of the proper warranted rate of growth rate for each period: $G_{PW,t}=s_t/C_t$. The natural rate (G_N) is just the rate of growth of potential output at each period. The output gap is simply the percentage by which actual output fall short of potential output each period: $Gap_t = (Y_t^{pot} - Y_t)/ Y_t^{pot}$.

By regression analysis, He obtains the downward-sloping regression line.

$$Gap = -5.9(G_N - G_{PW})+16.1. \qquad (4)$$

$R^2=0.40$

His Figures 5[4] shows that "this is indeed the case: when the U.S. economy has a proper warranted rate of growth that was low relative to the natural rate, it has operated near to full potential."(p.22)

IV. Hoover curve of Japan

I follow Hoover's methods and apply it to the Japanese economy. I first constructed a potential output series for the Japan 1956-2007 (The detail of the data

[4] His Figure5, in Hoover(2005), is reproduced at Appendix1. Prof. Hoover added the years and gave the figure to me.

construction are found in the appendix 2). And the potential GDP and is estimated. Then GDP gap, and G_N and G_{PW} are estimated for the period between 1957 and 2007. By regression analysis the following equation is estimated:

$$Gap = -2.023(G_N - G_{PW}) + 7.039. \qquad (5)$$

$$(-4.286) \qquad (3.6249)$$

R^2=0.2727 AR^2=0.2578 DW=0.13059

The determinant co-efficient is rather small. A scattered diagram with connected lines and years is shown in Figure1. Looking at the time configuration of *Gap* and the difference of G_N and G_{PW} , some structural change must have occurred since 1970's. It is well known that the 1970 was a turning point and the end of rapid growth in the Japanese economy.[5]

I add a dummy variable during 1972 and 2007 and estimate again as follows:

$$Gap = -4.066(G_N - G_{PW}) + 10.545 - 15.117\text{Dummy}. \qquad (6)$$

$$(-9.786) \qquad (7.585) \qquad (-7.686)$$

R^2=0.67389 AR^2=0.6603 DW=0.6415

The determinant co-efficient is improved and the estimated values are all sufficiently significant in a statistics. The two estimated lines are showed in Figure 2. It means that the Japanese economy had structurally changed in the early 70's. The estimation line moves to the left with around 3.7% points. It means that G_{PW} fall from 7.7% to 4.0%.[6]

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[5] See Nakamura (1994), Yoshikawa (1995) , Boyer and Yamada (2000) and Chapter 2 of Okisho (1992) on Japanese economy from 1950's to 1990's.

[6] Hoover curve, $Gap = -a(G_N - G_{PW}) + b$, is shifted down if G_{PW} is smaller.
Let new Hoover curve as $Gap = -a(G_N - (G_{PW} - \beta)) + b$ and G_{PW}=7.7215%, then from (6), the value of β is calculated as:

$$\beta = 15.11726/4.40026 = 3.71824.$$

It would reflect the decline in the proper warranted rate of growth, $G_{PW}=s/C$, with smaller net saving rate, s, and/or larger capital-output ratio, C, in the long run. In open economy, from saving and investment balances, it follows that

$$s=s_p(1-d-t)-(g-t)-(x-m), \qquad (7)$$

where s_p: private net saving ratio of NDP, d: depreciation rate of GDP, g: government expenditure rate of GDP, t: tax rate, x: export rate, m: import rate. An decrease in s caused by 1. decline in s_p, 2. increase in d, 3. increase in tax rate, t, 4. increase of full employment budget deficits ratio, g-t, 5. increase in foreign trade surplus rate ($x-m$).These changes in the Japanese economy have been already pointed out by many authors.[7]

Figure 3 and 4 show *Gap* of GDP and the gap between G_N and G_{PW} in time series. As noticing ups and downs of these figures, the following periods are easily distinguished and corresponded to the economic topics:

- 1957-1970 high-speed economy:

  A decline in GDP gap and a zero gap in 1970. An increase in G_N. the difference between G_N and G_{PW} disappeared in 1970.

- 1971- 1973 full employment and yen-evaluation:

  Almost zero GDP gap (i.e. full employment). G_N exceeded G_{PW}.

- 1974-1985 oil shocks, stable growth:

  Sudden increase in GDP gap. After then the gap was relatively constant. Sudden fall of G_N below G_{PW}. After then a stable difference because of a falling G_N and a

[7] See Uni(2000) for trend of C, and Ihori (2007, p.109) and Hiromatsu, Dornbush, and Fisher (1998, p.110) for full employment budget deficits.

shift down of G_{PW}.

- 1986-1990 bubble economy:

  Diminishing GDP gap. Fluctuated difference between G_N and G_{PW}.

- 1991-2001 lost 10years:

  Increasing GDP gap and the peak in 2002. Enlarged difference between G_N and G_{PW} owing to decline in G_N.

- 2002-2007 Koizumi structural reform:

  Diminishing GDP gap but still high level. Small recover of difference between G_N and G_{PW}.

Without saying, it needs more investigations to analyze empirically the Japanese economy in detail. But the Hoover curve means the fact that Harrodian dynamic theory should have a strong empirical evidence and be effective to analyze economic growth and trade cycles.

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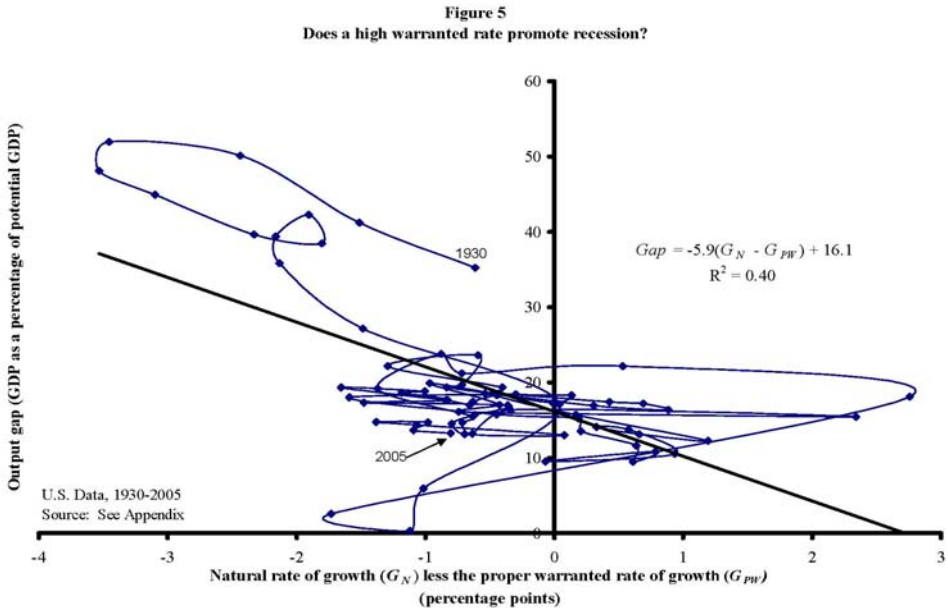
Appendix 1: **Hoover's Fig.5 with years**



Appendix2: Japanese Data Construction

1)Data Source

ESRI(economic and Social Research Institute) Website
(http://www.esri.cao.go.jp/jp/sna/menu.html)
Gross domestic product (*NY)*
Real gross domestic product (Y), fixed-based approach, 2000
Compensation of Employees
Private capital stock (K)

National Account for 2007(benchmark year=2000, 93SNA):
(http://www.esri.cao.go.jp/jp/sna/h19-kaku/21anual-report-j2.html)
real GDP, nominal GDP and Compensation of employees for 1980-2007
National Account for 2000
(http://www.esri.cao.go.jp/jp/sna/h12-nenpou/12anual-report-j.html)
real GDP, nominal GDP and Compensation of employees for 1955-1998
a series of real GDP(benchmark year=2000) is made from connecting two series at the 1990.

Ministry of International Affairs and Communications Website
(http://www.stat.gp.jp/data/roudou/index.htm)
Lobour Force (LF): Source: Labour Force Survey

(http://www.stat.gp.jp/data/longtime/03roudou.htm)

2) Construction of the Output Gap
Labor share in output(α): α_t= compensation of employees/NY_t.
α=the mean of α_t =0.485446

note: Hoover (2008) subtracted proprietors' income from *NY* at the deflator.

Total-factor productivity(A): $A_t=Y_t/\ (LF_t^{\alpha} K_{t-1}^{(1-\alpha)})$
where K_{t-1} means the value of capital at the binging end of the year *t*.

note: Hoover defined capital input as the K_t (the value of capital at the end of year *t*).

At the above definition, the use of the labor force (*LF*) rather than employment has the effect of incorporating the "inefficiency" of the unemployment into A. Since we are ultimately interested in efficient envelope of the A_t, there is not misleading.

Full employment total-factor productivity (A^{full}): an exponential trend is fitted to *A* by ordinal least squares:
$\ln A_t$=0.00073time+0.372548.
And then,
$\ln A_t^{full} = \ln \hat{A}_t + 0.160187457153429$.
The additional constant has the effect of shifting the whole path of $\boldsymbol{A_t}$ so that it forms the outer envelope of the A_t.

note: Hoover used a quadratic trend on estimation of *A*

potential GDP(Y_t^{pot}): $Y_t^{pot} = A_t^{full} LF_t^{\alpha} K_{t-1}^{(1-\alpha)}$.
GDPGap(Gap$_t$*)*:$Gap_t = (Y_t^{pot} - Y_t)/ Y_t^{pot}$.

3) Constiruction of the Natural Rate of groth.
Natural Rate of Growth(G$_N$*)*: $G_N=(Y_t^{pot} - Y_{t-1}^{pot})/ Y_{t-1}^{pot}$

4) Construction of the Proper Warranted Rate of Growth.
Trend capital (*K*): An exponential trend is fitted to *K* by ordinary least square:
$\ln K_t$= 0.074379time - 134.815
Capital-output Ratio (*C*): $C_t = K_{t-1}/Y_t^{pot}$.
Net Saving Rate(*s*): $s_t = (K_t - K_{t-1})/Y_t^{pot}$.
Proper Warranted Rate of Growth(G_{PW}): $G_{PW,t} = s_t/C_t = (\hat{K}_t - \hat{K}_{t-1})/ \hat{K}_{t-1}$=7.7215%,

Figure 1

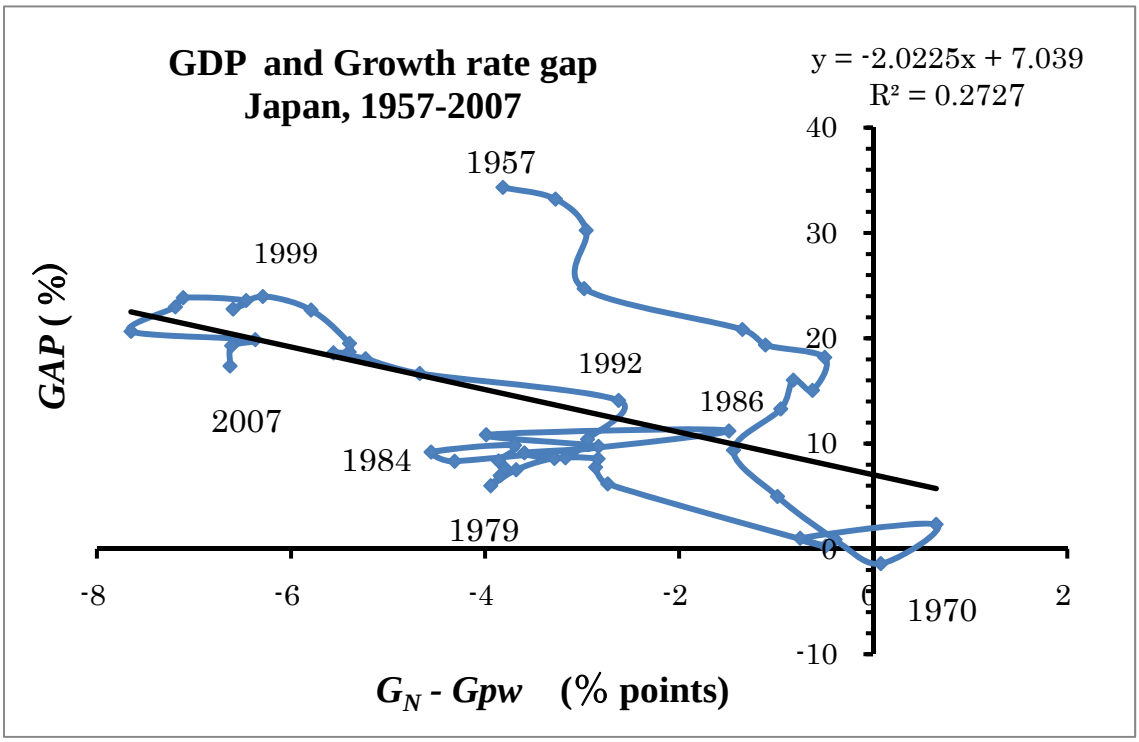


Figure 2

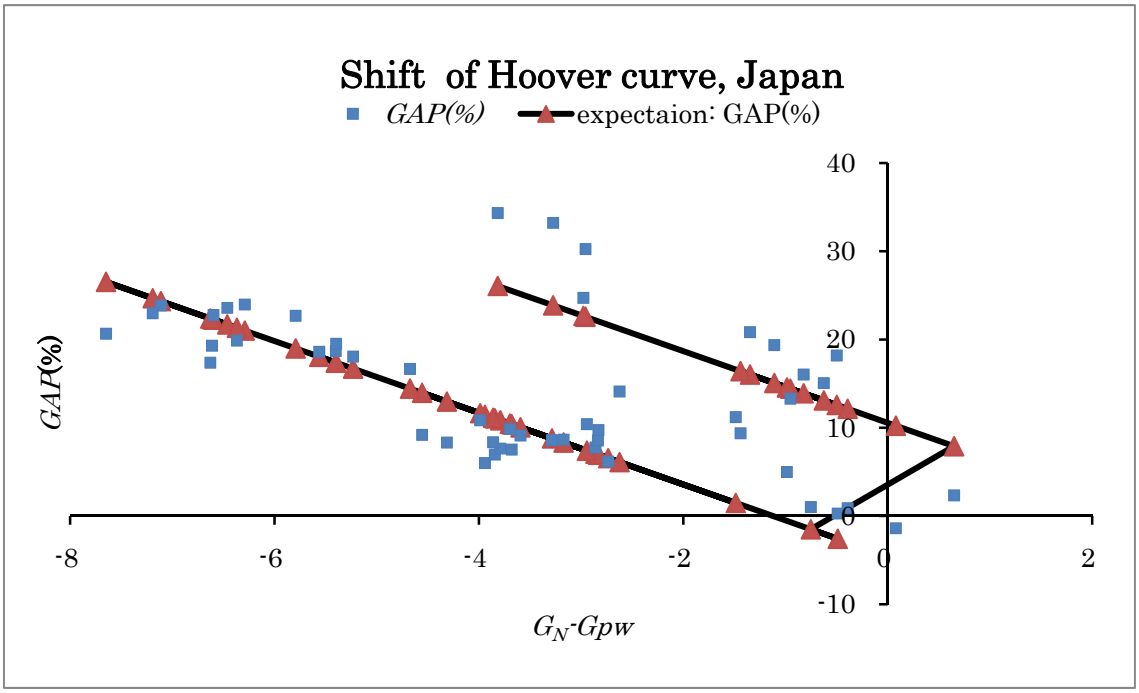


Figure 3

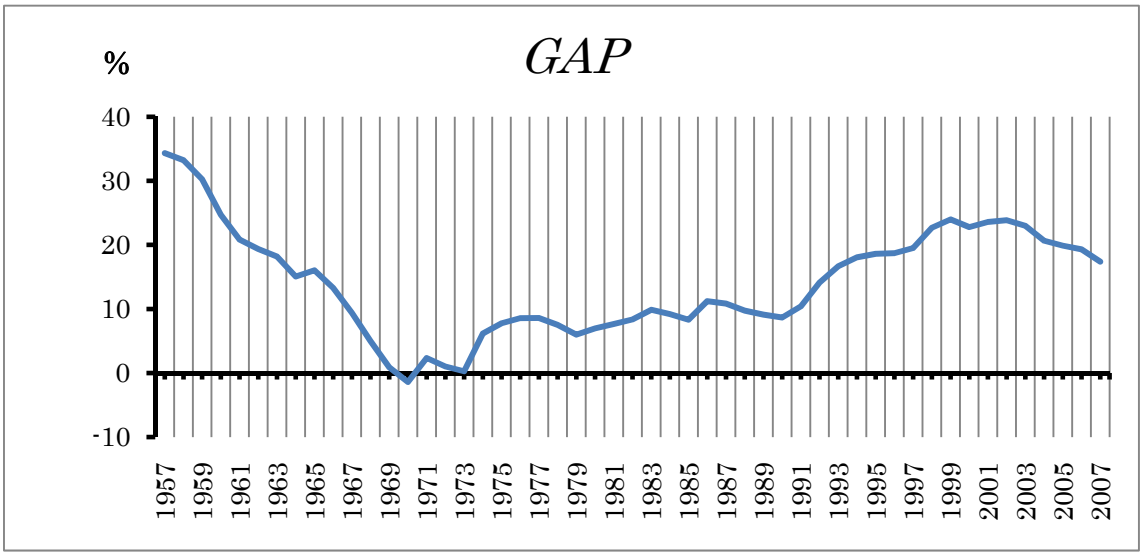


Figure 4